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# Present Bias in the Labor Market – When it Pays to be Naive

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# Present Bias in the Labor Market – When it Pays to be Naive*

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**Abstract**

We study optimal employment contracts for present-biased employees if firms cannot commit to long-term contracts. Assuming that an employee's effort increases his chances to obtain a future benefit, we show that individuals who are naive about their present bias will actually be better off than sophisticated or time-consistent individuals. Moreover, firms might benefit from being ignorant about the extent of an employee's naiveté. Our results also indicate that naive employees might be harmed by policies such as employment protection or a minimum wage, whereas sophisticated employees are better off.

**JEL Codes:** D21, D90, J31, J32
**Keywords:** Present bias, labor markets, on-the-job search, moral hazard

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# 1 Introduction

People suffer from self-control problems which are often caused by inconsistent time preferences. A huge literature has explored how firms lure consumers and employees into inefficient "exploitative contracts" and thereby extract substantial rents from those who are naive about their present bias.[1] Most of these contributions are based on firms offering future contracts which seemingly do not maximize profits, thus rely on firms' commitment power.

In this paper, we focus on *limited commitment* and explore the role of present bias in employment relationships. We show that employees who are unaware of their time-inconsistency can actually benefit from such naiveté. As a consequence, firms' profits are lower when hiring individuals they know to be naive. Furthermore, a firm's ability to commit harms naive, but may benefit sophisticated employees. This result has implications for the impact of policies such as employment protection or the introduction of a minimum wage. These policies reduce misperceptions and can make sophisticated individuals better off at the expense of naifs. Finally, being ignorant about an employee's naiveté may increase a firm's profits.

We derive these results within a three-period model in which a Principal can hire an agent without being able to commit to long-term contracts. The agent is present biased, hence puts extra weight on immediate consumption compared to future benefits. Moreover, the agent can either be sophisticated or naive (Laibson, 1997, O'Donoghue and Rabin, 1999b).[2] Whereas the sophisticated agent perfectly anticipates his future present bias, the naive agent expects to be time-consistent later on. In periods 1 and 2, the Principal makes an employment offer to the agent which contains a (possibly negative) wage payment. If he rejects the offer, the game ends. If he accepts, the agent chooses his effort level. Higher effort is associated with higher costs for the agent and increases the likelihood of receiving a benefit in the *subsequent* period. We first assume that effort generates no direct gain for the Principal, and that the agent's benefit is exogenously given. Importantly, the chance to exert effort is tied to the job, thus the possibility to collect the benefit can be viewed as a non-pecuniary advantage of employment.

We characterize profit-maximizing contracts that allow the Principal to make take-it-or-leave it offers in each period.[3] In the second period, the wage offer

[1] See DellaVigna (2009), Kőszegi (2014), or Heidhues and Kőszegi (2018) for overviews about the behavioral IO literature; Gilpatric (2008) or Englmaier et al. (2016) explore employment relationships with present-biased individuals.

[2] In our main specification, we assume that the Principal can observe the extent of the agent's present bias as well as naiveté. We relax this assumption in Section 6.3 and allow for uncertainty about the extent of naiveté, as in Eliaz and Spiegler (2006).

[3] Indeed, Dube et al. (2020), Manning (2021) and Card (2022) find that firms have considerable

therefore extracts the value that the agent assigns to this period's effort. Crucially, a stronger present bias not only reduces the agent's effort, but also the feasible wage reduction for a given effort level. Second-period outcomes are the same for a naive and a sophisticated agent. The same holds for first-period effort which is solely determined by the prospect of obtaining the benefit in period 2. The first-period wage, however, depends on the agent's naiveté. This is because his value of being employed is not only reflected in the potential benefit of period-1 effort, but also in an "extra rent" of period-2 effort, evaluated in period 1: Discounting between the second and the third period is larger from the perspective of period 2 than from the perspective of period 1. This extra rent – which is not extracted by the second-period wage – is not anticipated by the naive agent. Thus, only the sophisticated agent accepts an additional wage reduction. All this implies that the long-run utility, i.e., the utility evaluated an imaginary period before the game starts, of a naive agent exceeds the utility of a sophisticated agent.

Ultimately, the agent's present bias has two consequences. First, it increases the discounting between two periods. This implies that a time-consistent agent who discounts the future exponentially exerts more effort and accepts a larger wage reduction for a given effort level. Second, the agent's time-inconsistency generates the extra rent of future effort from the perspective of earlier periods. The extra rent of period-1 effort if assessed in the imaginary pre-game period cannot be extracted by the Principal, though. Therefore, the long-run utility not only of the naive, but also of the sophisticated, agent exceeds the time-consistent agent's value.

Next, we consider two applications that generate additional insights into the conse- quences of present bias in the labor market. First, we interpret effort as on-the-job search and the benefit as the value of receiving an outside offer.[4] In contrast to the benchmark model, successful search does not generate a one-time benefit, but permanently increases the agent's outside option because of a better position on the labor market.[5] As before, the naive agent's first-period reservation wage does not incorporate the extra rent of second-period search, which increases his long-run utility above the level of the sophisticated agent. In addition, he searches more than the sophisticated agent. The reason is that the extra rent caused by the agent's time inconsistency increases in future effort, which is higher after search has not been successful. Since only the sophisticated agent takes this link into account, his

wage-setting power.

[4]Our assumption that effort can only be exerted while employed requires on-the-job search to be more effective than search out of unemployment. Indeed, Biewen and Steffes (2010), Mueller (2010), or Cingano and Rosolia (2012) present evidence that this is the case.

[5]Note that we allow for both possibilities, that the agent moves to another employer or that he is retained by the Principal.

perceived utility of *not* receiving an offer is higher and thus his first-period search effort lower.

In the second application, we assume that effort benefits the Principal. It increases the probability with which some verifiable output is realized, thus the Principal can set a performance-based bonus. Then, the naive agent overestimates his second-period bonus and underestimates his second-period wage. Consequently, the agent's first-period reservation wage and his long-run utility again exceed the levels of the sophisticated agent. Besides the first-period wage difference, the realized contracts and effort choices of both the naive and sophisticated agent turn out to be identical, a result which is the same as in the baseline model.[6] Therefore, the Principal's profit with a sophisticated agent is higher than with a naive agent.

Given that the naive agent's misperception about his future contract drives our results, we explore a number of extensions that affect this misperception. First, we consider (partial) commitment by the Principal as a means to influence the naive agent's expectations. Specifically, we assume that she can announce a second-period contract already in period 1, but may terminate the relationship later on at some separation cost. Higher separation costs are equivalent to stronger commitment and allow the Principal to credibly offer a second-period wage that exceeds the amount which the naive agent would otherwise expect. This allows the Principal to reduce the naive agent's first-period wage and consequently his long-run utility. In contrast, the sophisticated agent holds correct beliefs irrespectively. If separation costs are sufficiently high, the naive agent will eventually be *worse off* than the sophisticated agent. This is because he overestimates his second-period effort as well as his second-period rent, which lets him accept an even larger wage reduction in the first period. We argue that more stringent employment protection reflects an increase in separation costs and may therefore harm the naive agent.

Next, we explore how a minimum wage affects the naive agent's perception of future offers and consequently his first-period reservation wage. We show that a minimum wage can harm the naive agent if it is below the payment he ends up receiving, but above the (lower) wage he expects to be paid in the future. Then, the minimum wage leads to a correction of the agent's misperception, which again allows the Principal to lower the wage in the first period. This result indicates that a non-binding minimum wage might *reduce* wages; indeed, evidence for such negative spillover effects has been presented by Neumark et al. (2004), Stewart (2012), and Hirsch et al. (2015). In contrast, a minimum wage always makes the sophisticated and time-consistent agent better off.

[6] The reason is that here and in the baseline model, the bonus is a one-time payment, while in the on-the-job search application the bonus yields a permanent increase in the outside option.

In a final extension to the baseline model, we consider asymmetric information in the sense that the Principal is not able to observe whether the agent is naive or sophisticated. As Eliaz and Spiegler (2006), we assume that the agent's present bias is common knowledge. Different from previous research, we find that the Principal's profits can actually be larger if she cannot observe the agent's naiveté. Then, if the naive agent perceives other agents to be time-inconsistent in the future, he anticipates a higher second-period wage offer than under symmetric information and consequently accepts a lower wage in the first period.[7]

Our paper is organized as follows. We provide a literature review in Section 2. The theoretical setup for the baseline model is described in detail in Section 3, which is then followed by the statement of and intuition behind our main results in Section 4. We turn to the analysis of the two applications in Section 5, and cover the extensions in Section 6. Finally, in Section 7, we discuss the robustness of our results with respect to partial naiveté, discounting between wage payment and effort choice, and extending the time horizon. All proofs are in Appendix Section A.

# 2 Literature Review

Contracting with present biased and potentially naive agents has been analyzed in the context of consumption as well as labor contracts.[8] Heidhues and Kőszegi (2010, 2017) or Gottlieb and Zhang (2021) demonstrate how firms design exploitative contracts for consumers. Gottlieb and Zhang (2021) show that such contracts do not depend on the consumer's extent of naiveté, whose harm vanishes as the number of periods goes to infinity.

Moreover recent observations suggest that time-inconsistent preferences matter in the workplace (Kaur et al., 2010, 2015). Theoretical contributions explo- ring the role of present bias in labor relationships also assume commitment. O'Donoghue and Rabin (1999b) derive the optimal incentive scheme for the comple- tion of a single task. Yılmaz (2013) considers optimal effort choice in a moral hazard setting and compares a sophisticated with a time-consistent agent. Englmaier et al. (2016) show that the optimal exploitative menu of contracts for the Principal consists of a virtual contract which the naive agent intends to select in the future, and a real contract which he ends up choosing. Again, naiveté harms agents, which also holds in Eliaz and Spiegler (2006) and Gilpatric (2008) where an optimal screening contract exploits naive, but not sophisticated, agents.

Different from the aforementioned papers, we consider a Principal-agent setting

[7] Such interplayer perceptions have received empirical support, see Fedyk (2018).

[8] See Kőszegi (2014) and Grubb (2015) for reviews.

were the Principal may not be able to commit to future labor contracts. Then, a naive agent is better off than a time-consistent agent, a result that holds for any number of periods and does not vanish as the number of periods goes to infinity.

Only few papers consider the effect of inconsistent time preferences on job search behavior. Exceptions are DellaVigna and Paserman (2005) and Paserman (2008) who, among others, have recently incorporated behavioral assumptions into job search models. They show that more present biased agents search less and set lower reservation wages. In our application in Section 5.1, we derive similar results for on-the job search.

We also contribute to the literature on the relationship between present bias and commitment, which however mostly focuses on the sophisticated agent's ability to commit. The preference description by O'Donoghue and Rabin (1999a) of the present-biased but sophisticated agent reveals that he might benefit from limiting his future set of feasible choices. More recently, Amador et al. (2006) and Bond and Sigurdsson (2018) have analyzed commitment contracts as a tool to help individuals. Kaur et al. (2015) model and show empirically that the demand for commitment in a labor market context is indeed affecting choices of employees, leading them to choose steep incentives for themselves. We complement these approaches and analyze commitment by the Principal, not the agent. Our results imply that commitment benefits the Principal and may benefit the sophisticated agent, whereas naive agents are harmed.

# 3 Baseline Model

## Environment, Technology & Contracts

There is one Principal (she) and one agent (he). We analyze a game with three periods, $t = 1, 2, 3$.[9] At the beginning of the first and the second period, the Principal can make a take-it-or-leave-it employment offer to the agent, which consists of an upfront payment $w_t \in \mathbb{R}$. If the agent rejects the offer, the game ends and the agent consumes his outside option which is normalized to zero. Upon acceptance, the agent chooses an effort level $e_t \in [0, 1]$ associated with effort costs $e_t^2/2$. Moreover, $e_t$ equals the probability with which the agent receives some benefit $b > 0$ *at the beginning of the next period*. Thus, the possibility to exert effort and potentially receive the benefit are tied to employment. Moreover, players are not active in the third period, only the agent potentially receives the benefit $b$.

[9]Results are robust to considering a large finite number of periods as well as an infinite time horizon; see Appendix Section C.3.

