DEPARTMENT OF ECONOMICS
JOHANNES KEPLER UNIVERSITY OF LINZ

How effective is emissions pricing?
The role of firm-product-level adjustment

by

Karin MAYR-DORN

Working Paper No. 2410
September 2024

Johannes Kepler University of Linz
Department of Economics
Altenberger Strasse 69
A-4040 Linz - Auhof, Austria
www.econ.jku.at

karin.mayr-dorn@jku.at

How effective is emissions pricing? The role of firm-product-level adjustment*

Karin Mayr-Dorn†

September 30, 2024

Abstract

Climate change is one of the pressing issues of our time, and carbon emissions caused by industrial production are among its most important drivers. This paper analyses how multi-product firms adjust to an increase in the cost of emissions (e.g. due to the introduction of emissions pricing) in terms of their output, product mix, and technology, and how their emissions change in response, depending on firm-specific production patterns and cost structures. My model delivers a (qualitative and quantitative) assessment of changes in aggregate emissions via conventional margins of firm adjustment that have not been sufficiently studied in the literature so far. In numerical simulations, I find that negative effects of emissions pricing on emissions of multi-product firms can be sizeable.

JEL codes: Q54, Q56, D24.
Keywords: emissions, carbon tax, multi-product firms, product mix, technology.

*I am grateful to Michael Irlacher for helpful comments.
†Department of Economics, Johannes Kepler University Linz; e-mail: karin.mayr-dorn@jku.at.

1 Introduction

Climate change is one of the pressing issues of our time. Human emissions of greenhouse gases, such as carbon emissions, are the most important driver of climate change today (IPCC, 2021). They come primarily from industrial sources, being emitted during production processes in particular in energy-intensive industries such as the iron and steel industry, the cement, paper, minerals, chemical, and particleboards industries as well as power plants and refineries.

In order to attenuate the projected huge social and economic costs related to climate change, policymakers are increasingly aiming at regulating the amount of emissions, for example via emissions trading schemes such as the European Union Emissions Trading System (EU ETS).[1] These schemes put a cap on the amount of emissions and distribute emission allowances among polluting firms. In doing so, they create a market for these allowances and effectively put a price on emissions. Imposing a price on emissions essentially increases input costs, to which firms can respond in different ways: they can adjust their output (scale effect), their product mix (composition effect), or their technology (technology effect), see, e.g., Copeland and Taylor (1994).[2] As a result, the overall amount of emissions is expected to fall. Little is known, however, about whether and to what extent it is optimal for firms to reduce emissions in the presence of such schemes. In particular, if firms produce more than one product, they may adjust not only the technology and scale of a single product, but the technology, scale, and scope of all their products, potentially to different degrees. If emission intensity (i.e. emissions per unit of output) varies across products, changes in the product mix may affect average emission intensity in the firm. More importantly, firm-level emissions may increase or decrease, depending on the extent of changes in technology across products with different scales, and changes in product scale across products with different emission intensities. As multi-product firms are responsible for a predominant share of production worldwide[3], such adjustments across products within firms are likely to be important for aggregate emissions in the economy.

In this paper, I investigate the way in which multi-product firms adjust to the imposition of emission costs in a state-of-the-art equilibrium model of multi-product firms with endogenous product-specific technology and product mix (product scope and scale) based on Eckel

[1] The EU Emissions Trading Scheme (ETS), introduced in the European Union in 2005, was the first-ever large-scale cap-and-trade system. Together with other emissions trading schemes, which followed worldwide, it currently covers around 20% of worldwide greenhouse gas emissions (World Bank (2023)).

[2] Moreover, firms may choose to relocate (parts of) their production to regions or countries where environmental regulations are less stringent ('pollution havens'). This would reduce firms' domestic output, but not necessarily their global output, which is the relevant outcome for climate policy.

[3] See, e.g., Irlacher (2024).

and Neary (2010), and Dhingra (2013) and Flach and Irlacher (2018). In this framework, I address the following two questions: i) 'In response to an increase in the price of carbon emissions, how do firms adjust their production and technology?' and ii) 'What is the potential impact of introducing an emissions trading scheme on total emissions?' The answers to these questions are likely to depend on firms' production patterns and cost structures. In taking a firm-level approach, I address a gap in the literature, which abstracts from differences in firms' production technologies when evaluating the impact of emissions regulations on aggregate emissions.[4] I find that, in contrast to single-product firms, emissions do not necessarily decrease in multi-product firms in response to emissions pricing. This is because the effect of changes in product scale on firm emissions is ambiguous. In numerical simulations, I find that it is negative for chosen parameter values.

Much of the existing literature links changes in emissions to changes at the national and industry levels. A couple of more recent papers acknowledge the importance of changes within industries and firms, and decompose these changes into changes in technology and product composition. In particular, they recognize that industry- or firm-level changes in emissions do not necessarily result from changes in the emission intensity of individual products but may, instead, be due to changes in the scope and scale of products with different emission intensities within the firm (e.g., Cherniwchan et al. (2017), Barrows and Ollivier (2018), Shapiro and Walker (2018)). I contribute to this literature and analyze the sign and size of these different margins of adjustment within multi-product firms. In doing so, I open the black box of changes in firm-level emissions and re-evaluate the extent to which emissions pricing may be effective in reducing aggregate emissions.

I also contribute to a number of papers that investigate empirically the impact of emissions regulations on emissions as well as economic outcomes at the firm or plant level. For example, Greenstone (2002) and Greenstone et al. (2012) find negative effects of the U.S. Clean Air Act on output and total factor productivity. Ryan (2012) finds that the Clean Air Act reduced profits via an increase in sunk entry costs and capacity adjustment costs. Becker and Henderson (2000) find that negative effects of the Clean Air Act on emissions were mitigated by shifts in industrial structure towards less regulated firms. Similarly, Fowlie et al. (2016) find that emissions regulation in the U.S. cement industry reduced domestic emissions but also resulted in offsetting effects due to leakage and changes in market power. Walker (2013) estimates transitional costs of the Clean Air Act in the form of nonemployment and reduced earnings, as production is relocated. Martin et al. (2014) find that the introduction of a carbon tax in the U.K. reduced energy intensity by around 20% and had no impact on

[4]Barrows and Ollivier (2018) use a model with heterogeneous multi-product firms based on Mayer et al. (2014) to investigate the effects of competition and trade liberalisation on emission intensity. In their model, emissions are treated as a separate input, the use of which decreases in firm productivity.

employment, revenue, or plant exit. Looking at the same carbon tax, Abrell et al. (2022) find that it reduced emissions by 6% on average, and that the impact varied substantially depending on relative fuel prices. Fowlie et al. (2012) find that an NOx trading program in Southern California reduced emissions by 20%. He et al. (2020) find that water quality regulation in China reduced emissions as well as total factor productivity. Regarding the EU ETS, Jonghe et al. (2020) find improvements in emission efficiency, while Calel (2020) finds no effect on (short-term) emission intensity. Existing evidence on the effects of the EU ETS on emissions is ambiguous. Specifically, Klementsen et al. (2020) find that emissions decreased in Norway, while Jaraite and Maria (2016) find that emissions increased in Lithuania. Wagner et al. (2020) find inconclusive evidence on emissions of firms in Denmark, Finland, France, Germany, Lithuania, Norway and Sweden. Dechezleprêtre et al. (2023) find that the EU ETS reduced emissions in France, the Netherlands, Norway and the United Kingdom by about 10% and had no significant impact on profits. Most recently, Colmer et al. (2024) find that the EU ETS reduced emissions in France by 14-16% via targeted investments. They find no evidence of reductions in output or outsourcing to unregulated firms or markets, which implies that the emission intensity of production decreased.

These empirical findings are consistent with theoretical results in my paper, according to which emission intensity decreases in response to emissions pricing, as firms invest in abatement technologies. My results can also explain the ambiguous empirical findings regarding effects on output, profits, and emissions. This is because in multi-product firms, optimal product scale depends not only on the absolute but also on the relative marginal cost of production across products within the firm. In consequence, firms may increase output of certain products, and decrease output of other products. While firm emissions decrease via the technology effect, they may increase via changes in product composition in my model, depending on how products rank in terms of their emission intensity and scale. I find that firm emissions are more likely to decrease via changes in the product mix, if the emission intensity is greater for products with smaller scale (i.e., sales). Given information about how emission intensity varies with sales, my model delivers a (qualitative and quantitative) assessment of changes in aggregate emissions via conventional margins of firm adjustment that have not been sufficiently studied in the literature so far. In numerical simulations, using parameter values and information about product-level emission intensities and sales from the literature, I find that overall, emissions decrease mainly due to the fact that firms drop their dirtiest products. In addition, there is a smaller positive effect on emissions due to an increase in product scale, and an even smaller negative effect on emissions due to abatement investment, in the products that firms continue to produce. I also provide conditions for greater or smaller cuts in emissions, together with a decomposition into the various channels via which these cuts are achieved. This is important because it allows to distinguish true technology effects from pure compositional effects, and thus helps to improve the cost-effectiveness of climate policies.

2 The Model

In this section, I analyse how firms adjust optimally to an environmental policy that increases their cost of emissions. I use a standard model of multi-product firms that produce according to a flexible manufacturing technology in an oligopolistic market a la Eckel and Neary (2010).[5] My model accounts for endogenous changes in firm-level product scope, product scale, and product technology. Throughout my analysis, I focus on intra-firm adjustment and assume that firms are symmetric within a given industry[6] but may vary in terms of their production structure across industries. In this framework, I compare the impact of emissions pricing on product scope, scale, and technology in industries with different production structures. In the next section, I will analyze the changes in firm emissions that result from these adjustments in production structures and discuss how emissions pricing affects aggregate emissions in the economy as a result.