We allow for several interpretations of the agent's effort associated with different specifications of the benefit $b$. First, in the baseline setting, the agent's effort leads to an exogenous one-time benefit which either does not benefit the Principal directly or cannot be incentivized. We might interpret this as effort for the accomplishment of private projects, for which the agent requires resources or reputation only the employer can provide. An example would be a researcher who is intrinsically motivated to conduct some research project, but who needs data provided by the employer. The benefit of a successful project is personal satisfaction which the employer cannot directly influence. This is the setting we explore in the current section.

Second, a successful project leads to better job opportunities, thus effort corresponds to on-the-job search. This interpretation is not captured by our baseline model because a success permanently changes the agent's utility: He will either switch to a better-paying employer, or the current employer has to keep up with the outside offer if she wants to retain him. This setting is explored in more detail in Section 5.1. Third, in Section 5.2, we show that our results also apply to a more standard moral hazard setting, where the employer benefits from the employee's effort, thus wants to provide incentives and sets a bonus endogenously.

**Contracting** Effort $e_t$ is the agent's private information and thus not contractible. Employing the agent has some inherent value for the Principal, for which we provide specific examples later on. Crucially, the Principal can only offer *short-term employment contracts*. In particular, in the first period she is not able to commit to any second-period wage. We discuss the importance of this assumption and the consequences of allowing for partial commitment in Section 6.1. If the agent is employed in the first period, the benefit from $e_1$ is not tied to employment in period 2.

## Preferences

The agent is risk neutral and discounts future costs and future utilities in a quasi-hyperbolic way according to Laibson (1997) and O'Donoghue and Rabin (1999a). Immediate utilities are not discounted. Utilities at the next stage of a period are discounted with a factor $\beta\delta$ and utilities after $t$ periods are discounted with a factor $\beta\delta^t$, where $\beta \in (0, 1)$. Hence, an agent's preferences are dynamically inconsistent. We normalize $\delta$ to 1 as it has no qualitative effect on our results.

This implies that, conditional on accepting the Principal's offers, the agent's

utility at the beginning of period $t = 1$ equals

$$U_1 = w_1 - \frac{1}{2}e_1^2 + \beta\left\{e_1 b + \left[w_2 - \frac{1}{2}e_2^2 + e_2 b\right]\right\}.$$

The agent's utility at the beginning of period $t = 2$ equals

$$U_2 = e_1 b + w_2 - \frac{1}{2}e_2^2 + \beta e_2 b.$$

A comparison between $U_1$ and $U_2$ reveals the agent's time inconsistency. Whereas there is no discounting between periods 2 and 3 from the perspective of period 1, the effective discount factor falls to $\beta$ if evaluated from the perspective of period 2.[10]

The Principal is not present biased. We describe the Principal's preferences and problem in more detail for the case where effort directly benefits the Principal and $b$ is a performance-based bonus (Section 5.2).

## Perceptions

We assume that the agent might be sophisticated or (fully) naive concerning his future present bias.[11] A naive agent expects his present bias to disappear and to discount the future exponentially from the next period on. In contrast, a sophisticated agent perfectly anticipates his future present bias and thus also his future behavior.

Concerning inter-player perceptions, we assume common knowledge about the Principal's time preferences. Moreover, the Principal is aware of the agent's present bias as well as whether he is naive or sophisticated.[12] However, whereas the Principal anticipates potential contradictions between planned and realized actions, the agent thinks that the Principal shares his own perception regarding his future preferences.

## Equilibrium

Following O'Donoghue and Rabin (1999a) and Englmaier et al. (2016), our equilibrium concept is perception-perfect equilibrium. There, a player's strategy maximizes expected payoffs in all subgames, given one's present preferences, and given one's

[10] In Appendix Section C.2 we discuss the implications of the present bias referring to *all* subsequent actions or outcomes. Then, upon receiving the period-$t$ wage, the agent would already discount this period's search effort with $\beta$.

[11] In Appendix Section C.1, we explore partial naiveté and show that our results continue to hold.

[12] In Section 6.3, we assume that the Principal cannot observe the extent of the agent's naiveté.

perceptions of one's own future behavior as well as of the others'. This equilibrium concept enables us to support strategies that are built on a naive agent's inconsistent beliefs.

# 4 Results

In the following, we solve for a perception-perfect equilibrium that maximizes the Principal's profits. Since the bonus is exogenous in the baseline setting, this is equivalent to finding the agent's reservation wage in every period. Moreover, the Principal's inability to commit to long-term contracts implies that we have to apply backwards induction to solve for equilibrium outcomes. As a benchmark, we will start with the time-consistent agent. We then characterize equilibria for sophisticated and fully naive agents separately and subsequently compare outcomes.

## Benchmark: Time-Consistent agent

As a benchmark, we will first derive outcomes for a time-consistent agent (which is equivalent to setting $\beta = 1$).

In the second period, conditional on having accepted the Principal's employment offer, the time-consistent agent chooses effort to maximize $-e_2^2/2 + e_2 b$, which yields an effort level

$$e_2^{TC} = b.$$

Since the total utility of exerting effort, $-(e_2^{TC})^2/2 + e_2^{TC} b = b^2/2$, is strictly positive, and since the agent can only exert effort if he is employed by the Principal, the period-2 reservation (and thus offered) wage equals

$$w_2^{TC} = \frac{1}{2}(e_2^{TC})^2 - e_2^{TC} b = -\frac{1}{2}b^2 < 0.$$

Taking into account $w_2^{TC}$ and expected net benefits of exerting effort, period-2 utility equals the agent's outside utility of zero. Hence, the situation in the first period is equivalent to the second period, which implies that outcomes coincide as well. Lemma 1 collects the results for the case of a time-consistent agent.

**Lemma 1** *A time consistent agent*

- *exerts the same effort in periods 1 and 2, i.e.* $e_1^{TC} = e_2^{TC} = b$
- *receives the same wage in periods 1 and 2, i.e.* $w_1^{TC} = w_2^{TC} = -b^2/2$.

## 4.1 Sophisticated agent

Now, we analyze outcomes for a present biased but sophisticated agent. In the second period, having accepted the Principal's employment offer he chooses effort to maximize $-e_2^2/2 + \beta e_2 b$, which yields an effort level

$$e_2^S = \beta b.$$

As with the time-consistent agent, the period-2 wage $w_2^S$ takes into account that the agent can only exert effort and subsequently collect $b$ if he is employed, thus is set to satisfy $U_2^S = w_2^S - (e_2^S)^2/2 + \beta e_2^S b = 0$. Therefore,

$$w_2^S = \frac{1}{2}(e_2^S)^2 - \beta e_2^S b = -\frac{1}{2}(\beta b)^2 < 0.$$

The Principal can reduce the wage below the agent's outside option and extract the agent's rent from his effort.

In period 1, the agent's effort determines his chances to receive $b$ in the subsequent period 2, thus his incentives to exert effort are the same in both periods and ($e_1^S = e_2^S$). Moreover, from the perspective of period 1, there is no discounting between periods 2 and 3, whereas the potential third-period benefit $b$ is discounted with $\beta$ from the perspective of period 2. This changes the relative assessment of costs and benefits of period-2 effort. Thus, although $w_2^S$ fully extracts the agent's net utility from effort in period 2, it does so only *from the perspective of period 2*. From the perspective of period 1, the agent's period-2 net utility from effort is higher, which gives the Principal additional, inter-temporal, opportunities to reduce wages: Plugging $w_2^S = \frac{1}{2}(e_2^S)^2 - \beta e_2^S b$ into the agent's period-1 utility yields

$$U_1^S = w_1 - \frac{1}{2}(e_1)^2 + \beta \left[ e_1 b + \beta(1-\beta)b^2 \right].$$

There, the last term, $\beta(1-\beta)b^2$, captures the "extra rent" of period-2 effort when assessed from the perspective of earlier periods.

Finally, note that, from the perspective of period 1, the agent's period-2 effort is low for his own taste ($b$ versus $\beta b$).

**Lemma 2** *Assume the agent is sophisticated. Then,*

- *effort in the first period is the same as in the second period, i.e.,* $e_1^S = e_2^S = \beta b$
- *the period-1 wage is* lower *than the period-2 wage, i.e.,* $w_1^S = w_2^S - (1-\beta)(\beta b)^2 < w_2^S = -(\beta b)^2/2$.

## 4.2 Naive agent

Now, we assume that the agent is naive about his present bias which implies that, in period 1, he wrongly believes to be time-consistent in the second period. Therefore, we have to distinguish between his realized and his anticipated second-period effort.

Having accepted the Principal's employment offer, the naive agent's realized effort in period $t = 2$ also maximizes $-(e_2)^2/2 + \beta e_2 b$, yielding an effort level

$$e_2^N = \beta b.$$

Furthermore,

$$w_2^N = \frac{1}{2}(e_2^N)^2 - \beta e_2^N b = -\frac{1}{2}(\beta b)^2 < 0,$$

thus $w_2^N = w_2^S$ and $e_2^N = e_2^S$. From the perspective of period 1, the agent anticipates to maximize $-\frac{1}{2}(e_2)^2 + e_2 b$ and choose an effort level

$$\tilde{e}_2^N = b.$$

Because $\tilde{e}_2^N > e_2^N$, the agent overestimates his future effort. As a consequence, in period 1 the naive agent *underestimates* his period-2 wage. He expects to be offered a wage $\tilde{w}_2^N = (\tilde{e}^N)^2/2 - \tilde{e}^N b = -(1/2)b^2$, which is smaller than the period-2 wage he is effectively willing to accept, $w_2^N$.

The naive agent's behavior in $t = 1$ is thus determined by his perceptions of future outcomes, not their true realizations:

**Lemma 3** *Assume the agent is naive. Then,*

- *efforts in the first and second period are equal, i.e.,* $e_1^N = e_2^N = \beta b$
- *the period-1 wage is equal to the period-2 wage, i.e.,* $w_1^N = w_2^N = -(\beta b)^2/2$.

## 4.3 Comparison

Finally, we compare outcomes of a time-consistent, a naive, and a sophisticated agent. First, recall that $e_2^S = e_2^N < e_2^{TC}$ and $w_2^S = w_2^N > w_2^{TC}$. Therefore, realized outcomes in period 2 are identical for a sophisticated and a naive agent, but both exert less effort and receive a higher wage than a time-consistent agent. However, the naive agent expects to exert the same effort and receive the same wage as the time-consistent agent, i.e., $e_2^S < \tilde{e}_2^N = e_2^{TC}$ and $w_2^S > \tilde{w}_2^N = w_2^{TC}$. This lets period-1 wages of a naive and a sophisticated agent differ.

**Proposition 1** *In the first period,*

- *effort of a naive agent and a sophisticated agent are the same, and both are lower than the effort of a time-consistent agent, i.e.,* $e_1^S = e_1^N < e_1^{TC}$
- *the wage of the naive agent is higher than the wage of a sophisticated agent, which in turn is higher than the wage of a time-consistent agent , i.e.,* $w_1^{TC} < w_1^S < w_1^N$.

From the perspective of period 1, a sophisticated agent perceives his period-2 net utility from being employed to be positive, whereas a naive agent wrongly perceives it to be zero. Thus, a sophisticated agent is willing to accept a lower wage than a naive agent. The wage of a time-consistent agent is even lower, which however is solely driven by more first-period effort and the resulting higher rent that can be extracted.

Next, we show that the naive agent's higher wage also translates into a higher utility, compared to a sophisticated and a time-consistent agent. There, we follow the literature (see O'Donoghue and Rabin, 2001; DellaVigna and Malmendier, 2004; Gottlieb and Zhang, 2021) and compare *long-run realized* utility levels. We take the perspective of the period before the game starts, a "period 0", and denote the respective utilities by $\hat{U}_0$.

**Proposition 2** *Whereas both naive and sophisticated present-biased agents have a larger long-run utility than a time-consistent agent, a naive agent has a larger long-run utility than a sophisticated agent, i.e.,* $\hat{U}_0^{TC} = 0 < \hat{U}_0^S < \hat{U}_0^N$.

For a time-consistent agent, $\hat{U}_0^{TC}$ coincides with his period-1 utility, thus equals zero. The long-run utilities of a (naive or sophisticated) time-inconsistent agent, however, are strictly positive. This is because, from the perspective of period 0, there is no discounting between periods 1 and 2, whereas period-2 payoffs are discounted with $\beta$ from the perspective of period 1. Thus, a time-inconsistent agent enjoys an extra long-term rent from his period-1 effort, which the Principal cannot extract. Moreover, as discussed above, this extra rent also materializes from period-2 effort if the perspective of period 1 (or 0) is taken. While the Principal can extract this rent from a sophisticated agent, this is not possible with a naive agent, whose long-run utility therefore is even higher.

The cause of the difference between naive and sophisticated agent is the Principal's lack of commitment. The literature usually assumes that the Principal can commit to long-term contracts. Then, naive agents who are overoptimistic about their future actions can "pay" to change them once the future materializes. In our setting, in addition, naive agents are overpessimistic about future wages which more than compensates for the overoptimism about effort and allows the naive agent to

keep the extra rent generated by his time inconsistency. To fully grasp the role of commitment, assume that the second-period wage is given and the same for all agents. Then, the naive agent would not need to form expectations about future wages, but only about future effort. Since he is over-optimistic about his period-2 effort, he would accept a lower first-period wage than the sophisticated agent (see Section 6.1 for formal results) and consequently be worse off.

# 5 Applications

We have shown that naiveté may, in fact, protect an agent from exploitation if the Principal is not able to commit to long-term contracts. Now, we introduce two examples where our mechanism may be relevant. First, we assume that effort captures the agent's on-the-job search activities to find a better-paid occupation. Second, we regard $b$ as a performance-based bonus offered by the Principal who benefits from the agent's non-observable effort. We discuss both applications in turn and use them to derive further implications.

## 5.1 Effort as On-the-Job Search

A large numbers of job-to-job transitions indicate that on-the-job search is a significant force behind labor market dynamics. For example, Bjelland et al. (2011) find that employer-to-employer flows accounted for around 4 percent of total employment in the US between 1991 and 2003; see Fallick and Fleischman (2001) or Nagypál (2008) for further evidence. In line with these findings, we now interpret effort as reflecting the agent's on-the-job search activities. We assume that effort is identical to the probability with which the agent receives an outside job offer. This offer guarantees the agent a minimum payoff of $b$ in all subsequent periods, which thus constitutes his future outside option and reflects the agent's improved position on the labor market.[13] $b$ is drawn from an exogenous offer distribution $F(b)$ with support $[0, \overline{b}]$, $\overline{b} > 0$, similar to DellaVigna (2009).[14] Consequently, if the agent receives an outside offer in the first period, he will only benefit from a second-period offer if it exceeds the first-period offer, which we denote by $b_1$. Note that we do not take a stand on whether the agent actually moves to another employer or is retained by the Principal. We merely assume that an offer serves as a signal to the labor market that the agent's value is higher than previously assessed, and that he can keep on searching even after having received an offer.

[13] If $b$ instead constituted a one-time rent, the analysis would be equivalent to the baseline setting.
[14] To always guarantee an interior solution, we assume $\mathbb{E}(b) < 1$.

Before analyzing this case formally, note that it relies on on-the-job search being more effective than search out of unemployment. Indeed, a number of reasons have been identified for why this might be the case: A social stigma effect (see Biewen and Steffes, 2010, for evidence), a missing network (see Cingano and Rosolia, 2012, for evidence), the decay of human capital (Pissarides, 1992), or a higher likelihood of job termination by unemployed individuals (Nagypál, 2005) may reduce their chances of receiving a job offer. Generally, Mueller (2010) provides evidence that job search is more effective when being employed. Thus, on-the-job search can be viewed as a non-pecuniary benefit of being employed by the Principal. For simplicity, we thus assume that search out of unemployment is not feasible, or alternatively that its benefit is zero.

In the second period, a time-inconsistent agent's optimal search effort, conditional on having received an offer $b_1 \geq 0$ in the first period (receiving no offer is equivalent to setting $b_1 = 0$), is given by:

$$\begin{aligned} e_2^k(b_1) &= \beta \left( \int_{b_1}^{\bar{b}} b dF(b) - b_1(1 - F(b_1)) \right) \\ &= \beta \int_{b_1}^{\bar{b}} (b - b_1) dF(b), \end{aligned}$$

for $k \in \{S, N\}$. Again, a time-consistent agent would search more and therefore, for a given $b_1 \geq 0$,

$$e_2^{TC}(b_1) > e_2^k(b_1) \qquad \text{and} \qquad w_2^{TC}(b_1) < w_2^k(b_1).$$

In contrast to the sophisticated agent, the naive agent perceives his period-2 preferences to be time-consistent, thus $\tilde{e}_2^N(b_1) = e_2^{TC}(b_1)$ and $\tilde{w}_2^N(b_1) = w_2^{TC}(b_1)$.

As before, in period 1 the naive agent's misperception of his future time preferences prevents the Principal from extracting the extra rent stemming from his future time inconsistency. In contrast to the baseline model, however, this does not only affect the period-1 wage, but also period-1 effort which is determined by the utility difference between receiving and not receiving an outside offer: First, a potential search benefit mechanically decreases over time, since it can be consumed in fewer remaining periods. Thus, the naive and the time-consistent agent search more and are paid a lower wage in the first than in the second period. Second, the sophisticated agent's first-period effort depends on the extra rent of period-2 search from the perspective of $t = 1$. This rent increases in (expected) future search and thus is smaller after a first-period success because then a second-period success only benefits the agent if it exceeds $b_1$. Therefore, the sophisticated agent's motivation to

exert search effort is smaller compared to the naive and the time-consistent agent. The moderating effect of the extra rent on first-period effort also implies that the sophisticated agent's first-period wage might be higher or smaller than the naive agent's wage: While awareness of the extra rent reduces his first-period wage, a lower first-period effort increases it.

This leads to the following implications for agents' search effort and long-run utilities:

**Proposition 3** *Consider the on-the-job search application. Then in the first period,*

- *the sophisticated agent exerts less effort than the naive agent, who in turn exerts less than the time-consistent agent, i.e.,* $e_1^S < e_1^N < e_1^{TC}$.
- *the naive agent's wage is higher than the time-consistent agent's; both wages might be higher or lower than the sophisticated agent's, i.e.,* $w_1^{TC} < w_1^N$.
- *the naive agent's utility exceeds the sophisticated agent's utility, which in turn is larger than the time-consistent agent's utility of zero, i.e.,* $\hat{U}_0^{TC} = 0 < \hat{U}_0^S < \hat{U}_0^N$.

Hence, an agent's long-run utility is determined by the same factors as in the baseline model: An agent's time-inconsistency allows him to capture a rent if the Principal cannot commit to long-term contracts, and this rent his higher for the naive agent.

To conclude, whereas time inconsistency generally reduces search, it is the sophisticated agent who searches the least and therefore forgoes higher payoffs from better employment. The naive agent searches more *and* is better off.

## 5.2 $b$ as Performance-Based Bonus

In our second application we assume that the agent's effort benefits the Principal but is non-contractible. Moreover, effort is identical to the probability with which a positive outcome of value $\theta$ for the Principal is realized, with $\theta \in (0, 1)$. This event is verifiable, thus $b$ corresponds to a performance-based bonus. The previous analysis can be applied to this case for a given, exogenous bonus. Here, we incorporate the possibility that the Principal is free to set $b$, and show how present bias and naiveté affect the Principal's profit. Formally, the employment offer consists of $(w_t, b_t) \in \mathbb{R}^2$, where $w_t$ is an upfront wage and $b_t$ the bonus that can be obtained by period-$t$ effort. There, note that the outcome realization happens at the beginning of period $t+1$, which implies that also the bonus is effectively paid out one period after effort has been exerted. We argue that this does not contradict our no-commitment

assumption because the performance contract is based on a verifiable measure and simply executed in the subsequent period. Alternatively, we could divide a period into two sub-stages, in which the first stage contains the wage payment and the agent's effort choice, while the second stage contains the realization of the output and the payment of the bonus. Then, if sufficient time passes between the two stages, the bonus is discounted with $\beta$ from the perspective of stage 1, and our results are the same as in the present setting. Finally, if the agent rejects the offer in any period, he still consumes his outside option utility of 0 from then on.

In period 2, the Principal's maximization problem with either the naive or the sophisticated present-biased agent is given by

$$\begin{aligned} \max_{w_2, b_2} \quad & \pi + e_2(\theta - b_2) - w_2 \\ \text{s.t.} \quad & w_2 - \frac{1}{2}e_2^2 + \beta e_2 b_2 \geq 0, \end{aligned}$$

taking into account that effort is determined by

$$e_2 = \beta b_2.$$

$w_2$ is chosen to keep the agent to his outside option of zero, which yields a profit-maximizing bonus $b_2^k = \frac{\theta}{2-\beta}$, $k \in \{S, N\}$. In contrast, the optimal contract for a time-consistent agent involves a higher bonus $b_2^{TC} = \theta$. Moreover,

$$e_2^{TC} > e_2^k,$$

and

$$w_2^{TC} < w_2^k.$$

Note that, although the time consistent agent's upfront wage is smaller, his total expected compensation $w_2 + e_2 b_2$ exceeds the level of a time-inconsistent agent.

From the perspective of the first period, a time-inconsistent agent again enjoys an extra rent from period 2 effort. The Principal can only extract this rent from the sophisticated agent, the naive misperceives his second-period preferences, consequently also his offered bonus ($\tilde{b}_2^N = b_2^{TC}$), effort ($\tilde{e}_2^N = e_2^{TC}$), and upfront wage ($\tilde{w}_2^N = w_2^{TC} < w_2^S$). An agent's perception of his second-period rent has no effect on his effort, though, thus also not on the optimal first-period bonus. Therefore, $e_1^S = e_1^N = \beta b$ and $b_1^S = b_1^N = \theta/(2-\beta)$, and the respective values are identical to their second-period counterparts. Nevertheless, whereas the naive agent expects to receive a period-2 utility of zero and does not accept a wage below $w_1^N = w_2^N$, the sophisticated agent suffers from an additional wage reduction, and $w_1^S < w_2^S$.

This yields the following implications regarding agents' long-run utilities and the Principal's profits, where we denote the Principal's present value of profits in period 1 by $\Pi$.

**Proposition 4** *Assume that $b$ is a performance-based bonus and can be set by the Principal. Then,*

- *the naive agent's first-period wage exceeds the sophisticated agent's first-period wage, which exceeds the time-consistent agent's first-period wage i.e., $w_1^N > w_1^S > w_1^{TC}$; the time-consistent agent's first-period bonus exceeds the naive and sophisticated agent's first period bonus, which are identical: $b_1^{TC} > b_1^S = b_1^N$.*
- *the naive agent's utility exceeds the sophisticated agent's utility, which in turn is larger than the time-consistent agent's utility of zero, i.e., $\hat{U}_0^{TC} = 0 < \hat{U}_0^S < \hat{U}_0^N$.*
- *the Principal's profits are highest with a time-consistent agent; profits with a sophisticated agent exceed profits with a naive agent, i.e., $\Pi^{TC} > \Pi^S > \Pi^N > 0$.*

If the Principal can set a bonus $b$, the same relationship of effort, wages, and long-run realized utilities between the different types of agents holds as with an exogenously given benefit.

The Principal's profits are highest with a time-consistent agent. This is because he does not discount the bonus, thus it is cheaper to motivate him all else equal. Consequently, incentives are stronger and the generated surplus is higher, which is completely extracted by the Principal. If the agent is time-inconsistent, the Principal's profits are higher with a sophisticated agent who receives a lower first-period wage, but whose contract is otherwise identical to the contract of a naive agent.

# 6 Extensions

In this section, we shed more light on key assumptions and explore the consequences of having alternative set-ups. In particular, we focus on the fact that our main results are driven by the naive agent's misperception of the future. Thus, the following extensions affect the extent of this misperception, and we demonstrate that a reduction generally benefits the Principal but harms the naive agent. First, we allow the Principal to partially commit to a future contract, which we argue also captures the consequences of employment protection. Second, we show that a

moderate minimum wage may reduce the naive agent's misperception and consequently increase the rent that can be extracted from him. Third, we allow the agent to have private information regarding his naiveté. This might reduce the naive agent's misperception about his future wage and consequently harm him. In addition, it may have negative spillover effects on the sophisticated agent.