2.1 Setup

Demand. I consider a closed economy with L consumers whose utility depends on their consumption of a homogeneous good, q_0, and differentiated varieties, $q_s(i_s)$ (or $q_s(i'_s)$), produced in industry s, as follows:

$$U[\{u_s\}] = q_0 + \sum_{s=1}^{S} u_s, \tag{1}$$

where

$$u_s = a_s Q_s - \frac{1}{2} b_s \left((1-\epsilon_s) \int_{i_s \in \Omega_s} q_s(i)^2 di + \epsilon_s Q_s^2 \right). \tag{2}$$

Equations (1)-(2) describe linear preferences across industries and quadratic utilities across product varieties within an industry. The representative consumer allocates expenditure across the homogeneous good, $q_0 > 0$[7], and varieties of differentiated goods i from the measure Ω_s of goods produced in industry s. Furthermore, $a_s > 0$ is the consumer's

[5] Multi-product firms are generally considered to be large in their markets and, as a result, behave like oligopolists and internalize demand linkages between their own products (cannibalization effect). Accordingly, I assume a finite number of firms in my decomposition of aggregate emissions in Section 3 below.

[6] This is approximately true, if the definition of industries is sufficiently narrow.

[7] This ensures positive consumption of the differentiated goods.

maximum willingness to pay, $b_s > 0$, $\epsilon_s \in (0,1)$ is an inverse measure of product differentiation, and $Q_s = \int_{i_s \in \Omega_s} q_s(i)di$ denotes total consumption over all varieties indexed by $i_s \in \Omega_s$. Consumers maximize utility in Equations (1)-(2) subject to the budget constraint:

$$q_0 + \sum_{s=1}^{S} \int_{i_s \in \Omega_s} p_s(i_s) q_s(i_s) di_s = I, \tag{3}$$

where $p_s(i_s)$ is the price of variety i_s and I is individual income. I assume that $p_0 = 1$.[8] This results in inverse market demand given by:

$$p_s(i_s) = a_s - b'_s \left[(1 - \epsilon_s) x_s(i_s) + \epsilon_s \int_{i_s \in \Omega_s} x_s(i_s) di_s \right]. \tag{4}$$

where $b'_s \equiv b_s / L$ is an inverse measure of market size and $x_s(i_s) = Lq_s(i_s)$ denotes market demand for variety i_s by all consumers.

Market structure and technology. The economy consists of a homogeneous good industry and differentiated goods industries indexed by $s \in (1, S)$. Labor is the only factor of production. It is mobile across industries and inelastically supplied in a competitive market. The homogeneous good is produced under constant returns to scale at unit cost under perfect competition, such that the wage is equal to unity. The differentiated goods are produced in oligopolistic markets with symmetric firms. Following Eckel and Neary (2010), I assume that firms produce an endogenous number of goods according to a flexible manufacturing technology. This technology is characterized by a core competence $i = 0$ that is the most efficient variety in each firm's portfolio, and potential additional varieties whose marginal cost increases in distance from a firm's core competence. The rank of products in terms of their private marginal cost may differ from their rank in terms of their total marginal cost, which is equal to the sum of private marginal cost and the cost of emissions per unit (external marginal cost). In the following, I consider in turn the two cases where i) there is no emissions pricing, and firms take into account only the private cost of production, and ii) there is emissions pricing, and firms take into account both the private and the external cost of production.[9] I denote product rank in the former case by i_s and in the latter case by i'_s and distinguish between a marginal cost of production $c_s(i)$ and $c'_s(i')$, respectively, where the latter includes the cost of emissions per unit but the former does not. Firms can

[8]That is, the homogeneous good serves as the numéraire. As preferences according to Equation (2) are quasi-linear, all income effects are absorbed by the numéraire good and any potential wage changes would not affect consumption of the differentiated varieties. I can therefore assume that $w = 1$.

[9]In cap-and-trade systems, firms have to pay a strictly positive price for their emissions, if emission allowances are sufficiently small.

reduce marginal costs via product-specific investments in technology.[10] Investments $k_s(i)$ and $k'_s(i')$ reduce private costs, and investment $l_s(i')$ reduces emission costs at a decreasing rate, respectively. The marginal cost of product i in the case without emissions pricing is given as:

$$c_s(i_s) = c_s + i_s^{\alpha_s} - 2k_s(i_s)^{0.5}, \tag{5}$$

where $c_s > 0$ and $\alpha_s > 0$ $\forall s$. In the case with emissions pricing, it is:

$$c'_s(i'_s) = d_s + i_s'^{\beta_s} - 2k'_s(i'_s)^{0.5} - 2\gamma_s l_s(i'_s)^{0.5}, \tag{6}$$

where $d_s > 0$, $\gamma_s > 0$ and $\beta_s > 0$, $\forall$ s. γ_s is a measure of the relative efficiency of capital investment in reducing emission costs (abatement investment) compared to investment in the reduction of private costs. Goods are ordered in increasing distance from the core competence. The parameters α_s and β_s are the scope elasticities of marginal costs in the case without and with emissions pricing, respectively, and they determine how fast sales drop in the distance of a product from the firm's core competence (as shown below). The relative size of the scope elasticities α_s and β_s determines the extent to which private and external marginal costs are increasing in scope, respectively. In particular, the greater β_s relative to α_s, the greater is the difference in emission cost relative to the difference in production cost between any two products i'_s and j'_s, $j'_s > i'_s$.

Emission intensity. The unit costs of producing each variety given in Equations (5)-(6) are functions of the amounts of labor and emissions required to produce one unit of output, $L(i)$ and $e(i)$ (technological components), together with the unit-costs of labor and emissions, w and p_e (factor-cost components). The unit cost of emissions is assumed to be constant and equal to zero in the case without emissions pricing, and equal to $p'_e = 1$ in the case with emissions pricing.[11] Technological investments, $k_s(i_s)$, $k'_s(i'_s)$, $l_s(i'_s)$, reduce marginal costs of production via reducing the technological components $L_s(i_s)$, $L_s(i'_s)$, $e_s(i'_s)$, such that marginal costs can be re-written as:

$$c_s(i_s) = w\left[L_s(i_s) - 2k_s(i_s)^{0.5}\right], \tag{7}$$

$$c'_s(i'_s) = w\left[L_s(i'_s) - 2k'_s(i'_s)^{0.5}\right] + p'_e\left[e_s(i'_s) - 2\gamma l_s(i'_s)^{0.5}\right], \tag{8}$$

where $w = p'_e = 1$. Putting together Equations (5)-(8), we can express the emission

[10] This corresponds to process innovation as modelled in Dhingra (2013) and Flach and Irlacher (2018).

[11] In this case, the relative price of emissions is equal to one, $p'_e/w = 1$. This has no effects on qualitative results, as shown below.

intensity of a given product with rank i' as follows:

$$e_s(i'_s) = \frac{z_s(i'_s)}{x_s(i'_s)} = d_s - c_s + i'^{\beta_s}_s - i'^{\alpha_s}_s, \tag{9}$$

where $z_s(i'_s)$ is emissions and $x_s(i'_s)$ is output of variety $i'_s < M'_s$, respectively.[12] Equation (9) shows how emission intensity varies with product rank in terms of total (private plus external) marginal cost. Depending on parameter values, emission intensity may increase (if $\beta_s > \alpha_s$), decrease (if $\beta_s < \alpha_s$), or be constant in product rank (if $\beta_s = \alpha_s$). The change in emission intensity in response to the introduction of emissions pricing is given by:

$$de_s(i'_s) = d\left(\frac{z_s(i'_s)}{x_s(i'_s)}\right) = -2\gamma l_s(i'_s)^{0.5}. \tag{10}$$

Firm behavior. Firms maximize profits given by:

$$\Pi_s = \int_0^{M_s} \left[p_s(i_s) - c_s(i_s)\right] y_s(i_s) di_s - \int_0^{M_s} r_k k(i_s) di_s \tag{11a}$$

and

$$\Pi'_s = \int_0^{M'_s} \left[p'_s(i'_s) - c'_s(i'_s)\right] y'_s(i'_s) di'_s - \int_0^{M'_s} r_k k'(i'_s) di_s - \int_0^{M'_s} r_l l(i'_s) di'_s, \tag{11b}$$

in the case without and with emissions pricing, where $y_s(i_s) = x_s(i_s)$ and $y'_s(i'_s) = x'_s(i'_s)$ due to market clearing, and r_k and r_l is the marginal cost of non-abatement technology and abatement technology, respectively. I next determine how firms simultaneously choose optimal output of each variety, $x_s(i_s)$ (scale), the optimal number of products, M_s (scope), and optimal capital investment per product, $k_s(i_s)$ (technology), in the case without emissions pricing (Section 2.2.1) as well as the optimal choice of $x'_s(i'_s)$, M'_s, $k'_s(i'_s)$ and $l_s(i'_s)$ in the case with emissions pricing (Section 2.2.2).

2.2 Output, product mix, and technology

In the following, I first determine firm-product-level production and technology choices in equilibrium in the case without emissions pricing, where firms take into account only the private cost of production. I then discuss the case with emissions pricing, where firms take into account both the private cost and the emission cost of production. I drop the industry subscript s in the following for purposes of readability unless required.

[12] Optimal output and, therefore, emissions, of the marginal variety $i'_s = M'_s$ is equal to zero (see below).

2.2.1 Without emissions pricing

To determine optimal product scale (output per product), firms maximize profits in Equation (11a) with respect to $x(i)$, with production costs given by $c(i)$ according to Equation (5). This results in

$$x(i) = \frac{a - c(i) - b'\epsilon(X + Y)}{2b'(1 - \epsilon)}, \tag{12}$$

where $X = \int_0^M x(i)di$ is total firm output (total scale) and $Y = LQ = \int_{i\in\Omega} x(i)di = NX$ is industry-wide output of all N producers. According to Equation (12), the optimal scale of any given variety i decreases in total industry output, Y, (competition effect) and in total firm output, X, (cannibalization effect) - the more so the smaller the extent of product differentiation (greater e).

To determine optimal product scope (range of products), firms maximize profits in Equation (11a) with respect to M. This implies that the output of the marginal product is equal to zero (see Eckel and Neary (2010)):

$$x(M) = 0. \tag{13}$$

From Equation (13), it follows that $x(i) = x(i) - x(M)$. Therefore, we can express output by product as

$$x(i) = \frac{M^\alpha - i^\alpha + 2k(i)^{0.5}}{2b'(1 - \epsilon)} \tag{14}$$

I use this to solve for optimal capital investment next, before solving for M explicitly in the final step.

To determine optimal capital investment, firms maximize profits with respect to $k(i)$ and $l(i)$, respectively. At the optimum, savings due to the decrease in production cost are equal to the cost of technology, such that[13]

$$k(i)^{0.5} = \frac{x(i)}{r_k}. \tag{15}$$

Substituting in Equation (15) for $x(i)$ using Equation (14), I derive:

$$k(i) = \frac{(M^\alpha - i^\alpha)^2}{[2(r_k b'(1 - \epsilon) - 1)]^2} \tag{16}$$

According to Equation (16), the capital allocated to a given product within the firm

[13]The second-order condition is negative, as required: $\frac{\partial^2 \pi}{\partial k(i)^2} = -0.5k(i)^{-1.5}x(i) < 0$.

is greater the closer it is to the product with the smallest total marginal cost, i.e. the core variety, $i = 0$. Thus, the core product receives the greatest capital investment, while investment in the marginal product $i = M$ is zero.

Substituting this expression for $k(i)$ in Equation (14), I derive:

$$x(i) = \frac{(M^\alpha - i^\alpha)}{2[b'(1-\epsilon) - \frac{1}{r_k}]}. \tag{17}$$

To ensure that $x(i) = x(i) - x(M) = (c(M) - c(i))/(2b'(1-\epsilon)) > 0$, I need to restrict parameter values such that the cost of capital investment is sufficiently large:

Condition 1. *To ensure that the cost of the marginal product M is greater than the cost of any infra-marginal product i,* $c(M) - c(i) = \frac{(M^\alpha - i^\alpha) r_k b'(1-\epsilon)}{r_k b'(1-\epsilon) - 1} > 0$*, I restrict parameter values such that*

$$r_k > \frac{1}{b'(1-\epsilon)}.$$

Integrating the expression for $x(i)$ according to Equation (17), I get:

$$X = \frac{\alpha M^{1+\alpha}}{2(1+\alpha)[b'(1-\epsilon) - \frac{1}{r_k}]}. \tag{18}$$

Finally, I can determine optimal product scope, using Equation (12) together with $x(M) = 0$ (see Equation (13)) and $k(M) = 0$ (see Equation (16)) as well as the symmetry condition $Y = NX$. This results in:

$$M = (a - c - b'\epsilon(1+N)X)^{\frac{1}{\alpha}}. \tag{19}$$

Equations (18)-(19) determine the symmetric industry equilibrium with firm-level scale X and scope M. According to Equation (18), firm scale, X, increases in product scope, M. According to Equation (19), an increase in firm scale reduces product scope, due to the cannibalization effect. Optimal product-level investment, $k(i)$, and product-level scale, $x(i)$, follow from optimal firm-level scope, M, according to Equations (16) and (17), respectively. The equilibrium is unique, as $\partial X/\partial M > 0$ and $\partial M/\partial X < 0$. It is illustrated in Figure 1 below.

2.2.2 With emissions pricing

So far, I have assumed that emissions are costless. In the following, I analyze optimal firm decisions on product scale, scope, and technologies when there is emissions pricing. In this case, firms maximize profits given in Equation (11b), where marginal costs are given by $c'(i')$

according to Equation (6). I derive optimal scale, $x'(i')$, capital investment, $k'(i')$, $l(i')$, and scope, M', analogously to Section 2.2.1. As shown in Appendix A.1, the symmetric industry equilibrium with emissions pricing is given by:

$$X' = \frac{\beta M'^{1+\beta}}{2(1+\beta)[b'(1-\epsilon) - \frac{1}{r_k} - \frac{\gamma^2}{r_l}]} \tag{20}$$

and

$$M' = (a - d - b'\epsilon(1+N)X')^{\frac{1}{\beta}}. \tag{21}$$

In turn, optimal capital investment is given by

$$k'(i') = \frac{(M'^{\beta} - i'^{\beta})^2}{4[b'(1-\epsilon)r_k - \gamma^2 r_k/r_l - 1]^2} \tag{22}$$

and

$$l(i') = \frac{\gamma^2(M'^{\beta} - i'^{\beta})^2}{4[b'(1-\epsilon)r_l - r_l/r_k - \gamma^2]^2}, \tag{23}$$

and optimal product scale is given by

$$x'(i') = \frac{M'^{\beta} - i'^{\beta}}{2[b'(1-\epsilon) - \frac{1}{r_k} - \frac{\gamma^2}{r_l}]}. \tag{24}$$

As before, the cost of capital investment must be sufficiently large:

Condition 2. *To ensure that the cost of the marginal product M is greater than the cost of any infra-marginal product i, I restrict parameter values such that*

$$r_k > \frac{1}{b'(1-\epsilon) - \gamma^2/r_l}.$$

2.3 Firm-product-level effects of emissions pricing

A comparison of the equilibria derived in Section 2.2 shows that an introduction of emissions pricing may affect product scope, product scale, and product technology in firms. First, it may change product scope, M, and firm scale, X, via a change in marginal costs ($c \neq d$, $\alpha \neq \beta$, $\gamma \neq 0$).[14] Second, it may change product scale, $x(i)$, via three channels: i) a change in the scope elasticity ($\alpha \neq \beta$), ii) a change in product scope ($M \neq M'$), and iii) a change in the rank of a given product in terms of its marginal cost ($i \neq i'$).[15] Third, it may change

[14]Compare Equations (20)-(21) to Equations (18)-(19).
[15]Compare Equation (24) to Equation (17).

a product's emission intensity via abatement investment ($l_s(i'_s) \neq 0$).[16] In this section, I derive predictions about the effects of an introduction of emissions pricing on production at the firm-product level. In the next section, I derive implied effects on firm emissions. I also report numerical simulation results in Appendix A.2.

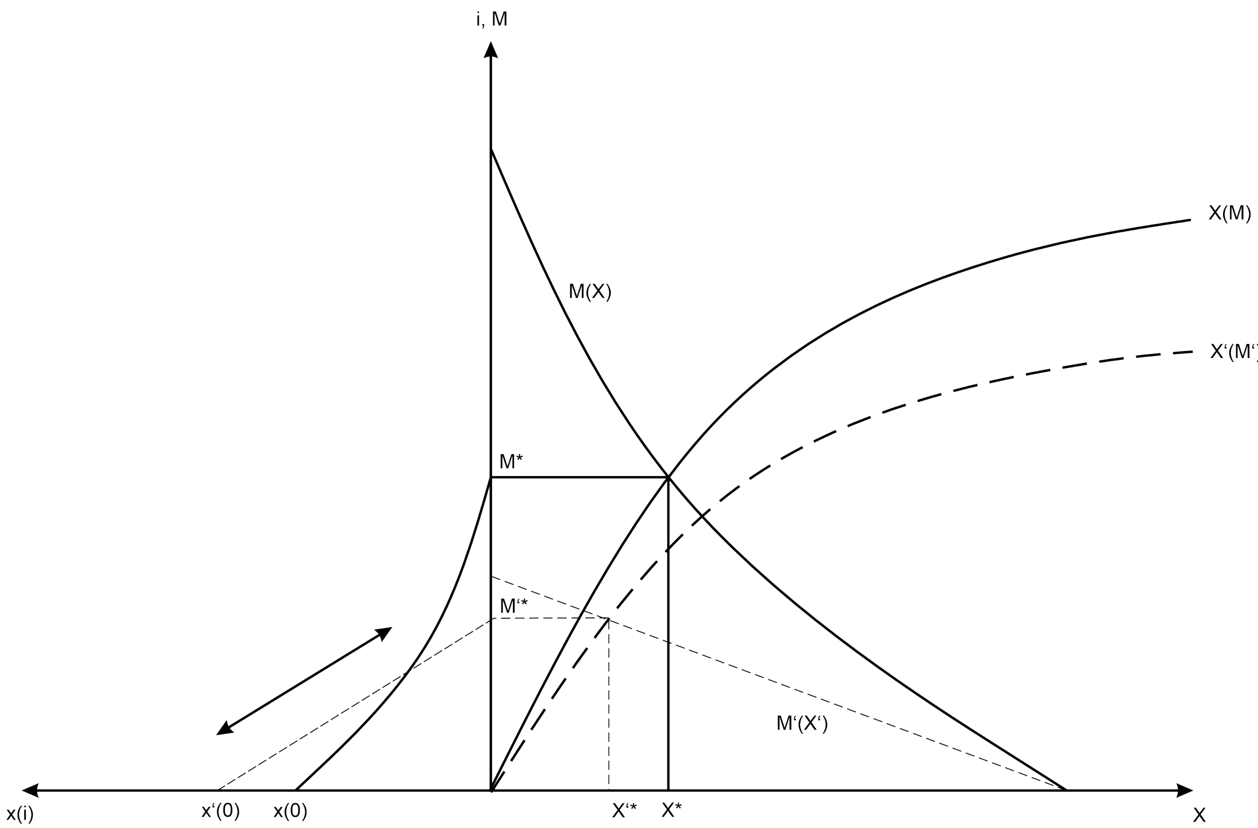

Figure 1: Scale and scope effects of an increase in the scope elasticity of marginal cost at the firm-product level

Product scope and firm scale

An increase in the scope elasticity increases total firm scale, X, for any given product scope, M, according to Equation (18). This is because the scale of any given variety i increases, as it gets cheaper relative to the marginal product M. In turn, the increase in firm scale, X, reduces product scope, M, due to the cannibalization effect, according to Equation (19). Furthermore, the increase in scope elasticity reduces product scope M, given X, according to Equation (19). In Figure 1, the $X(M)$-curve rotates clockwise, while the $M(X)$-curve rotates counter-clockwise (see right panel) such that, in equilibrium, firm scope decreases, while firm scale may increase or decrease (or remain unchanged). And vice versa.