## 6.1 Partial Commitment and Employment Protection

Now, we assume that the Principal can commit to future wages but may fire (and rehire) the agent at cost $k$, after the agent has potentially consumed the benefit $b$. The agent is still free to leave at any time. A higher $k$ might reflect more stringent employment protection or, more generally, labor market regulation that protects workers. As long as $b$ is exogenous and the Principal only cares about wage payments, $k$ is immaterial for the sophisticated and time-consistent agent. A higher $k$ harms the naive agent, however, because it allows the Principal to credibly commit to a second period wage that exceeds the level that maximizes profits from the perspective of the agent's first-period self. Recall that this wage equals $-b^2/2$, whereas the actually paid wage amounts to $-(\beta b)^2/2$. Now, assume that the Principal promises a second period wage $\tilde{w}_2^N > -b^2/2$. Then, the agent takes into account that the Principal could fire him at cost $k$ and afterwards make a new wage offer $-b^2/2$.[15] Paying a higher wage than promised is always possible, irrespective of the size of $k$.

Thus, from the perspective of the agent's first-period self, an offer $\tilde{w}_2^N$ made in period 1 is only credible if it satisfies

$$\tilde{w}_2^N \leq k + \left(-\frac{1}{2}b^2\right),$$

and the Principal can credibly commit to the actual wage $-(\beta b)^2/2$ if

$$k \geq \frac{b^2\left(1-\beta^2\right)}{2}.$$

Otherwise $\tilde{w}_2^N = k - b^2/2$. This yields the following implications for the naive agent's first-period wage and his long-run utility.

**Proposition 5** *Assume that the Principal can commit to future contracts but deviate*

[15] Alternatively, the agent could expect the Principal to "renegotiate" the contract and pay $k$ to the agent in exchange for accepting the respective wage cut

*at cost $k$. Then, in the first period she offers a second-period wage*

$$\tilde{w}_2^N = min\left\{-\frac{1}{2}(\beta b)^2, k - \frac{1}{2}b^2\right\},$$

*which is increased to $w_2^N = -(\beta b)^2/2$ at the beginning of the second period. Moreover, the naive agent's first-period wage and his long-run utility*

- *(weakly) decrease in $k$,*
- *are smaller than the values of the sophisticated agent if $k$ is sufficiently large,*
- *are always larger than the values of the time-consistent agent.*

The naive agent is harmed by higher termination costs $k$ because the extent to which he underestimates the second-period wage is reduced. Thus, he is willing to accept a larger wage reduction in the first period. If $k$ is sufficiently large, he is even worse off than the sophisticated agent. This is because the naive agent also overestimates his second-period effort and thus the rent he is then going to capture. The time-consistent agent anticipates the correct second-period wage and thus is not affected by separation costs.

Additional forces are at play if the Principal can influence the agent's effort by endogenously setting the bonus. Then, commitment also affects the Principal's relationship with the sophisticated agent. In fact, both the sophisticated agent and the Principal benefit from a higher $k$. The reason is that their interests are partially aligned: Both benefit from a higher effort of the sophisticated agent's future self. The demand for commitment by the sophisticated agent due to the difference in short-run and long-run preferences has already been described in the literature, see Amador et al. (2006), Bond and Sigurdsson (2018) or Kaur et al. (2015). Our result complements these findings as it shows that allowing the Principal to commit to a bonus which exceeds the myopically optimal level may generate similar gains even without commitment by the agent. The naive agent is still harmed by commitment if the Principal can set the bonus. A formal characterization of the consequences of partial commitment on outcomes with an endogenous bonus can be found in Appendix Section D.1.

## 6.2 Minimum Wage

In this section, we show that our main mechanism can generate new insights on the effects of a minimum wage. Whereas a sufficiently high minimum wage in our setup benefits all agents, an intermediate level can actually harm a naive agent. Assume there is a minimum wage that exceeds the period-2 wage a naive agent expects to

be paid, but is below the wages he is paid in the first (and second) period. Then, a naive agent wrongfully anticipates a rent in the second period and consequently accepts a lower wage in the first. A sophisticated agent, on the other hand, would at all levels benefit from a minimum wage. These results are collected in the following proposition.

**Proposition 6** *Assume there is a minimum wage* $\overline{w} \geq \tilde{w}_2^N = -b^2/2$. *Then, a higher minimum wage has the following effects on wages and payoffs:*

- *For the naive agent, there exists a a threshold* $\overline{w}^* \leq -(\beta b)^2/2$ *such that both the first-period wage and long-run utility decrease in the minimum wage if* $\overline{w} \in \left[\tilde{w}_2^N, \overline{w}^*\right)$. *If* $\overline{w} \geq \overline{w}^*$, *then long-run utility increases in the minimum wage.*

- *For the sophisticated and time-consistent agent, the first-period wage and long-run utility increase with a higher minimum wage.*

The naive agent can be harmed by a higher minimum wage because he underestimates his period-2 wage. Thus, it is possible that he wrongly expects the minimum wage to bind in the second period. In turn, he is willing to accept a lower wage in the first period and experiences an effective utility reduction. The sophisticated agent can only benefit from a minimum wage; it increases his wages and limits the Principal's ability to extract second-period rents. Similarly, the time-consistent agent's wages go up.

The result for the naive agent indicates that a non-binding minimum wage might have *negative* spillover effects on higher wages. Previous literature has mostly focused on explanations for observed positive spillover effects, which include forces outside our model, such as firms wanting to preserve their wage distribution. However, there is evidence that spillover effects can indeed be negative. For example, Stewart (2012) examines the consequences of the British minimum wage. He finds that the growth of wages slightly above the minimum wage generally is *smaller* than what would have been expected without a minimum wage. Neumark et al. (2004) observe that, although immediate spillover effects are positive, lagged effects are strongly negative. Finally, Hirsch et al. (2015) provide indicative evidence that wages above the minimum wage increase less strongly than they would have without the minimum wage.

To conclude this section, note that a minimum wage also generates new insights in our applications, in particular on effort as on-the-job search. Then, a minimum wage is less likely to bind after a success. Therefore, we would expect a minimum wage to reduce search effort – and consequently turnover – because the benefits of

the current employment relationship go up. This holds in particular for the naive agent, who might wrongly perceive a minimum wage to bind in the second period. There is abundant evidence for negative turnover effects. A number of theoretical explanations have been provided, which however only consider *binding* minimum wages. We predict negative turnover effects of a *non-binding* minimum wage, which the naive agent wrongly perceives to bind in the future. To the best of our knowledge, this aspect has not yet been explored empirically. Some indicative evidence is given by Hirsch et al., 2015, who find that the negative effects of a minimum wage on turnover are not necessarily increasing in the extent to which minimum wages bind.

## 6.3 Asymmetric Information

Our main results rely on the Principal knowing the agent's extent of naiveté and, closely related, on the naive agent's belief that the Principal shares his own perception of his future self. We now refer to those baseline model assumptions as *symmetric* information.[16] In this section, we show that the Principal may benefit from not knowing the agent's extent of naiveté. For formal statements and proofs, we refer the reader to Appendix Section B.

We consider the case of asymmetric information in the sense that the Principal is not able to observe the agent's naiveté, neither at the time of contracting nor at any later point. We focus on time-inconsistent agents and, as Eliaz and Spiegler (2006), assume the level of $\beta$ to be common knowledge, but allow for uncertainty about the agent's extent of naiveté. We assume that the Principal is randomly matched with an agent before the employment relationship starts, and that the agent is naive with some probability $\alpha$ and sophisticated with probability $1-\alpha$, where $\alpha$ is known to the Principal. Moreover, if the Principal abstains from making an offer to the agent, or if the agent does not accept her offer, she cannot employ him in later periods.

In our setting without long-term commitment, asymmetric information about the agent's naiveté implies that, to anticipate wages, the agent not only has to form beliefs about his own future present bias, but also about the distribution of other agents' present bias.[17] We thus build upon games of present-biased players where such beliefs have been formulated (Sarafidis, 2006, Akin, 2007). There,

[16] Note that our previous analysis does not assume symmetric information in the strict sense since the naive agent does not share the Principal's belief about his own future preferences. Rather, it represents a form of non-common priors, as stated by Eliaz and Spiegler (2006).

[17] Such beliefs do not have to be specified in Principal-agent models with full commitment where the Principal can guarantee a stream of contingent future payments which only depend on the agent's own actions and thus his own present bias (see Eliaz and Spiegler, 2006; Englmaier et al., 2016; Gottlieb and Zhang, 2021).

it is commonly assumed that a sophisticated agent – just like the Principal – knows that all agents are time-inconsistent and is aware of the share of naive and sophisticated agents in the population. Thus, in our setting, the reservation wage of the sophisticated agent is always the same as under symmetric information. There is less consensus on a naive agent's beliefs. His (inter-player) expectations are crucial for the results, though, as they determine the wage he expects in the second period. We consider two polar cases: Either, the naive agent perceives all other agents to be time-consistent, or he perceives all others to be time-inconsistent in the future. Note that we always stick to the assumption that the naive agent believes the Principal to share his own perception.

If the naive agent thinks all other agent are time consistent in the future, he expects to receive the second-period offer intended for a time-consistent agent. His first-period reservation wage thus is the same as under symmetric information. As the principal cannot distinguish between naive and sophisticated agents, she faces a trade-off: Either she offers the reservation wage of the naive agent which is also accepted by the sophisticated agent, or the lower reservation wage of the sophisticated agent which is rejected by the naive agent. She is only willing to do the former if the share of naive agents is sufficiently large. In any case, the Principal is worse off than with symmetric information.

If the naive agent perceives all other agents to be time-inconsistent in the future, he expects to be offered $w_2^S$ because the principal cannot single him out. However, he overestimates his search and thus has overoptimistic beliefs about his future payoff. This implies that his reservation wage in period 1 is lower than under symmetric information, and it is lower than the sophisticated agent's first-period wage. Thus, the Principal has to choose whether to offer the reservation wage of the naive agent, which will be rejected by the sophisticated agent, or the higher reservation wage of the sophisticated agent which will also be accepted by the naive agent. She will do the former if the share of naifs in the population is sufficiently large. Either way, the Principal will always be *better off* than under symmetric information.

# 7 Discussion and Conclusion

We have shown that present-biased agents can benefit from being naive, if firms are *not* able to commit to long-term contracts. This result suggests that the extent to which an employee's naiveté can be exploited depends on the employer's commitment power. However, even if the latter is high, our insights continue to be relevant if other forces limit the benefits of commitment. For example, a firm facing uncertainty regarding its future prospects may want to retain the flexibility

to adjust its workforce and not commit itself to long-term employment contracts.

Finally, we discuss the relevance of further assumptions for our results. First, our focus on full naiveté versus full sophistication is not restrictive. To the contrary, in Section C.1 of the Appendix, we allow for partial naiveté and show that being more naive about his present bias makes the agent better off. Note that this stands in contrast to much of the IO literature where the extent of naiveté does not affect outcomes, a result that again relies on firms having commitment power. Second, we relax the assumption that the agent assesses a period's wage and effort costs simultaneously at the beginning of a period. Indeed, there is evidence that present bias only relates to very close events (O'Donoghue and Rabin, 2015). For example, Augenblick (2018) shows that the $\beta$ discounts consumption already a few hours away and that consumption more than a few days away is not included in the "present" an individual is biased towards. In Appendix Section C.2, we split each period into two steps and discount period-$t$ effort costs with $\beta$ already *at the beginning* of period $t$, after receiving $w_t$. Then, the long-run utility of a naive agent can still be larger than the long-run utility of a sophisticated agent, if $\beta$ is sufficiently large. Finally, in Section C.3 of the Appendix, we show that our results do not rely on a particular number of periods and also hold within an infinite time horizon. In the latter case, an interesting twist occurs in our endogenous bonus application. There, the Principal is able to endogenously commit to a future bonus that exceeds the myopic level for a sufficiently high discount factor $\delta$. This yields results similar to those under formal commitment as in Section 6.1.

Concluding, the role of commitment of firms who deal with present-biased individuals has so far only played a limited role in the literature. We hope that our paper can serve as one step towards a better understanding of the importance of commitment, in particular in labor markets.