Product scale

[16] Compare Equation (23).

In response to an increase in the scope elasticity, the scale of any product with given rank i increases relative to that of a product with a comparatively higher rank (i.e. the firm focuses more on core products) (left panel). There are two opposing effects of an increase in the scope elasticity of marginal costs on product-level scale (see Equation (17)). First, there is a positive effect due to the decrease in production cost relative to the marginal product for given product scope. Second, there is a negative effect due to the decrease in scope and, thus, the relative production cost of the marginal product. According to my numerical results in Appendix A.2, where $\beta > \alpha$, the positive effect is larger relative to the negative one for varieties that are closer to the core variety. Note, however, that the result for product scale only holds conditional on product rank. If we allow product rank to change, the effect of emissions pricing on product scale is not necessarily monotonic in distance from the core any more. This is illustrated in the left panel of Figure 1, where I have assumed that $\alpha < \beta = 1$. For any given rank, the scale of variety i increases more (decreases less), the closer a variety is to the core, as can be seen by comparing the horizontal distances between the solid and dashed lines. Moreover, any change in rank corresponds to a movement along the dashed curve and, thus, an additional increase in output (if rank decreases) or decrease in output (if rank increases).

Product technology

Abatement investment in a product i', $l_s(i')$, reduces the cost of emissions per unit of output via a reduction in per-unit emissions, $z_s(i')/x_s(i')$. Therefore, it reduces the emission intensity of product i'. The reduction is larger, the closer the product is to the core product (see Equation (23)). The following three propositions summarize my findings regarding the effect of emissions pricing on product scope, firm scale, product scale, and product emission intensity in multi-product firms.

Proposition 1. *The effect of emissions pricing on product scope is negative (positive), if the scope elasticity of marginal cost increases (decreases). The effect on firm scale is ambiguous.*

Proposition 2. *The effect of emissions pricing on product scale is ambiguous.*

Proposition 3. *The effect of emissions pricing on product emission intensity is negative. The effect is more negative, the closer a product is to the core product.*

My findings above have interesting implications regarding the effectiveness of emissions pricing policies.[17] They imply that, in multi-product firms, the output of products with high

[17] Also, note that Proposition 1 implies that firm productivity may increase in the case where the scope

emissions per unit (i.e. high emission-intensity) does not necessarily decrease relative to the output of products with lower emissions per unit (i.e. lower emission-intensity). This is because optimal product scale within the firm depends on how products compare in terms of their total marginal cost, and the ranking of products in terms of total marginal cost does not necessarily coincide with their ranking in terms of emissions cost. In other words, products that rank low in terms of production costs may be relatively emission-intensive (and vice versa). This implies that a firm's total emissions do not necessarily decrease, as emissions pricing is introduced. Emissions will increase, if the increase in emissions of products whose output increases dominates the decrease in emissions of products whose output decreases. The net effect depends i) on the amount of emissions per unit (emission intensity) of each product and ii) on the size of the change in output of each product. The latter depends on how products rank in terms of their total cost (and, therefore, sales).[18]

3 The environmental impact of emissions pricing

The economic literature largely attributes changes in aggregate emissions to three different channels: a change in the scale of real output, a change in the composition of output, and a change in production technique (Copeland and Taylor (1994), Grossman and Krueger (1995)). This type of decomposition is typically performed at the industry level. However, more recent work has extended this exercise to the firm level and, further, to the product level. It thus considers not only changes in the mix and technology of industries, but also of firms within industries and, further, changes in the mix of products at each firm as well as the emission intensity of each product (Cherniwchan et al. (2017), Barrows and Ollivier (2018), Shapiro and Walker (2018)). Such disaggregation allows to attribute changes in emissions more precisely to their true sources, and thus avoids potentially interpreting compositional changes falsely as changes in technology, i.e. the emission intensity of single firms or products. Not accounting for these sources may result in a misidentification of the way in which technology responds, e.g., to changes in environmental policies. Moreover, the way in which firms adjust to policy changes is likely to be important for the effect of policies on total emissions.

As shown in detail in Appendix A.3, we can express the percentage change in total emissions in an economy with industries $s \in \{1, S\}$, each with a discrete number of symmetric

elasticity of marginal cost increases and marginal products are dropped. This is consistent with empirical findings regarding a positive effect of the EU ETS on firm productivity, e.g. in Colmer et al. (2024).

[18] In the special case where emission intensity (and, thus, emission costs per unit) are the same across products, total emissions increase, if total firm scale increases (and vice versa).

firms $n \in \{0, N_s\}$ that produce a continuum of products $i \in [0, M_{ns}]$, respectively, as follows:

$$\hat{Z} = \underbrace{\hat{Y}}_{\text{aggregate scale}} + \underbrace{\sum_{s=1}^{S} \frac{N_s Z_s}{Z} \widehat{\Theta}_s}_{\text{industry composition}} \tag{25}$$
$$+ \sum_{s=1}^{S} \frac{N_s Z_s}{Z} \underbrace{\int_0^{M_s} \frac{z_s(i)}{Z_s} \hat{\theta}_s(i)\, di}_{\text{product scale}} + \sum_{s=1}^{S} \frac{N_s Z_s}{Z} \underbrace{M_s \left(\frac{z_s(M_s)}{Z_s} - \theta_s(M_s) \right) \hat{M}_s}_{\text{product scope}}$$
$$+ \sum_{s=1}^{S} \frac{N_s Z_s}{Z} \underbrace{\int_0^{M_s} \frac{z_s(i)}{Z_s} \hat{e}_s(i)\, di}_{\text{product emission intensity}},$$

where $\Theta_s = Y_s/Y$ is the output share of industry s, $\theta_s(i) = N_s x_s(i)/Y_s = x_s(i)/X_s$ is the share of product i in output of (a representative firm in) industry s, with $x_{ns}(i) = x_{ms}(i) \equiv x_s(i)$, $X_s(n) = X_s(m) \equiv X_s$, $M_{ns} = M_{ms} \equiv M_s$ is the product range of a firm in industry s, $z_{ns}(i) = z_{ms}(i) \equiv z_s(i)$, $Z_s(n) = Z_s(m) \equiv Z_s$ and $e_{ns}(i) = e_{ms}(i) \equiv e_s(i) = z_s(i)/x_s(i)$ are product- and firm-specific emissions and emission intensities of product i produced in a firm in industry s, respectively, and $\hat{Y}$ is the percentage change in Y, etc. Overall, emissions may decrease, if aggregate output decreases, if relatively dirty industries lose market share, or if, within industries, the share of relatively dirty products decreases, if the dirtiest products are dropped, or if product-level emission intensities decrease due to a change in technology.

In my model in Section 2 above, I analyzed the way in which firms may respond to emissions pricing along all three different margins of adjustment within firms, which correspond to terms three (product scale), four (product scope) and five (product emission intensity) in Equation (25). Given information about how emission intensity varies with sales rank, my model delivers a (qualitative and quantitative) assessment of how multi-product firms respond to emissions pricing via the different channels, which allows me to predict the impact of such a policy on firm emissions and, in turn, on emissions in the economy overall. The following three propositions summarize my findings regarding the effect of emissions pricing on aggregate emissions via changes in product scope, product scale, and product emissions intensity in multi-product firms.

Proposition 4. *The effect of a change in product scope on emissions in response to an introduction of emissions pricing is zero.*

The effect of a change in product scope on aggregate emissions is given by term four in

Equation (25):

$$\sum_{s=1}^{S} \frac{Z_s}{Z} M_s \left(\frac{z_s(M_s)}{Z_s} - \theta_s(M_s) \right) \hat{M}_s,$$

which is equal to zero, since $x_s(M_s) = 0$ and $z_s(M_s) = x_s(M_s)e_s(M_s) = 0$.

Proposition 5. *The effect of a change in product scale on emissions in response to an introduction of emissions pricing is ambiguous.*

The change in aggregate emissions via a change in product mix is given by term three in Equation (25):

$$\sum_{s=1}^{S} \frac{Z_s}{Z} \int_0^{M_s} \frac{z_s(i)}{Z_s} \hat{\theta}_s(i)\, di. \tag{26}$$

As shown in Appendix A.4, Equation (26) can be rewritten as follows:

$$\frac{1}{Z_s} \int_0^{M_s} \left(\frac{z_s(i)}{x_s(i)} - \frac{Z_s}{Y_s} \right) dx_s(i)\ di. \tag{27}$$

This expression may be positive or negative, depending on the variance of product-specific emission intensities and changes in scale across products with different rank i. Product scale changes are ambiguous and depend on parameter values according to Proposition 2. In the next Section 3.1, I provide numerical simulations for the effect of changes in product scale on aggregate emissions.