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# A Omitted Proofs

**Proof of Lemma 1.** Conditional on having accepted the Principal's employment offer at the beginning of $t = 1$, the agent chooses effort to maximize,

$$-\frac{1}{2}(e_1)^2 + \left[e_1 b + w_2^{TC} - \frac{1}{2}(e_2^{TC})^2 + e_2^{TC} b\right].$$

Since effort does not affect the expected wage in the second period, the problem boils down to maximizing $-\frac{1}{2}(e_1)^2 + e_1 b$, yielding $e_1^{TC} = b$, as well as $w_1^{TC} = -\frac{1}{2}b^2 < 0$, hence $e_1^{TC} = e_2^{TC}$ and $w_1^{TC} = w_2^{TC}$. ■

**Proof of Lemma 2.** $e_1^S$ maximizes

$$-\frac{1}{2}e_1^2 + \beta\left[e_1 b + w_2 - \frac{1}{2}e_2^2 + e_2 b\right]$$

hence

$$e_1^S = \beta b.$$

It follows that $e_1^S = e_2^S = \beta b$.

Plugging in $w_2^S = -\frac{1}{2}(\beta b)^2$ and $e_2^S = \beta b$, $w_1^S$ is set to satisfy $U_1^S = w_1^S - \frac{1}{2}(e_1^S)^2 + \beta\left[e_1^S b + (1-\beta)\beta b^2\right] = 0$, hence

$w_1^S = \frac{1}{2}(e_1^S)^2 - \beta\left[e_1^S b + (1-\beta)\beta b\right] < \frac{1}{2}(e_1^S)^2 - \beta e_1^S b = -\frac{1}{2}(\beta b)^2$. ■

**Proof of Lemma 3** A fully naive agent perceives his first-period utility to be

$$\tilde{U}_1^N = w_1 - \frac{1}{2}(e_1)^2 + \beta\left[e_1 b + \tilde{w}_2^N - \frac{1}{2}(\tilde{e}_2^N)^2 + \tilde{e}_2^N b\right].$$

Making use of $\tilde{w}_2^N = \frac{1}{2}(\tilde{e}_2^N)^2 - \tilde{e}_2^N b$, this becomes

$$\tilde{U}_1^N = w_1 - \frac{1}{2}(e_1)^2 + \beta e_1 b.$$

The Lemma immediately follows. ■

**Proof of Proposition 1.** As shown in Lemmas 2 and 3, period-1 efforts of a sophisticated agent and naive agent equal

$$e_1^S = \beta b = e_1^N.$$

As shown in Lemma 1, period-1 effort of a time-consistent agent equals

$$e_1^{TC} = b.$$

Hence, the first part of the Proposition immediately follows.

As to the second part, from Lemmas 2 and 3, it follows that

$$w_1^S = -\frac{1}{2}(\beta b)^2(3-2\beta) < -\frac{1}{2}(\beta b)^2 = w_1^N$$

Moreover, $w_1^{TC} < w_1^S$ since

$$\begin{aligned} -\frac{1}{2}b^2 &< -\frac{1}{2}(\beta b)^2(3-2\beta) \\ 1 &> \beta^2(3-2\beta) \end{aligned}$$

where the second line follows from multiplying the first by $-2$ and dividing it by $b^2$. The inequality is true because the right hand side is strictly increasing in $\beta$ and $\lim_{\beta\to 1}\beta^2(3-2\beta) = 1$.

■

**Proof of Proposition 2.** Any agent's long-run realized utility is given by:

$$\hat{U}_0 = w_1 - \frac{1}{2}e_1^2 + \left[e_1 b + w_2 - \frac{1}{2}e_2^2 + e_2 b\right]$$

Evaluating the realized utility at the equilibrium contract and effort level, $w_t^{TC}, e_t^{TC}$, $t \in \{1,2\}$, of the time-consistent agent, we get

$$\hat{U}_0^{TC} = -\frac{1}{2}b^2 - \frac{1}{2}b^2 + b^2 - \frac{1}{2}b^2 - \frac{1}{2}b^2 + b^2 = 0$$

Similarly, evaluating the realized utility at the equilibrium contract and effort level, $w_t^S, e_t^S$, $t \in \{1,2\}$, of the sophisticated agent, we get

$$\begin{aligned} \hat{U}_0^S &= -\frac{1}{2}(\beta b)^2(3-2\beta) - \frac{1}{2}(\beta b)^2 + \beta b^2 - \frac{1}{2}(\beta b)^2 - \frac{1}{2}(\beta b)^2 + \beta b^2 \\ &= -(\beta b)^2(2-\beta) + \beta(2-\beta)b^2 \\ &= (2-\beta)\beta(1-\beta)b^2 > 0. \end{aligned}$$

Finally, the realized utility at the equilibrium contract and effort level, $w_t^N, e_t^N$,

$t \in \{1, 2\}$, of the naive agent, amounts to

$$\begin{aligned}\hat{U}_0^N &= -\frac{1}{2}(\beta b)^2 - \frac{1}{2}(\beta b)^2 + \beta b^2 - \frac{1}{2}(\beta b)^2 - \frac{1}{2}(\beta b)^2 + \beta b^2 \\ &= 2\beta(1-\beta)b^2 > (2-\beta)\beta(1-\beta)b^2 > 0.\end{aligned}$$

The first inequality is intuitive as the only difference between the contract of the naive and the sophisticated agent is that the sophisticated receives a smaller wage in the first period.

■

**Proof of Proposition 3 (Effort as On-the-job Search)** For the following analysis, we do not specifically analyze the Principal; a successful counteroffer increases the agent's net utility from being employed by the same amount, no matter whether he stays with the Principal or moves to another employer.

For the equilibrium outcomes in stage 2, we start with the analysis of the time-inconsistent agent. Conditional on having a per-period outside option $b_1$, the sophisticated as well as naive agent's search level maximizes

$$-\frac{1}{2}e_2^2 + \beta\left[e_2\left(\int_{b_1}^{\bar{b}} b dF(b) + b_1 F(b_1)\right) + (1-e_2)b_1\right].$$

Thus, the agent's optimal period-2 effort $e_2^k(b_1)$, $k \in \{S, N\}$, is

$$\begin{aligned}e_2^k(b_1) &= \beta\left(\int_{b_1}^{\bar{b}} b dF(b) - b_1(1-F(b_1))\right) \\ &= \beta\int_{b_1}^{\bar{b}}(b-b_1)dF(b).\end{aligned}$$

In the remainder of this proof we denote the censored expected difference between a new offer and an old offer conditional on the old offer granting payoff $b_1$ by $C(b_1) = \int_{b_1}^{\bar{b}}(b-b_1)dF(b)$. Note that $C(0) = \mathbb{E}(b)$, while $C(b_1) < \mathbb{E}(b)\ \forall b_1 > 0$. The agent only accepts an offer in period 2 if the following condition is satisfied:

$$\begin{aligned}&w_2^k - \frac{1}{2}e_2^k(b_1)^2 + \beta\left[e_2^k(b_1)\left(\int_{b_1}^{\bar{b}} b dF(b) + b_1 F(b_1)\right) + (1-e_2^k(b_1))b_1\right] \geq b_1 + \beta b_1 \\ &\Rightarrow w_2^k(b_1) = b_1 + \frac{1}{2}e_2^k(b_1)^2 - \beta e_2^k(b_1)C(b_1) = b_1 - \frac{1}{2}\left[\beta C(b_1)\right]^2.\end{aligned}$$

For the period-2 wage and effort of the time-consistent agent, it is sufficient to replace $\beta$ with 1 in the above expressions, hence $e_2^{TC}(b_1) = C(b_1)$ and $w_2^{TC}(b_1) = b_1 - \frac{1}{2}C(b_1)^2$.

Taking those values as given, it is instructive to calculate the long-run realized period-2 utility for a job offer $b_1$. Note that, for the time-consistent agent, this realized long-run utility is necessarily equal to the outside option, i.e. $2b_1$. Since the equilibrium outcomes for the naive and the sophisticated agents are the same, so is the long-run period-2 utility, namely:

$$2b_1 + \beta(1-\beta)C(b_1)^2 > 2b_1.$$

For the analysis of the equilibrium contract in period 1, we start with the **naive agent**. Since the naive agent believes to act like a time-consistent agent, i.e. $\tilde{e}_2^N(b_1) = e_2^{TC}(b_1)$ and $\tilde{w}_2^N(b_1) = w_2^{TC}(b_1)$, he also believes that his utility from employment in period 2 will be that of a time-consistent agent. Hence his perceived utility from employment in the first period is given by:

$$U_1^N = w_1 - \frac{1}{2}e_1^2 + \beta\left[e_1\mathbb{E}(2b) + (1-e_2)0\right] \tag{1}$$

$$= w_1 - \frac{1}{2}e_1^2 + e_1 2\beta\mathbb{E}(b) \tag{2}$$

Consequently, optimal search and wage for the naive agent will be:

$$e_1^N = 2\beta\mathbb{E}(b) \qquad \text{and} \qquad w_1^N = \frac{1}{2}(e_1^N)^2 - \beta e_1^N 2\mathbb{E}(b) = -\frac{1}{2}(2\beta\mathbb{E}(b))^2.$$

This leads to the following long-run realized utility from employment:

$$\begin{aligned}\hat{U}_0^N &= w_1^N - \frac{1}{2}(e_1^N)^2 + \left[e_1^N\left(\mathbb{E}(2b) + \beta(1-\beta)\mathbb{E}(C(b)^2)\right) + (1-e_1^N)\beta(1-\beta)\mathbb{E}(b)^2\right] \\ &= (1-\beta)e_1^N 2\mathbb{E}(b) + e_1^N\beta(1-\beta)\mathbb{E}(C(b)^2) + (1-e_1^N)\beta(1-\beta)\mathbb{E}(b)^2 > 0\end{aligned}$$

In contrast, the **sophisticated agent** correctly anticipates his future present bias, hence also that his long-run utility will be larger than the outside option:

$$U_1^S = w_1 - \frac{1}{2}e_1^2 + \beta\left[e_1\left(\mathbb{E}(2b) + \beta(1-\beta)\mathbb{E}(C(b)^2)\right) + (1-e_2)\beta(1-\beta)\mathbb{E}(b)^2\right] \tag{3}$$

Therefore, optimal effort of the sophisticated agent in period 1 is:

$$\begin{aligned}e_1^S &= \beta\left[\mathbb{E}(2b) + \beta(1-\beta)\mathbb{E}(C(b)^2) - \beta(1-\beta)\mathbb{E}(b)^2\right] \\ &= 2\beta\mathbb{E}(b) - \beta(1-\beta)\left[\mathbb{E}(b)^2 - \mathbb{E}(C(b)^2)\right] < 2\beta\mathbb{E}(b) = e_1^N\end{aligned}$$

Moreover, the Principal offers him the following wage:

$$\begin{aligned} w_1^S =& \frac{1}{2}(e_1^S)^2 - \beta e_1^S 2\mathbb{E}(b) - \beta \left( e_1^S \beta(1-\beta)\mathbb{E}(C(b)^2) \right. \\ & \left. + (1-e_1^S)\beta(1-\beta)\mathbb{E}(b)^2 \right) \\ <& \frac{1}{2}(e_1^S)^2 - \beta e_1^S 2\mathbb{E}(b) \end{aligned}$$

Summarizing, the Principal can push the period-1 wage of the sophisticated agent below the (negative) net surplus of search not only due to the "extra" utility of assessing period-2 search from the perspective of period 1 for the case that search has not been successful, but also for the case of receiving a better offer. Thus, the long-run realized utilty of the sophisticated agent is:

$$\begin{aligned} \hat{U}_0^S &= w_1^S - \frac{1}{2}(e_1^S)^2 + e_1^S \left[ \mathbb{E}(2b) + \beta(1-\beta)\mathbb{E}(C(b)^2) \right] + (1-e_1^S)\beta(1-\beta)\mathbb{E}(b)^2 \\ &= (1-\beta)e_1^S 2\mathbb{E}(b) + (1-\beta)e_1^S \beta(1-\beta)\mathbb{E}(C(b)^2) + (1-\beta)(1-e_1^S)\beta(1-\beta)\mathbb{E}(b)^2 \\ &< (1-\beta)e_1^S 2\mathbb{E}(b) + e_1^S \beta(1-\beta)\mathbb{E}(C(b)^2) + (1-e_1^S)\beta(1-\beta)\mathbb{E}(b)^2 \\ &< (1-\beta)e_1^N 2\mathbb{E}(b) + e_1^N \beta(1-\beta)\mathbb{E}(C(b)^2) + (1-e_1^N)\beta(1-\beta)\mathbb{E}(b)^2 = \hat{U}_0^N \end{aligned}$$

The inequality in the third line follows from $(1-\beta) < 1$ and the inequality in the second line follows from $e_1^S < e_1^N$ and:

$$2\mathbb{E}(b) + \beta\mathbb{E}(C(b)^2) > \beta\mathbb{E}(b)^2$$

as $\mathbb{E}(b) < 1$. Note that from the second line it is evident that $\hat{U}_0^S > 0$.