Proposition 6. *The effect of a change in product emission intensity on emissions in response to an introduction of emissions pricing is negative.*

Aggregate emissions decrease via a change in technology, if term five in Equation (25) is negative:

$$\sum_{s=1}^{S} \frac{Z_s}{Z} \int_0^{M_s} \frac{z_s(i)}{Z_s} \hat{e}_s(i)\, di,$$

where $\hat{e}_s(i) = d(z_s(i)/x_s(i))/(z_s(i)/x_s(i))$. According to Proposition 3, emission intensity decreases for all products, $\hat{e}_s(i) < 0\ \forall\ i$.

■

As firms invest in abatement technology, which reduces emissions per unit of each product, aggregate emissions decrease, for any given product scope and scale. Note that investment is greater, the closer a product is to the core product. For $\beta > \alpha$, this implies that emission intensity is reduced more for cleaner products (i.e. products with smaller emission intensity).

This is because the objective of the firm is not to minimize (the cost of) emissions but total costs.

3.1 Quantitative results

Table 1 shows how firms adjust to an introduction of emissions pricing, which affects the scope elasticity of their marginal costs, via three different channels: product scale, product scope, and product emission intensity. More precisely, in rows 1-3 I report simulated numerical values of the following terms[19]:

$$\int_0^M \frac{z(i)}{Z}\frac{d(x(i)/X)}{x(i)/X}\,di = \int_0^M \frac{z(i)}{Z}\left(\frac{dx(i)}{x(i)} - \frac{dX}{X}\right)\,di, \tag{product scale}$$

$$\left(\frac{z(M)}{Z} - \frac{x(M)}{X}\right)dM, \tag{product scope}$$

$$\int_0^M \frac{z(i)}{Z}\frac{d(z(i)/x(i))}{z(i)/x(i)}\,di. \tag{product emission intensity}$$

According to Equation (9), product-level emission intensity is given by

$$\frac{z(i')}{x(i')} = d - c + i'^{\beta} - i'^{\alpha}.$$

Given parameter values for $\beta > \alpha$ according to Table 1, this implies that product emission intensity is approximately linearly increasing in product rank, consistent with evidence in Barrows and Ollivier (2018).[20] In turn, this implies that product rank in terms of sales remains unchanged, $i' = i$.[21] Furthermore, if $\beta > \alpha$, then $M' < M$ (see Proposition 1). It

[19] The extent to which these firm level changes translate into a change in emissions in the economy overall depends on the share of emissions of the firm's industry in aggregate emissions. Compare terms three, four and five in Equation (25).

[20] Other parameter values are chosen in accordance with Conditions 1 and 2, and the existing literature (Chan et al. (2022)). Interest rates are chosen according to the current (September 2024) value of the U.S. Federal Reserve funds interest rate equal to 5% (https://tradingeconomics.com/united-states/interest-rate). See table notes.

[21] Also see footnote 27 below.

follows that the change in product scale in response to the introduction of emissions pricing is equal to

$$dx(i) = \begin{cases} \frac{M'^{\beta} - i'^{\beta}}{2[b'(1-\epsilon) - \frac{1}{r_k} - \frac{\gamma^2}{r_l}]} - \frac{(M^{\alpha} - i^{\alpha})}{2[b'(1-\epsilon) - \frac{1}{r_k}]} & \text{if } 0 \le i \le M' \\ -x(i) & \text{if } M' < i \le M \end{cases} \tag{28}$$

The changes in product scope and firm scale are equal to

$$dM = (a - d - b'\epsilon(1+N)X')^{\frac{1}{\beta}} - (a - c - b'\epsilon(1+N)X)^{\frac{1}{\alpha}} \tag{29}$$

and

$$dX = \frac{\beta M'^{1+\beta}}{2(1+\beta)[b'(1-\epsilon) - \frac{1}{r_k} - \frac{\gamma^2}{r_l}]} - \frac{\alpha M^{1+\alpha}}{2(1+\alpha)[b'(1-\epsilon) - \frac{1}{r_k}]}. \tag{30}$$

And the change in product-level emission intensity is given by Equation (10), where I use $i' = i$:

$$d\left(\frac{z(i)}{x(i)}\right) = \begin{cases} -2\gamma l(i)^{0.5} & \text{if } 0 \le i \le M' \\ -\frac{z(i)}{x(i)} & \text{if } M' < i \le M \end{cases} \tag{31}$$

Table 1: Effects of Binding Emissions Pricing via Different Channels of Within-Firm Adjustment

	$\beta = 0.7$	$\beta = 1$	$\beta = 1.5$
Product scale	-0.06	-0.13	-0.28
(intensive+extensive)	(0.21-0.27)	(0.28-0.42)	(0.31-0.59)
Product scope	0	0	0
Product emission intensity	-0.37	-0.51	-0.66
(intensive+extensive)	(-0.11-0.25)	(-0.11-0.4)	(-0.10-0.56)

Note: Parameter values: $\alpha = 0.5$, $\gamma = 1$, $c = 10$, $d = 16$, $\epsilon = 0.7$, $b = 200$, $L = 1$, $a = 100$, $N = 10$, $r_k = r_l = 0.05$.

Row one in Table 1 (product scale) reports the percentage change in industry emissions due to the change in the product scale. It is calculated as a weighted sum of percentage changes in product shares, where the weights are given by the shares of product emissions in total (industry) emissions. I find a negative effect on emissions that is greater in absolute size the greater the increase in the scope elasticity of marginal cost. Row two splits up the product scale effect on emissions into the effect due to changes in the product scale of i) infra-marginal products (intensive margin) and ii) products that are dropped (extensive margin).

As the scope elasticity increases and marginal products are dropped, the product shares of remaining products increase, and so does their contribution to firm emissions. At the same time, however, the product shares of dropped products go to zero, which reduces firm emissions - the more so, the greater the increase in the scope elasticity. Row three (product scope) shows that the effect on emissions via the change in product scope is zero. This is because output and, therefore, emissions, of the marginal product is equal to zero. Row four (product emission intensity) reports the percentage change in industry emissions due to changes in product-level emission intensities, weighted by the emission shares of products. The effect is negative because i) firms invest in abatement once emissions are priced (intensive margin) and ii) marginal products, which are relatively dirty, are dropped (extensive margin). Row five shows the extent to which the intensive and extensive margins contribute to the decrease in emissions via the decrease in product emission intensities. Product-specific abatement increases in product scope according to Equation (23). In consequence, the decrease in emissions due to abatement becomes smaller in absolute terms, as the scope elasticity increases, and product scope decreases (see Table A2). In turn, as product scope decreases, more of the relatively dirty products are dropped, and the resulting decrease in emissions becomes greater in absolute terms.

4 Extensions

In my main analysis above, I focus on changes in the emissions of given number of homogeneous multi-product firms. In this section, I discuss the implications of allowing for free entry of firms, firm heterogeneity, and the coexistence of single- and multi-product firms for the effect of emissions pricing on aggregate emissions.

Free entry

In my main analysis above, I investigate effects on aggregate emissions due to product-level adjustments in production and technology within firms, given by terms five, six and seven in Equation (A.16). In addition, emissions may change due to changes in aggregate output (term one), $\hat{Y}$, changes in the industry composition of output (term two),

$$\sum_{s=1}^{S} \frac{H_s}{Z} \widehat{\Theta}_s, \tag{32}$$

where $\Theta_s = Y_s/Y$ is the share of industry s in aggregate output, and changes in the firm composition of industries (terms three and four):

$$\sum_{s=1}^{S}\frac{H_s}{Z}\sum_{n=1}^{N_s}\frac{Z_s(n)}{H_s}\hat{\theta}_s(n)+\sum_{s=1}^{S}\frac{H_s}{Z}N_s\left(\frac{Z_s(N_s)}{H_s}-\theta_s(N_s)\right)\hat{N}_s, \tag{33}$$

where $\theta_s(n) = X_s(n)/Y_s$ is the share of firm n in output in industry s.

Emissions pricing may increase or decrease firm profits (see Table A2 in Appendix A.2) and, therefore, result in firm entry or exit. In turn, firm entry (exit) may increase aggregate emissions, if it increases aggregate output, or increases output relatively more in dirtier industries. The firm composition of industries does not affect aggregate emissions due to our assumption of symmetry. That is, the terms in expression (33) are equal to zero, since emission intensity is the same for all firms in an industry:

$$\frac{Z_s(n)}{X_s(n)}=\frac{H_s}{Y_s}\quad \forall\, n\in\{1,...,N_s\}.$$

Heterogeneous firms

According to my analysis above, emissions pricing affects industry outputs, $Y_s = N_s X_s$, and, therefore, aggregate output, $Y = \sum_{s=1}^{S} Y_s$, via a change in the cost parameter α:

$$dY_s = N_s dX_s \tag{34}$$

and

$$dY = \sum_{s=1}^{S} dY_s. \tag{35}$$

According to Equation (30), dX_s differs depending on the cost structure of firms, which is heterogeneous across industries. If dX_s is smaller (larger) in industries that contribute more to aggregate emissions[22], the term in expression (32) above decreases (increases), and so does the change in aggregate emissions.[23]

Coexistence of single- and multi-product firms

Single-product firms may respond to emissions pricing in two ways: via reductions in

[22] That is, $\hat{\Theta}_s$ is smaller (greater) in industries with a greater share of emissions, H_s/Z.

[23] Industry equilibrium can also be computed, if firms are non-symmetric, i.e. heterogeneous within industries (compare Eckel and Neary (2010), Section 5.2). In this case, firm responses in terms of product scope and emission intensity are qualitatively identical to average responses (see Propositions 4 and 6). Firm responses in terms of product scale may differ qualitatively depending on their cost structures (see Proposition 5).

output or via investment in abatement technology[24], both of which will reduce emissions.[25] Multi-product firms choose optimal scale and technology not only for a single product but for multiple products simultaneously. In the presence of multiple products, it is no longer clear whether adjustment will serve to reduce firm emissions. This is because firms do not necessarily reduce output more, or invest in abatement more, for products that are more rather than less emission-intensive. Instead, they produce more of, and invest more in, the products that are most efficient in terms of their total marginal cost, and not necessarily the products that are most efficient in terms of their emission intensity.

Let us assume that emissions are generated by both multi- and single-product firms, and that industries indexed $s \in \{1, R\}$ are multi-product firms, while industries $s \in \{R+1, S\}$ are single-product firms. Then, I can write the economy-wide change in emissions (compare Equation (25)) as follows:

$$\begin{aligned}\hat{Z} = {} & \underbrace{\hat{Y}}_{\text{aggregate scale}} + \underbrace{\sum_{s=1}^{S} \frac{N_s Z_s}{Z} \widehat{\Theta}_s}_{\text{industry composition}} \\ & + \sum_{s=1}^{R} \frac{N_s Z_s}{Z} \underbrace{\int_0^{M_s} \frac{z_s(i)}{Z_s} \hat{\theta}_s(i)\, di}_{\text{product scale}} + \sum_{s=1}^{R} \frac{N_s Z_s}{Z} \underbrace{M_s \left(\frac{z_s(M_s)}{Z_s} - \theta_s(M_s) \right) \hat{M}_s}_{\text{product scope}} \\ & + \sum_{s=1}^{R} \frac{N_s Z_s}{Z} \underbrace{\int_0^{M_s} \frac{z_s(i)}{Z_s} \hat{e}_s(i)\, di}_{\substack{\text{product emission intensity} \\ \text{in multi-product firms}}} + \sum_{s=R+1}^{S} \frac{N_s Z_s}{Z} \underbrace{\hat{e}_s(0)}_{\substack{\text{product emission intensity} \\ \text{in single-product firms}}} .\end{aligned} \tag{36}$$

Changes in emissions due to changes in product scale and product scope stem only from multi-product firms. This is because $\hat{\theta}_s(0) = 0$, and $z_s(0)/Z_s = x_s(0)/X_s = 1$ in the case of single-product firms, which only product one product $i = 0$. Whether emissions decrease more or less in the presence of single-product firms depends on the sign of the product scale effect, and the relative size of the (average) reduction in emission intensities in single- and multi-product firms.

[24]Compare Colmer et al. (2024).

[25]Output may also be reduced via relocation or leakage effects. In this case, it reduces local (but not global) emissions.

5 Conclusion

Attenuation of the expected huge costs of climate change requires significant changes in industrial production. In turn, the effectiveness of existing emissions regulation policies such as the EU Emission Trading Scheme highly depends on how firms adjust their production processes in response. This paper shows that adjustment depends on firm-specific cost structures and does not unambiguously serve to reduce emissions. I find, first, that multi-product firms decrease their product scope, if emission intensity decreases in product scale, and vice versa. I also find, second, that changes in product scale may increase or decrease emissions, depending on how emission intensity varies with product scale. Third, I find that multi-product firms increase abatement investment, the more so the closer a product is to the core. In numerical simulations, I find that these adjustments serve to reduce emissions in the economy for chosen parameter values. This is to a large extent due to the fact that firms drop their dirtiest products, and to a smaller extent due to abatement investment in remaining products. More generally, my model allows to compute changes in aggregate emissions for different levels of emission intensities across products. It shows that changes in the product mix of a firm can be important for changes in aggregate emissions, and that policies that target emission-intensive products directly may be more effective than general emissions pricing.

A Theoretical Appendix

A.1 Equilibrium with emissions pricing

In the following, I derive optimal scale, $x'(i')$, capital investment, $k'(i')$, $l(i')$, and scope, M', in the presence of emissions pricing. Analogously to Section 2.2.1, I derive optimal scale:

$$x'(i') = \frac{a - c'(i') - b'\epsilon(X' + Y')}{2b'(1-\epsilon)}. \tag{A.1}$$

Taking into account the fact that $x(M') = 0$, I can express this as:

$$x'(i') = \frac{M'^{\beta} - i'^{\beta} + 2k'(i')^{0.5} + 2\gamma l(i')^{0.5}}{2b'(1-\epsilon)}. \tag{A.2}$$

The first-order conditions for optimal capital investments imply that

$$k'(i')^{0.5} = \frac{x'(i')}{r_k}, \tag{A.3}$$

$$l(i')^{0.5} = \frac{\gamma x'(i')}{r_l}, \tag{A.4}$$

and

$$l(i')^{0.5} = \frac{r_k}{r_l}\gamma k'(i')^{0.5}. \tag{A.5}$$

Using Equation (A.5) to substitute for $l(i')$ in Equation (A.2), and substituting the resulting expression for $x'(i')$ in Equation (A.3), I derive:

$$k'(i') = \frac{(M'^{\beta} - i'^{\beta})^2}{4[b'(1-\epsilon)r_k - \gamma^2 r_k/r_l - 1]^2}, \tag{A.6}$$

and, using Equation (A.5),

$$l(i') = \frac{\gamma^2(M'^{\beta} - i'^{\beta})^2}{4[b'(1-\epsilon)r_l - r_l/r_k - \gamma^2]^2}. \tag{A.7}$$

As before, the capital allocated to a given product within the firm is greater the closer it is to the product with the smallest total marginal cost ($i' = 0$). The share of investment in the reduction of emission costs (abatement investment) in total capital investment per product, $\frac{l(i')}{l(i')+k'(i')} = \frac{\gamma^2 r_k^2}{\gamma^2 r_k^2 + r_l^2}$, increases in the relative efficiency of the abatement technology,

γ.[26]

Using Equations (A.3)-(A.4) to substitute for $k'(i')$ and $l(i')$ in Equation (A.2), I derive:

$$x'(i') = \frac{M'^{\beta} - i'^{\beta}}{2[b'(1-\epsilon) - \frac{1}{r_k} - \frac{\gamma^2}{r_l}]} \tag{A.13}$$

Integrating this expression for $x(i')$, I get:

$$X' = \frac{\beta M'^{1+\beta}}{2(1+\beta)[b'(1-\epsilon) - \frac{1}{r_k} - \frac{\gamma^2}{r_l}]}. \tag{A.14}$$

From Equation (A.1) together with $k(M') = l(M') = 0$, $x(M') = 0$, and $Y' = NX'$, it follows that:

$$M' = (a - d - b'\epsilon(1+N)X')^{\frac{1}{\beta}}. \tag{A.15}$$

Equations (A.14) and (A.15) determine the unique symmetric industry equilibrium in the presence of emissions quota, with firm-level scale X' and scope M'. These, in turn, determine optimal capital investments, according to Equations (A.6) and (A.7), and optimal product scale, according to Equation (A.13).

A.2 Numerical appendix

In the following, I investigate numerically the effects of an introduction of emissions pricing, and a resulting discrete change in the scope elasticity of marginal cost, on production and investment at the firm and firm-product level. In Table A2, I simulate changes in firm-level equilibrium outcomes (scope, scale, and profit) for different values of the ex-post scope elasticity β (when $\alpha = 0.5$). Table A3 shows changes in firm-product-level equilibrium scale and investment for products with different rank i (when $\alpha = 0.5$ and $\beta = 1$).

Firm-level effects: product scope, output, and profit

Table A2 shows that product scope increases (decreases), if the slope elasticity of marginal cost decreases (increases). This is in line with Proposition 1. It also shows that the change in firm scale is close to zero, which indicates that, for chosen parameter values, the negative and positive effects of a change in the slope elasticity approximately balance out. Furthermore, the change in profits, which depends on the change in the price-cost mark-up as well as the change in scale across products, is negative.

Firm-product level effects: output and technology

[26] For $\gamma = 1$ and equal cost, $r_k = r_l$, abatement investment is equal to investment in the reduction of private costs.

Table A2: Effects of Emissions Quota for Different Slope Elasticities β

	$\beta = 0.2$	$\beta = 1$	$\beta = 1.5$
ΔM	2.68	-3.64	-4.05
ΔX	-0.0033	-0.0036	-0.0037
$\Delta \Pi$	-0.07	-0.39	-2.79

Note: Parameter values: $\alpha = 0.5$, $\gamma = 1$, $c = 10$, $d = 16$, $\epsilon = 0.7$, $b = 200$, $L = 1$, $a = 100$, $N = 10$, $r_k = r_l = 0.05$.

Table A3 shows that, conditional on product rank, product scale decreases more (increases less) with an increase in the slope elasticity, the greater the distance from the core.[27] This is in line with Proposition 2. I also find that capital investment in the reduction of private costs may increase or decrease, while abatement investment increases (from zero) - the more so the closer a product is to the core. In case of the marginal product, $i = M'$, both capital investment in the reduction of private costs and abatement investment are equal to zero.

Table A3: Effects of Emissions Quota for Different Product Ranks i

	$i = 1$	$i = 2$	$i = M'$
$\Delta x(i)$	0.009	-0.010	-0.011
$\Delta k(i)$	0.16	-0.058	-0.056
$\Delta l(i)$	0.28	0.0009	0

Note: Parameter values: $\alpha = 0.5$, $\beta = 1$, $\gamma = 1$, $c = 10$, $d = 16$, $\epsilon = 0.7$, $b = 200$, $L = 1$, $a = 100$, $N = 10$, $r_k = r_l = 0.05$, $M' = 2.06$.