Finally, for the time-consistent agent we have that $e_1^{TC} = 2\mathbb{E}(b) > e_1^N$ and $\hat{U}_0^{TC} = U_1^{TC} = 0$, which establishes the proposition. ■

**Proof of Proposition 4 ($b$ as Performance-Based Bonus)** Plugging $e_1^S = e_1^N = \beta b$ and $b_1^S = b_1^N = \theta/(2-\beta)$ into an agent's first-period utility and setting it equal to zero yields

$$w_1^S = -\frac{\left(\beta\frac{\theta}{2-\beta}\right)^2}{2} - \beta^2 \left(\frac{\theta}{2-\beta}\right)^2 (1-\beta) = -\left(\beta\frac{\theta}{2-\beta}\right)^2 \left[\frac{3-2\beta}{2}\right]$$

and $w_1^N = w_2^N (= -\frac{1}{2}\left(\frac{\beta}{2-\beta}\theta\right)^2)$.

Therefore,

$$\begin{aligned}\hat{U}_0^S =& w_1^S + e_1^S b_1^S - c(e_1^S) + w_2^S + e_1^S b_2^S - c(e_1^S)\\ =& \frac{(1-\beta)\,\beta\theta^2}{(2-\beta)}\\ \hat{U}_0^N =& 2\frac{(1-\beta)\,\beta\theta^2}{(2-\beta)^2} > \hat{U}_0^S\end{aligned}$$

Profits are

$$\begin{aligned}\Pi^N =& \theta^2 \frac{(3-2\beta)\,\beta}{(2-\beta)^2}\\ \Pi^S =& \frac{\theta^2\beta\,(3-2\beta)}{(2-\beta)^2} + \beta^2\left(\frac{\theta}{2-\beta}\right)^2(1-\beta) > \Pi^N\\ \Pi^{TC} =& \theta^2,\end{aligned}$$

with,

$$\begin{aligned}&\Pi^{TC} > \Pi^S\\ \Leftrightarrow& 4 - 2\beta - \beta\,(1+\beta) > 0.\end{aligned}$$

■

**Proof of Proposition 5 (Employment Protection)** The arguments in the text deliver that

$$\begin{aligned}\tilde{w}_2^N =& \min\left\{-\frac{1}{2}(\beta b)^2, k - \frac{1}{2}b^2\right\}\\ w_2^N =& -\frac{1}{2}(\beta b)^2.\end{aligned}$$

The naive agent's perceived period-1 utility equals

$$\begin{aligned}\tilde{U}_1^N =& w_1^N - \frac{1}{2}(e_1^N)^2 + \beta\left[e_1^N b + \tilde{w}_2^N + \frac{1}{2}b^2\right]\\ =& w_1^N + \frac{1}{2}(\beta b)^2 + \beta\left(\tilde{w}_2^N + \frac{1}{2}b^2\right),\end{aligned}$$

thus

$$w_1^N = \begin{cases} -\frac{(b\beta)^2}{2} - \beta k & \text{if } k < \frac{1}{2}b^2\left(1-\beta^2\right) \\ -\frac{b^2\beta(1+\beta(1-\beta))}{2} & \text{if } k \geq \frac{1}{2}b^2\left(1-\beta^2\right). \end{cases}$$

Clearly, if $w_1^N = -b^2\beta\left(1+\beta\left(1-\beta\right)\right)/2$, the naive agent's first period wage is lower than the sophisticated agent's first-period wage, $w_1^S = -(\beta b)^2(3-2\beta)/2$. However, it is still larger than the wage of the time-consistent agent $w_1^{TC} = -\frac{1}{2}b^2$.

Moreover, since $w_2^N = -(\beta b)^2/2$ and $\hat{U}_0^N = w_1^N + w_2^N + \beta b^2\left(2-\beta\right)$,

$$\hat{U}_0^N = \begin{cases} -\beta k + 2\beta b^2\left(1-\beta\right) & \text{if } k < \frac{1}{2}b^2\left(1-\beta^2\right) \\ \frac{b^2\beta\left((2-\beta)^2-1\right)}{2} & \text{if } k \geq \frac{1}{2}b^2\left(1-\beta^2\right) \end{cases}$$

It immediately follows that, if $\hat{U}_0^N = b^2\beta\left((2-\beta)^2-1\right)/2$, it is lower than the sophisticated agent's long-run utility, $\hat{U}_0^S = (2-\beta)\beta b^2(1-\beta)$, but larger than the level of the time-consistent agent. ■

**Proof of Proposition 6 (Minimum Wage)** To begin with, note that the Principal's lack of commitment rules out the use of a firing threat to provide incentives, which might otherwise be optimal to reduce the agent's first-period rent (for the same reasons as in moral hazard problems with limited liability).

Now, recall that, without a minimum wage,

$$\begin{aligned} \tilde{w}_2^N &= -\frac{1}{2}b^2 \\ <& w_1^N = w_2^N = w_2^S = -\frac{1}{2}(\beta b)^2. \end{aligned}$$

Assume $\overline{w} \geq -\frac{1}{2}b^2$, hence $\tilde{w}_2^N = \overline{w}$.

Furthermore, recall that the naive agent's perceived period-1 utility equals

$$\begin{aligned} \tilde{U}_1^N =& w_1^N - \frac{1}{2}(e_1^N)^2 + \beta\left[e_1^N b + \left(\overline{w} + \frac{1}{2}b^2\right)\right] \\ =& w_1^N + \frac{1}{2}(\beta b)^2 + \beta\left(\overline{w} + \frac{1}{2}b^2\right), \end{aligned}$$

thus $w_1^N$ is the lowest feasible wage that satisfies $\tilde{U}_1^N \geq 0$, or

$$w_1 = \max\left\{\overline{w};\ -\left[\frac{1}{2}(\beta b)^2 + \beta\left(\overline{w} + \frac{1}{2}b^2\right)\right]\right\}.$$

Moreover,

$$\begin{aligned}\overline{w} &< -\left[\frac{1}{2}(\beta b)^2 + \beta\left(\overline{w} + \frac{1}{2}b^2\right)\right] \\ \Leftrightarrow \overline{w} &< \overline{w}^* \equiv -\frac{1}{2}\beta b^2.\end{aligned}$$

Assume this condition holds (note that $\overline{w}^* < -\frac{1}{2}(\beta b)^2$, i.e., $w_2^N > \overline{w}^*$), then

$$\begin{aligned}\hat{U}_0^N =& w_1^N - \frac{1}{2}(e_1^N)^2 + e_1^N b + w_2^N - \frac{1}{2}(e_2^N)^2 + e_2^N b \\ =& w_1^N + \beta b^2 \frac{(4-3\beta)}{2} \\ =& \frac{(3-4\beta)}{2}\beta b^2 - \beta\overline{w},\end{aligned}$$

which is decreasing in $\overline{w}$.

Now, assume $\overline{w} \geq \overline{w}^*$, hence $w_1 = \overline{w}$.

For $\overline{w} \in \left[\overline{w}^*, -\frac{1}{2}(\beta b)^2\right]$,

$$\hat{U}_0^N = \overline{w} + \beta b^2 \frac{(4-3\beta)}{2},$$

for $\overline{w} > -\frac{1}{2}(\beta b)^2$,

$$\hat{U}_0^N = 2\overline{w} + \beta b^2 (2-\beta),$$

which both are increasing in $\overline{w}$.

The results for the time-inconsistent but sophisticated and time-consistent agents follow: They are only affected by a binding minimum wage; if the minimum wage exceeds $w_1^S$ ($w_1^{TC}$), the respective first-period wages without a minimum wage, $U_1^S$ ($U_1^{TC}$) and consequently $\hat{U}_0^S$ ($\hat{U}_1^{TC}$) go up.

■

# B Asymmetric Information

We consider two alternative assumptions on the naive agent's inter-player perception, while we always stick to the assumption that the naive agent believes the Principal to share his own perception:

**Assumption 1a (Interpersonal Naiveté)** *The naive agent believes that all other agents are time-consistent in the future.*

**Assumption 1b (Interpersonal Sophistication)** *The naive agent believes that all other agents are time-inconsistent in the future.*

Assumption 1a follows Sarafidis (2006), whose analysis of a bargaining game between time-inconsistent players is based on such *interpersonal naiveté*. The presumption that a naive agent believes all other agents to be time consistent in the future as well reflects a situation in which a naive agent does not perceive himself to be distinct. The contrary, *interpersonal sophistication*, is the foundation of Assumption 1b and follows Akin (2007). It is consistent with evidence that agents are over-confident (see Santos-Pinto and de la Rosa, 2020) and perceive themselves to be less present biased than others (see Fedyk, 2018).

Equipped with these assumptions, we use the solution concept of *Naive Backwards Induction* as propagated by Sarafidis (2006). There, naive agents' optimal strategies are based on their perceptions of other agents' preferences and thus actions.[18]

In the following, we characterize potential outcomes, how they rely on the share of naive agents, $\alpha$, and on the naive agent's beliefs about the present bias of other agents as described in Assumptions 1a and 1b.

We assume that the Principal's base profits when employing an agent are $\pi$ (for example generated by actions that can be contractually specified), and that her profits when not employing an agent are $\underline{\pi} < \pi$. Besides, she only cares about wage payments. Now, the symmetric-info result that realized outcomes in period 2, as well as the optimal effort level in period 1, solely rely on $\beta$ and not the agent's naiveté also extends to the present setting with asymmetric information. Only the naive agent's period-1 belief regarding period-2 outcomes depends on the above assumptions. We start with a characterization of the results under *interpersonal naiveté*, that is, we impose Assumption 1a.[19]

**Proposition 7a** *Consider the case with asymmetric information and suppose Assumption 1a holds. Then the Principal's expected profits are smaller than with symmetric information. Moreover, there exists a threshold $\overline{\alpha} \in (0,1)$ such that:*

- *If $\alpha \geq \overline{\alpha}$, the Principal employs both types of agents.*
- *If $\alpha < \overline{\alpha}$, the Principal employs only the sophisticated agent.*

[18] More precisely, applying naive backwards induction to games of time-inconsistent and potentially naive agents usually also requires the formulation of higher order beliefs. This is not necessary in our setting, thus we refrain from doing so.

[19] Note that a separating contract in which different types of agents accept different wages in the first period does not exist because the Principal cannot commit to condition her second-period wage offer on any first-period decision. Hence, we only consider pooling equilibria, where we impose as tie-breaking rule that, whenever the Principal is indifferent between employing both types of agents and employing only one type of agent, she chooses the former.

**Proof** Recall the second-period wage and first- as well as second-period search effort of a time-inconsistent agent:

$$w_2^k = -\frac{1}{2}(\beta b)^2 < 0 \text{ and } e_2^k = e_1^k = \beta b,$$

where $k \in \{S, N\}$. Since the sophisticated agent is fully aware of the future contract terms, his reservation wage will not change compared to the symmetric information case. The same is true for the naive agent: Since he believes everyone, including himself, to be time-consistent in the future, he anticipates a wage $\tilde{w}_2^N = -\frac{1}{2}b^2$ and hence also has the same reservation wage compared to the symmetric information case, $w_1^N$.

From the symmetric information case we know that $w_1^S < w_1^N$. Thus, the Principal faces two options: Either he offers $w_1^N$ and both types of agents accept, or he offers $w_1^S$ and only the sophisticated agent accepts. Employing both types of agents in the first period is optimal if:

$$3\pi - w_1^N - w_2^k - w_3^k \geq \alpha(3\underline{\pi}) + (1-\alpha)(3\pi - w_1^S - w_2^k - w_3^k)$$

Rearranging the inequality yields:

$$\begin{aligned} \alpha(3(\pi - \underline{\pi}) - w_1^S - w_2^k &\geq w_1^N - w_1^S \\ \alpha &\geq \frac{w_1^N - w_1^S}{3(\pi - \underline{\pi}) - w_1^S - w_2^k} = \overline{\alpha} \end{aligned}$$

Clearly, $\overline{\alpha}$ is strictly larger than 0 since $w_1^S < w_1^N$. Moreover, it is strictly smaller than 1 since $w_1^N < 0$, $w_2^k < 0$ and $\pi > \underline{\pi}$.

Now, either way the Principal's expected profits are lower than under symmetric information: If $\alpha \geq \bar{\alpha}$, then she employs both agents at a wage $w_1^N$ and cannot extract the sophisticated agent's rent (but there is always a small share of sophisticated agents because $\alpha < 1$. If $\alpha < \bar{\alpha}$, she decides to extract the sophisticated agent's rent, but at the cost of missing the employment opportunity when she is matched with a naive agent. ■

If we impose *interpersonal sophistication*, that is, Assumption 1b, results are as follows:

**Proposition 7b** *Consider the case with asymmetric information and suppose Assumption 1b holds. Then the Principal's profits are larger than with symmetric information. Moreover, there exists a threshold $\overline{\overline{\alpha}} \in (0,1)$ such that:*

- *If $\alpha > \overline{\overline{\alpha}}$, the Principal employs only the naive agent.*

- *If $\alpha \leq \overline{\overline{\alpha}}$, the Principal's employs both types of agents .*

**Proof** Remember the second-period wage and first- as well as second-period search effort of a time-inconsistent agent:

$$w_2^k = -\frac{1}{2}(\beta b)^2 < 0 \text{ and } e_2^k = e_1^k = \beta b$$

where $k \in \{S, N\}$. Since the sophisticated agent is fully aware of the future contract terms, his reservation wage will not change compared to the symmetric information case. Now, however, the naive agent perceives all other agents except himself to be time-inconsistent in the future, thus he anticipates a wage $w_2^S = -\frac{1}{2}(\beta b)^2$. Despite expecting the same wage as the sophisticated agent, the naive agent's reservation wage in period 1 is lower because he anticipates a larger effort $\tilde{e}_2^N = b$:

$$\begin{aligned} U_1^N &= w_1 - \frac{1}{2}(\beta b)^2 + \beta\left[\beta b^2 - \frac{1}{2}(\beta b)^2 - \frac{1}{2}b^2 + b^2\right] \geq 0 \\ \Rightarrow \hat{w}_1^N &= -\frac{1}{2}(\beta b)^2 - \frac{1}{2}\beta(1-\beta^2)b^2 < -\frac{1}{2}(\beta b)^2 - \beta^2(1-\beta)b^2 = w_1^S \end{aligned}$$

Hence, the Principal faces again a trade-off: Either he offers $\hat{w}_1^N$ and only the naive agent accepts, or he offers $w_1^S$ and both types of agents accept. Employing only naive agents is optimal if:

$$\alpha(3\pi - w_1^N - w_2^k - w_3^k) + (1-\alpha)3\underline{\pi} > 3\pi - w_1^S - w_2^k - w_3^k$$

Rearranging the inequality yields:

$$\alpha > \frac{3(\pi - \underline{\pi}) - w_1^S - w_2^k}{3(\pi - \underline{\pi}) - w_1^N - w_2^k} = \overline{\overline{\alpha}}$$

Clearly, $\overline{\alpha}$ is strictly larger than 0. Moreover, it is strictly smaller than one since $w_1^N < w_1^S < 0$.

The Principal's profits are larger than under symmetric information in both cases. First, consider the case when $\alpha \leq \overline{\overline{\alpha}}$. Then the Principal can employ both agents at the reservation wage of the sophisticated agent $w_1^S > w_1^N$, where $w_1^N$ is the reservation wage of the naive agent under symmetric information. Clearly, she is better off than under symmetric information in expectation because she can extract the same rent from both, sophisticated and naive agent instead of only the sophisticated (and there is always a small share of naive agents because $\alpha > 0$). Now, when $\alpha > \overline{\overline{\alpha}}$, the Principal decides to exclude the sophisticated agent because

her expected profits increase *above* what would be possible by employing both type of agents at $w_1^S$. ■

# C Robustness

## C.1 Partially Naive agent

While as in the main part of the paper, utilities in the next period are discounted with $\beta$, the agent expects to discount the future with $\tilde{\beta} \in (\beta, 1)$ from the next period onward. Sections 4.1 and 4.2 have therefore covered the cases $\tilde{\beta} = \beta$ and $\tilde{\beta} = 1$.

All realized outcomes in the second period, as well as effort in the first, are independent of $\tilde{\beta}$, hence are equivalent to the respective values with a sophisticated or fully naive agent. From the perspective of period 1, a partially naive agent expects to maximize $-(e_2)^2/2 + \tilde{\beta} e_2 b$ in period $t = 2$. This implies that the partially naive agent expects to choose an effort level $\tilde{e}_2^{PN}$ which is characterized by

$$\tilde{e}_2^{PN} = \tilde{\beta} b.$$

Furthermore, in period 1 the partially naive agent anticipates a second-period wage offer

$$\tilde{w}_2^{PN} = -\frac{1}{2}(\tilde{\beta} b)^2.$$

As before, the agent's first-period reservation wage is determined by his perceptions of future outcomes, not their true realizations. The following Lemma focuses on $\beta \geq \frac{1}{2}$ which is in line with empirical estimates of present bias which generally find values of $\beta$ that are not too small (see Imai et al. (2020))

**Lemma 4** *Assume the agent is partially naive with $\tilde{\beta} \in (\beta, 1)$ and $\beta \geq \frac{1}{2}$. Then,*

- *the period-1 wage that is lower than the period-2 wage, i.e. $w_1^{PN} = -\frac{1}{2}(\beta b)^2 - \beta\tilde{\beta}(1-\tilde{\beta})b^2 < w_2^{PN}$.*
- *A higher extent of naiveté increases the period-1 wage, i.e. $\frac{\partial w_1^{PN}}{\partial \tilde{\beta}} > 0 \ \forall \tilde{\beta} \in (\beta, 1)$. Moreover, $\lim_{\tilde{\beta} \to \beta} w_1^{PN} = w_1^S$ and $\lim_{\tilde{\beta} \to 1} w_1^{PN} = w_1^N$.*

**Proof** A partially naive agent perceives his first-period utility to be (already taking into account $\tilde{w}_2$ and $\tilde{e}_2$)

$$\tilde{U}_1^{PN} = w_1 - \frac{1}{2}(e_1)^2 + \beta \left[ e_1 b + \tilde{\beta}(1-\tilde{\beta}) b^2 \right].$$

Since, as before, the optimal effort level is given by $e_1^{PN} = \beta b$, the reserve wage is given by

$$\begin{aligned} w_1^{PN} &= \frac{1}{2}(e_1^{PN})^2 - \beta\left[e_1^{PN} b + \tilde{\beta}(1-\tilde{\beta})b^2\right] \\ &= -\frac{1}{2}(\beta b)^2 - \beta\tilde{\beta}(1-\tilde{\beta})b^2 < -\frac{1}{2}(\beta b)^2 = w_2^{PN} \end{aligned}$$

and

$$\frac{dw_1^{PN}}{d\tilde{\beta}} = -\beta(1-2\tilde{\beta}) > 0 \text{ since } \tilde{\beta} > \beta \geq \frac{1}{2}$$

■

The relationship between period-1 and period-2 wages is the same as for the sophisticated agent. Because the difference between the partially naive agent's current preference over future effort and the perceived future effort is monotonously and continuously decreasing in the extent of navieté, so is his expected wage offer and therefore his current reservation wage.[20]

Finally, when comparing a partially naive agent to a time-consistent agent in terms of long-run realized utility, a similar relationship emerges. Corollary 1 directly follows from combining Lemma 4 and Proposition 2, again focusing on the arguably more realistic case $\beta \geq \frac{1}{2}$.

**Corollary 1** *Assume the agent is partially naive with $\tilde{\beta} \in (\beta, 1)$ and $\beta \geq \frac{1}{2}$. $\hat{U}_0^{TC} = 0 < \hat{U}_0^S < \hat{U}_0^{PN} < \hat{U}_0^N$. Moreover, $\hat{U}_0^{PN}$ is strictly increasing in $\tilde{\beta}$.*

A larger extent of navieté makes a present-biased agent better off. This result differs from the behavioral IO literature. There, a discontinuity at $\tilde{\beta} = \beta$ is often observed. If firms have commitment power, (partially) naive agents have to "pay" for a reversion of their actions, and for their payoffs it only matters that a misperception of actions happens, not its extent.

## C.2 Discounting Between Each Step

We split each period into two steps and discount period-$t$ effort costs with $\beta$ already *at the beginning* of period $t$, after receiving $w_t$. For example, at the beginning of period 2 an agent's utility is

$$w_2 + \beta\left[-\frac{1}{2}e_2^2 + e_2 b\right],$$

[20]Note that, allowing for $\beta < \frac{1}{2}$ would only affect the comparative static result in Lemma 4: The wage of the partially naive agent decreases in $\tilde{\beta}$ for $\tilde{\beta} \in (\beta, \frac{1}{2})$, but then increases for $\tilde{\beta} \in [\frac{1}{2}, 1)$.

whereas at the time when the effort choice is made his utility amounts to

$$-\frac{1}{2}e_2^2 + \beta e_2 b.$$

Then, the naive agent also overestimates his effort in $t = 2$ when assessing the period's wage offer. He expects to exert effort $\tilde{e}_2^N = b$ and accepts a wage satisfying $w_2^N + \beta\left[-\tilde{e}_2^2/2 + \tilde{e}_2 b\right] = 0$. Thus, the naive agent's *realized* period-2 utility now is negative from the perspective of period 2 (whereas the sophisticated agent still only accepts a wage that yields a non-negative utility). It continues to be positive from the perspective of earlier periods, though. In total, discounting future payoffs after each step reduces the naive agent's utility compared to our main model because he overestimates his effort also at the beginning of a period and thus accepts a lower wage. Still, it is possible that his long-run utility exceeds the level of the sophisticated agent, which is captured in the following proposition.

**Proposition 7** *Assume that time-inconsistent agent discounts future payoffs also after receiving $w_t$ and before exerting effort. Then, the long-run utilities of both, sophisticated and naive agent, are positive. The naive agent's long-run utility exceeds the sophisticated agent's long-run utility if $\beta$ is sufficiently large.*

**Proof** We first compute outcomes for the *naive* agent. The discussion in the text implies that $e_2^N = \beta b$, $\tilde{e}_2^N = b$, thus he is willing to accept a wage $w_2^N = -\beta b^2/2$, whereas (in period 1) he anticipates a wage $\tilde{w}_2^N = -b^2/2$. Equivalently, $e_1^N = \beta b$ and $\tilde{e}_1^N = b$.

His first-period wage is determined by

$$w_1^N + \beta\left(\tilde{e}_1^N b - \frac{\left(\tilde{e}_1^N\right)^2}{2} + \tilde{w}_2^N + \tilde{e}_2^N b - \frac{\left(\tilde{e}_2^N\right)^2}{2}\right) = 0,$$

therefore

$$w_1^N = -\beta\frac{b^2}{2},$$

and

$$\begin{aligned}\hat{U}_0^N =& w_1^N + e_1^N b - \frac{\left(e_1^N\right)^2}{2} + w_2^N + e_2^N b - \frac{\left(e_2^N\right)^2}{2}\\ =& \beta b^2\left(1-\beta\right) > 0.\end{aligned}$$

Next, we compute outcomes for the *sophisticated* agent. His effort levels are

$e_2^S = e_1^S = \beta b$, his second-period wage solves $w_2^S + \beta\left(-(e_2^S)^2/2 + e_2^S b\right) = 0$, thus

$$w_2^S = -\frac{2-\beta}{2}(\beta b)^2 < 0.$$

Note that $w_2^S > w_2^N = -\beta b^2/2$.