A.3 Decomposition

Following Cherniwchan et al. (2017), I express the percentage change in total emissions in an economy with industries $s \in \{1, S\}$, each with a discrete number of firms $n \in \{1, N_s\}$ that

[27] Barrows and Ollivier (2018) provide evidence of the variation in emission intensity across products within firms. Using product-level input and output data from India, they estimate how emission intensity varies with distance from the core competency. They find that emission intensity is increasing in distance from the core. This finding has two implications that are important for the impact of emissions pricing in my model. It implies, first, that the scope elasticity of marginal cost increases, $\beta > \alpha$. Second, this implies, in turn, that product rank in terms of sales remains unchanged, $i' = i$.

produce a continuum of products $i \in [0, M_{ns}]$, respectively, as follows:

$$
\begin{aligned}
\hat{Z} = \hat{Y} + \sum_{s=1}^{S} \frac{H_s}{Z} \widehat{\Theta}_s + \sum_{s=1}^{S} \frac{H_s}{Z} \sum_{n=1}^{N_s} \frac{Z_s(n)}{H_s} \hat{\theta}_s(n) + \sum_{s=1}^{S} \frac{H_s}{Z} N_s \left(\frac{Z_s(N_s)}{H_s} - \theta_s(N_s) \right) \hat{N}_s \\
+ \sum_{s=1}^{S} \frac{H_s}{Z} \sum_{n=1}^{N_s} \frac{Z_s(n)}{H_s} \int_0^{M_{ns}} \frac{z_{ns}(i)}{Z_s(n)} \hat{\theta}_{ns}(i)\, di \\
+ \sum_{s=1}^{S} \frac{H_s}{Z} \sum_{n=1}^{N_s} \frac{Z_s(n)}{H_s} M_{ns} \left(\frac{z_{ns}(M_{ns})}{Z_s(n)} - \theta_{ns}(M_{ns}) \right) \hat{M}_{ns} \\
+ \sum_{s=1}^{S} \frac{H_s}{Z} \sum_{n=1}^{N_s} \frac{Z_s(n)}{H_s} \int_0^{M_{ns}} \frac{z_{ns}(i)}{Z_s(n)} \hat{e}_{ns}(i)\, di
\end{aligned}
\tag{A.16}
$$

where $Y = \sum_{s=1}^{S} Y_s$ is the economy-wide real output, $\Theta_s = Y_s/Y$ is the share of industry s in aggregate output, $\theta_s(n) = X_s(n)/Y_s$ is the share of firm n in output of industry s, $\theta_{ns}(i) = x_{ns}(i)/X_s(n)$ is the share of product i in output of firm n in industry s, N_s is the marginal firm in industry s, M_{ns} is the marginal product in firm n and industry s, Z, H_s, $Z_s(n)$, and $z_{ns}(i)$ are economy-wide, industry-specific, firm-specific, and product-specific emissions, $e_{ns}(i) = z_{ns}(i)/x_{ns}(i)$ is the emission intensity of product i in firm n and industry s, and $\hat{Z} = dZ/Z$, etc.

To derive this, consider an economy with aggregate emissions Z generated by S industries. Each industry produces real output Y_s and emits Z_s units of emissions. I write aggregate emissions as:

$$
Z = \sum_{s=1}^{S} Y_s E_s, \tag{A.17}
$$

where $E_s = Z_s/Y_s$ is the emission intensity of industry s.

To decompose changes in aggregate emissions into their different potential sources according to Equation (A.16), I totally differentiate Equation (A.17) and then divide through by Z to derive:

$$
\hat{Z} = \hat{Y} + \sum_{s=1}^{S} \frac{Z_s}{Z} \widehat{\Theta}_s + \sum_{s=1}^{S} \frac{Z_s}{Z} \hat{E}_s, \tag{A.18}
$$

where $\Theta_s = Y_s/Y$ is the share of industry s in aggregate output. The right-hand side of Equation (A.18) shows that the change in aggregate emissions is composed of a scale effect (first term), a composition effect (second term), and a technology effect (third term).

Next, I move to the firm level and express industry emission intensity, E_s, as a function of firm-level emissions, $Z_s(n)$, output, $X_s(n)$, and emission intensity, $E_s(n)$, across all firms

$n \in \ \{1, N_s\}$:

$$E_s = \frac{Z_s}{Y_s} = \sum_{n=1}^{N_s} E_s(n)\theta_s(n), \tag{A.19}$$

where

$$Z_s = \sum_{n=1}^{N_s} Z_s(n) = X_s \sum_{n=1}^{N_s} \frac{X_s(n)}{X_s}\frac{Z_s(n)}{X_s(n)},$$

$$X_s = \sum_{n=1}^{N_s} X_s(n),$$

$E_s(n) = Z_s(n)/X_s(n)$, and $\theta_s(n) = X_s(n)/X_s$.

Totally differentiating Equation (A.19) and dividing through by E_s, I can express the change in industry-level emission intensities, $\hat{E}_s$, as follows:

$$\hat{E}_s = \sum_{n=1}^{N_s} \frac{Z_s(n)}{Z_s}\hat{E}_s(n) + \sum_{n=1}^{N_s} \frac{Z_s(n)}{Z_s}\hat{\theta}_s(n) + N_s \left[\frac{Z_s(N_s)}{Z_s} - \theta_s(N_s)\right] \hat{N}_s. \tag{A.20}$$

Equation (A.20) decomposes the change in average industry emission intensity into three different changes at the firm level: a change in firm emission intensity (first term), a change in industry composition (second term), and firm entry and exit (third term). The effect of firm entry is positive, if the share of the marginal firm in industry emissions is greater than its share in industry output or, equivalently, if the entering firm is more emission-intensive than the average firm.

Finally, I move to the product level and express firm emission intensity, $E_s(n)$, as a function of product-level emissions, $z_{ns}(i)$, output, $x_{ns}(i)$, and emission intensity, $e_{ns}(i)$, across all products $i \in \{1, M_{ns}\}$ in firm n and industry s:

$$E_s(n) = \frac{Z_s(n)}{X_s(n)} = \int_0^{M_{ns}} e_{ns}(i)\theta_{ns}(i)di, \tag{A.21}$$

where

$$Z_s(n) = \int_0^{M_{ns}} z_{ns}(i)di = X_s(n)\int_0^{M_{ns}} \frac{x_{ns}(i)}{X_s(n)}\frac{z_{ns}(i)}{x_{ns}(i)}di,$$

$$X_s(n) = \int_0^{M_{ns}} x_{ns}(i)di,$$

$e_{ns}(i) = z_{ns}(i)/x_{ns}(i)$, and $\theta_{ns}(i) = x_{ns}(i)/X_s(n)$.

Totally differentiating Equation (A.21) and dividing through by $E_s(n)$, I can express the change in firm-level emission intensities, $\hat{E}_s(n)$, as follows:

$$\hat{E}_s(n) = \int_0^{M_{ns}} \frac{z_{ns}(i)}{Z_s(n)} \hat{e}_{ns}(i) di + \int_0^{M_{ns}} \frac{z_{ns}(i)}{Z_s(n)} \hat{\theta}_{ns}(i) di + M_{ns} \left(\frac{z_{ns}(M_{ns})}{Z_s(n)} - \theta_{ns}(M_{ns}) \right) \hat{M}_{ns}. \tag{A.22}$$

Equation (A.22) decomposes the change in average firm emission intensity into three different changes at the product level: a change in product emission intensity (first term), a change in product composition within firms (second term), and product entry and exit (third term). The effect of product entry is positive, if the share of the marginal product in firm emissions is greater than its share in firm output or, equivalently, if the additional product is more emission-intensive than the average product. Substituting for $\hat{E}_s$ in Equation (A.18) using Equations (A.20) and (A.22) gives the decomposition of changes in emissions in Equation (A.16).

The first term in Equation (A.16) is the aggregate scale effect, and the second term is the across-industry composition effect. The third and fourth terms capture the effects on industry emission intensities via the composition of firms within industries (term three) and via changes in firm entry and exit (term four). The remaining three terms all capture within-firm adjustments: the fifth and sixth terms are the effects via the composition of products within firms (term five) and via changes in product scope within firms (term six), and the seventh term captures the effects via changes in product emission intensities. In the early literature, terms three to seven together were subsumed in the classic technology effect, measured in terms of changes in industry emission intensities.[28] However, my more detailed decomposition allows me to account for composition effects not only within industries but also within firms, such that the residual effect, due to changes in the emission intensity of products, remains as the pure technology effect. The focus on firms and products rather than industries is relatively new to the literature.[29] It addresses the fact that products differ significantly in their emission intensities even within narrowly defined industries[30] and firms[31], and industry-level changes in emissions may be due either to changes in technology or in a reallocation of production from dirtier towards cleaner products.

Given my assumption that firms are symmetric within industries, terms three and four in the decomposition in Equation (A.16) drop out and changes in product scale, scope, and

[28] See, e.g., Cherniwchan et al. (2017).

[29] I am not the first to decompose emissions within industries and firms. For example, Cherniwchan et al. (2017) and Shapiro and Walker (2018) account for industry- and firm-level composition effects as well as changes in emission intensities at the plant (product) level. However, they do not analyse the effect of environmental policy on optimal product scope, product scale, or product emission intensity in multi-product firms.

[30] See, e.g., Shapiro and Walker (2018).

[31] See, e.g., Barrows and Ollivier (2018).

emission intensity (terms five, six and seven), experienced by all firms in an industry alike, are weighted by industry-specific shares in aggregate emissions, $N_s Z_s/Z$, to compute the average change in emissions across industries. I can then express the economy-wide change in emissions as follows:

$$\hat{Z} = \underbrace{\hat{Y}}_{\text{aggregate scale}} + \underbrace{\sum_{s=1}^{S} \frac{N_s Z_s}{Z} \widehat{\Theta}_s}_{\text{industry composition}}$$
$$+ \sum_{s=1}^{S} \frac{N_s Z_s}{Z} \underbrace{\int_0^{M_s} \frac{z_s(i)}{Z_s} \hat{\theta}_s(i)\, di}_{\text{product scale}} + \sum_{s=1}^{S} \frac{N_s Z_s}{Z} \underbrace{M_s \left(\frac{z_s(M_s)}{Z_s} - \theta_s(M_s) \right) \hat{M}_s}_{\text{product scope}}$$
$$+ \sum_{s=1}^{S} \frac{N_s Z_s}{Z} \underbrace{\int_0^{M_s} \frac{z_s(i)}{Z_s} \hat{e}_s(i)\, di}_{\text{product emission intensities}}, \tag{23}$$

where $\Theta_s = Y_s/Y$ is the output share of industry s, $\theta_s(i) = N_s x_s(i)/Y_s = x_s(i)/X_s$ is the share of product i in output of (a representative firm in) industry s, with $x_{ns}(i) = x_{ms}(i) \equiv x_s(i)$, $X_s(n) = X_s(m) \equiv X_s$, $M_{ns} = M_{ms} \equiv M_s$ is the product range of a firm in industry s, $z_{ns}(i) = z_{ms}(i) \equiv z_s(i)$, $Z_s(n) = Z_s(m) \equiv Z_s$ and $e_{ns}(i) = e_{ms}(i) \equiv e_s(i) = z_s(i)/x_s(i)$ are product- and firm-specific emissions and emission intensities of product i produced in a firm in industry s, respectively, and $\hat{Y}$ is the percentage change in Y, etc.

■

A.4 Product scale and emissions

In the following, I derive the expression for the change in aggregate emissions via changes in product scale given below Proposition 5 in Section 3. The respective change in emissions is given by term three in Equation (25):

$$\sum_{s=1}^{S} \frac{Z_s}{Z} \int_0^{M_s} \frac{z_s(i)}{Z_s} \hat{\theta}_s(i)\, di. \tag{A.24}$$

Using $\hat{\theta}_s(i) = d(x_s(i)/Y_s)/(x_s(i)/Y_s)$, I can rewrite $\int_0^{M_s} \frac{z_s(i)}{Z_s} \hat{\theta}_s(i)\, di$ as follows:

$$\int_0^{M_s} \frac{z_s(i)}{Z_s} \left(\frac{dx_s(i) Y_s - dY_s x_s(i)}{Y_s x_s(i)} \right) di \tag{A.25}$$

or, using $\int_0^{M_s} z_s(i)/Z_s = 1$,

$$\frac{1}{Z_s}\int_0^{M_s} \frac{z_s(i)}{x_s(i)} dx_s(i)\ di - \frac{dY_s}{Y_s}. \tag{A.26}$$

If emissions and output (and, therefore, emission intensity) were the same across products, $z_s(i) = z_s(j) = Z_s/M,\ i \neq j$, and $x_s(i) = x_s(j) = Y_s/M,\ i \neq j$, expression (A.26) would be equal to zero, since:

$$\frac{1}{Z_s}\int_0^{M_s} \frac{z_s(i)}{x_s(i)} dx_s(i)\ di = \frac{1}{Z_s}\int_0^{M_s} \frac{Z_s}{Y_s} dx_s(i)\ di = \frac{dY_s}{Y_s}. \tag{A.25}$$

In this case, changes in product composition would not affect aggregate emissions. Using Equation (A.25) to substitute for dY_s/Y_s, I can rewrite expression (A.26) as follows:

$$\frac{1}{Z_s}\int_0^{M_s} \left(\frac{z_s(i)}{x_s(i)} - \frac{Z_s}{Y_s}\right) dx_s(i)\ di. \tag{A.27}$$

This expression may be positive or negative, depending on parameter values.

References

Abrell, J., M. Kosch, and S. Rausch, "How effective is carbon pricing? A machine learning approach to policy evaluation," *Journal of Environmental Economics and Management*, 2022, *112.*

Barrows, G. and H. Ollivier, "Cleaner firms or cleaner products? How product mix shapes emission intensity from manufacturing," *Journal of Environmental Economics and Management*, 2018, *88*, 134–158.

Becker, R. and V. Henderson, "Effects of air quality regulations on polluting industries," *Journal of Political Economy*, 2000, *108* (2), 379–421.

Calel, R., "Adopt or innovate: Understanding technological choices under cap-and-trade," *American Economic Journal: Economic Policy*, 2020, *12*, 170–201.

Chan, J.M.L., M. Irlacher, and M. Koch, "Multiproduct mergers and the product mix in domestic and foreign markets," Working Paper 2203 2022.

Cherniwchan, J., B.R. Copeland, and M.S. Taylor, "Trade and the environment: New methods, measurements, and results," *Annual Review of Economics*, 2017, *9*, 59–85.

Colmer, J., R. Martin, M. Muûls, and U.J. Wagner, "Does pricing carbon mitigate climate change? Firm-level evidence from the European Union Emissions Trading System," *Review of Economic Studies*, 2024, *00*, 1–36.

Copeland, B.R. and M.S. Taylor, "North-South trade and the environment," *The Quarterly Journal of Economics*, 1994, *109* (3), 755–787.

Dechezleprêtre, A., D. Nachtigall, and F. Venmans, "The joint impact of the European Union emissions trading system on carbon emissions and economic performance," *Journal of Environmental Economics and Management*, 2023, *118* (102758).

Dhingra, S., "Trading away wide brands for cheap brands," *American Economic Review*, 2013, *103*, 2554–2584.

Eckel, C. and J.P. Neary, "Multi-product firms and flexible manufacturing in the global economy," *Review of Economic Studies*, 2010, *77*, 188–217.

Flach, L. and M. Irlacher, "Product versus process: Innovation strategies of multiproduct firms," *American Economic Journal: Microeconomics*, 2018, *10*, 236–277.

Fowlie, M., M. Reguant, and S. Ryan, "Market-based emissions regulation and industry dynamics," *Journal of Political Economy*, 2016, *124*, 249–302.

_ , S.P. Holland, and E.T. Mansur, "What do emissions markets deliver and to whom? Evidence from Southern California's NOx Trading Program," *American Economic Review*, 2012, *102* (2), 965–993.

Greenstone, M., "The impacts of environmental regulations on industrial activity: Evidence from the 1970 and 1977 Clean Air Act Amendments and the Census of M anufactures," *Journal of Political Economy*, 2002, *110* (6), 1175–1219.

_ , J. List, and C. Syverson, "The effects of environmental regulation on the competitiveness of US manufacturing," *NBER Working Paper*, 2012, (18392).

Grossman, G.M. and A.B. Krueger, "Economic growth and the environment," *Quarterly Journal of Economics*, 1995, *110* (2), 353–377.

He, G., S. Wang, and B. Zhang, "Watering down environmental regulation in China," *Quarterly Journal of Economics*, 2020, *135*, 2135–2185.

Irlacher, M., "Multi-product firms in international trade," *Oxford Research Encyclopedia of Economics and Finance, forthcoming*, 2024.

Jaraite, J. and C. Di Maria, "Did the EU ETS make a difference? An empirical assessment using Lithuanian firm-level data," *Energy Journal*, 2016, *37*, 1–23.

Jonghe, O. De, K. Mulier, and G. Schepens, "Going green by putting a price on pollution: Firm-level evidence from the EU," 2020. NBB Working Paper No. 390, National Bank of Belgium, Brussels.

Klementsen, M.E., K.E. Rosendahl, and A.L. Jakobsen, "The impacts of the EU ETS on Norwegian plants' environmental and economic performance," *Climate Change Economics*, 2020, *11.*

Martin, R., M. Muûls, L.B. de Preux, and U.J. Wagner, "Industry compensation under relocation risk: A firm-level analysis of the EU emissions trading scheme," *American Economic Review*, 2014, *104* (8), 2482–2508.

Mayer, T., M.J. Melitz, and G.I.P. Ottaviano, "Market size, competition, and the product mix of exporters," *American Economic Review*, 2014, *104* (2), 495–536.

Ryan, S.P., "The costs of environmental regulation in a concentrated industry," *Econometrica*, 2012, *80* (3), 1019–1061.

Shapiro, J.S. and R. Walker, "Why is pollution from U.S. manufacturing declining? The roles of environmental regulation, productivity, and trade," *American Economic Review*, 2018, *108* (12), 3814–3854.

Wagner, U.J., A. Gerster, M. Klementsen, J. Leisner, J.R. Munch, A.T. Nielsen, K.E. Rosendahl, D. Kassem, J. Jaraite-Kazukauske, M. Laukkanen, R. Martin, M. Muûls, L. de Preux, and S. Schusser, "Carbon footprints of European manufacturing jobs: Stylized facts and implications for climate policy," Discussion Paper 250 2020.

Walker, W.R., "The transitional costs of sectoral reallocation: Evidence from the Clean Air Act and the workforce," *Quarterly Journal of Economics*, 2013, *128*, 1787–1835.

World Bank, "State and Trends of Carbon Pricing 2023," Technical Report, http://hdl.handle.net/10986/39796 License: CC BY 3.0 IGO 2023.