His first-period wage is given by $w_1 + \beta\left(-(e_1^S)^2/2 + e_1^S b - \frac{2-\beta}{2}(\beta b)^2 - (e_2^S)^2/2 + e_2^S b\right) = 0$, thus

$$w_1^S = -\frac{(2-\beta)^2}{2}(\beta b)^2 < w_2^S.$$

Therefore, his long-run utility equals

$$\begin{aligned}\hat{U}_0^S =& w_1^S + e_1^S b - \frac{\left(e_1^S\right)^2}{2} + w_2^S + e_2^S b - \frac{\left(e_2^S\right)^2}{2} \\ =& \beta b^2 \frac{(2-\beta)^2(1-\beta)}{2} > 0.\end{aligned}$$

It follows that

$$\begin{aligned}&\hat{U}_0^N > \hat{U}_0^S \\ \Leftrightarrow& 2 > (2-\beta)^2\end{aligned}$$

which holds if and only if $\beta$ is sufficiently large. ■

## C.3 Infinite Time Horizon

Our setup is as before, only that we introduce "standard" exponential discounting between periods, captured by the discount factor $\delta$.[21]

Then, an agent's utility in a period $t$ equals

$$U_t = w_t - \frac{e_t^2}{2} + \beta\delta\left(e_t b + \hat{U}_{t+1}\right),$$

with

$$\hat{U}_{t+1} = \sum_{\tau=t+1}^{\infty} \delta^{\tau-(t+1)}\left(w_\tau - \frac{e_\tau^2}{2} + \delta e_\tau b\right).$$

As before, effort is only determined by the chance to obtain next period's bonus, therefore

$$e = \beta\delta b$$

[21] without such additional discounting, future payoffs would not be bounded.

in every period $t$. The naive agent, however, expects to exert effort

$$\tilde{e}^N = \delta b$$

in any future period.

For the following, note that stationary outcomes in which wages are the same in every period are optimal with an exogenous benefit $b$.[22] Thus we can omit time subscripts.

Now, the *naive* agent's misperception of his future time preferences induces him to believe that his future wage $\tilde{w}$ satisfies

$$\tilde{U}^N = \frac{\tilde{w}^N - \frac{\left(\tilde{e}^N\right)^2}{2} + \delta \tilde{e}^N b}{1-\delta} = 0,$$

and

$$\tilde{w}^N = -\frac{(\delta b)^2}{2}.$$

Therefore, in any period he anticipates a continuation utility of zero and does not accept a wage below

$$w^N = -\frac{(\beta \delta b)^2}{2},$$

which yields a long-run utility

$$\hat{U}^N = \frac{\beta \left(\delta b\right)^2 (1-\beta)}{1-\delta}.$$

The *sophisticated* agent correctly anticipates future outcomes. Therefore, his wage equals

$$w^S = -\frac{\left(\beta \delta b\right)^2}{2} \frac{1+\delta\left(1-\beta\right)}{1-\delta\left(1-\beta\right)},$$

which yields a long-run utility

$$\hat{U}^S = \frac{\beta \left(\delta b\right)^2 (1-\beta)}{1-\delta\left(1-\beta\right)} < \hat{U}^N.$$

All this implies that trade-offs do not change once the time horizon is infinite (or finite with any number of periods). The naive agent still underestimates his future wage, which makes it impossible for the Principal to extract the extra rent

[22] Realized and perceived future wages still differ for the naive agent; stationarity then means that all perceived future wages are the same, as well as all realized wages.

stemming from his time inconsistency, and is better off in the long run. Finally, a *time-consistent* agent's long-run utility is obtained by taking $\hat{U}^S$ and setting $\beta = 1$, thus equals zero.

Regarding our applications, an infinite time horizon would not affect the trade-offs associated with on-the-job search. With an endogenous bonus and a sophisticated agent, however, the Principal is able to *endogenously* commit to a future bonus that exceeds the myopic level. In Section 6.1, we have shown that this possibility exists with formal commitment, raising the extra rent of the agent's future time inconsistency and thus increasing profits and the agent's long-run utility. Without formal commitment but an infinite time horizon and a sufficiently high discount factor $\delta$, such an outcome can be sustained in an equilibrium in which any deviation is punished by a reversion to the myopic optimum in all future periods.[23] A formal derivation of such an equilibrium can be found in Section D.2 of Appendix B.

# D Supplementary Appendix

## D.1 Endogenous Bonus and Employment Protection

Introducing (limited) commitment has further implications for our application of an endogenous bonus because the Principal can also use her commitment to already set second-period incentives in the first period. Here, we first focus on the sophisticated agent. For him, the Principal might want to commit to a higher second-period bonus because higher effort increases the extra rent of second-period effort from the perspective of period 1. To see that, let us first assume that the Principal can fully commit to second-period outcomes, and in particular the bonus $b_2^S$. Then, it would still be optimal to set $w_2^S$ to fully extract the second-period rent, i.e., $w_2^S = -\left(\beta b_2^S\right)^2/2$. Moreover, first-period bonus and effort would be the same as without commitment, thus $U_1^S = w_1^S + \frac{1}{2}\left(\frac{\beta\theta}{(2-\beta)}\right)^2 + \left(\beta b_2^S\right)^2(1-\beta)$, which delivers

$$w_1^S = -\frac{1}{2}\left(\frac{\beta\theta}{(2-\beta)}\right)^2 - \left(\beta b_2^S\right)^2(1-\beta).$$

Therefore, the Principal maximizes

$$\Pi_1 = \frac{\beta\theta^2}{2(2-\beta)} + \left(\beta b_2^S\right)^2\frac{(3-2\beta)}{2} + \beta b_2^S\left(\theta - b_2^S\right),$$

[23] A similar mechanism is found in Fahn and Hakenes (2019), who show that teamwork allows time-inconsistent (and sophisticated) individuals to overcome their self-control problems.

and the optimal $b_2^S$ equals

$$b_2^S = \frac{\theta}{(2-\beta\,(3-2\beta))}.$$

This is larger than $\theta/\,(2-\beta)$, the second-period bonus without commitment.

All this yields

**Proposition 8** *Assume that, with an endogenous bonus, the Principal can commit to future contracts but deviate at cost $k$. With a sophisticated agent, she offers a second-period bonus which exceeds the optimal myopic level. Moreover, the Principal's long-run profits and the agent's long-run utility (weakly) increase in $k$*

**Proof:**

We have shown that the Principal would like to commit to a second-period bonus which exceeds the myopically optimal level, $\theta/(2-\beta)$. Generally, a bonus $b_2^S$ can only be credibly promised if

$$\beta b_2^S(\theta - b_2^S) + \frac{\left(\beta b_2^S\right)^2}{2} \geq \frac{\beta\theta^2}{2\,(2-\beta)} - k,$$

where $\pi_2 = \beta b_2^S(\theta - b_2^S) + \left(\beta b_2^S\right)^2/2$ are second-period profits for a general bonus, and $\beta\theta^2/\,[2\,(2-\beta)]$ is the maximum level of second-period profits.

If the bonus $b_2^S = \theta/\,(2-\beta\,(3-2\beta))$ (which is the optimal long-term bonus, as derived in the main text) satisfies this condition, it is offered. Otherwise, the bonus is determined by the binding condition, with

$$\frac{db_2^S}{dk} = -\frac{1}{\beta(\theta - 2b_2^S) + \beta^2 b_2^S} > 0,$$

where the inequality follows from the denominator being negative for $b_2^S > \theta/(2-\beta)$.

It also follows that long-term profits increase in $k$ as long as $b_2^S < \theta/\,(2-\beta\,(3-2\beta))$.

To determine the agent's long-run utility, we plug

$$w_1^S = -\frac{1}{2}\left(\frac{\beta\theta}{(2-\beta)}\right)^2 - \left(\beta b_2^S\right)^2(1-\beta)$$

into

$$\begin{aligned}\tilde{U}_0^S =& w_1^S + \frac{\beta\theta^2}{2\,(2-\beta)} + \beta\left(b_2^S\right)^2(1-\beta)\\ =& \frac{\beta\theta^2\,(1-\beta)}{(2-\beta)^2} + \beta\left(b_2^S\right)^2(1-\beta)^2.\end{aligned}$$

This increases in $b_2^S$ and consequently in $k$ (as long as $b_2^S < \theta/(2-\beta(3-2\beta))$). ■

A higher extra rent of second-period effort from the perspective of the first period benefits the Principal who can extract it. It also benefits the agent, though, because the Principal's extraction is only complete from the perspective of the first, not from the perspective of earlier periods (where there is no discounting between periods 1 and 2).

Now, we move on and analyze the optimal long-term contract for the naive agent if the Principal has limited commitment. Then, as before she would like to commit to a higher second-period wage and consequently reduce $w_1^N$. She will not offer the perceived surplus-maximizing bonus $b_2 = \theta$, though, and instead *not* specify a second-period bonus at all (or offer the actually optimal level $b_2^N = \theta/(2-\beta)$). This is because the agent anticipates a second-period bonus $\tilde{b}_2^N = \theta$ *anyway*, thus expects any lower bonus to be renegotiated upwards (to which she would agree because a higher bonus increases the agent's rent). If, however, the Principal specified $\tilde{b}_2^N = \theta$ already in the first-period contract, she would have to compensate the agent (or pay $k$) if she wanted to reduce $b_2^N$ to the actually optimal level $\theta/(2-\beta)$. Thus, the size of $k$ determines outcomes as specified by the following proposition.

**Proposition 9** *Assume that, with an endogenous bonus, the Principal can commit to future contracts but deviate at cost $k$. With a naive agent, she offers a second-period wage that exceeds the profit-maximizing level as perceived by the agent's first-period self. The Principal's profits (weakly) increase and the agent's long-run utility (weakly) decrease in $k$. If $k$ is sufficiently large, the agent's long-run utility is zero.*

**Proof:**

First-period bonus and effort levels are as without commitment. Moreover, let us assume that, in the first period, the Principal does not promise a second-period bonus $\tilde{b}_2^N$ (which is weakly optimal), only a wage $\tilde{w}_2^N$. Then, the agent expects $\tilde{b}_2^N = \theta$ to be offered at the beginning of the second period (and anticipates an effort choice $\tilde{e}_2^N = \theta$). Thus, for a given (credible) $\tilde{w}_2^N$, the agent's perceived first-period utility equals

$\tilde{U}_1^N = w_1^N + \frac{1}{2}\left(\frac{\beta\theta}{(2-\beta)}\right)^2 + \beta\left(\tilde{w}_2^N + \frac{\theta^2}{2}\right)$, thus

$$w_1^N = -\frac{1}{2}\left(\frac{\beta\theta}{(2-\beta)}\right)^2 - \beta\left(\tilde{w}_2^N + \frac{\theta^2}{2}\right),$$

and the Principal's profits are

$$\begin{aligned}\Pi_1 =& e_1^N\left(\theta - b_1^N\right) - w_1^N + e_2^N\left(\theta - b_2^N\right) - w_2^N\\ =& \beta\theta^2\frac{8-7\beta+\beta^2}{2\left(2-\beta\right)^2} + \beta\tilde{w}_2^N - w_2^N,\end{aligned}$$

where we have already taken into account that $b_2^N = \theta/\left(2-\beta\right)$ and $e_2^N = \beta b_2^N = \beta\theta/\left(2-\beta\right)$. As in the case with exogenous bonus, the Principal would like to commit to the wage she will eventually offer and that just extracts the second period surplus, i.e., to $\tilde{w}_2 = w_2^N = -\frac{(\beta\theta)^2}{2(2-\beta)^2}$. However, she is restricted by the condition $\tilde{w}_2^N \leq k - \frac{\theta^2}{2}$, where $-\theta^2/2$ is the wage the agent would expect without any commitment. All this implies

$$\begin{aligned}\tilde{w}_2^N =& \min\left\{-\frac{\left(\beta\theta\right)^2}{2\left(2-\beta\right)^2}, k - \frac{\theta^2}{2}\right\}\\ w_2^N =& -\frac{\left(\beta\theta\right)^2}{2\left(2-\beta\right)^2}.\end{aligned}$$

Thus, if $k \geq \frac{2(1-\beta)}{(2-\beta)^2}\theta^2$ and $\tilde{w}_2^N = w_2^N = -\frac{(\beta\theta)^2}{2(2-\beta)^2}$,

$$\Pi_1 = \beta\theta^2\frac{8-7\beta+\beta^2}{2\left(2-\beta\right)^2} + \frac{\left(\beta\theta\right)^2}{2\left(2-\beta\right)^2}\left(1-\beta\right)$$

and

$$\begin{aligned}\hat{U}_0^N =& \frac{\beta^2\theta^2}{2}\frac{\left(1-\beta\right)}{\left(2-\beta\right)^2} - \beta\tilde{w}_2^N + w_2^N\\ =& 0.\end{aligned}$$

Otherwise, $\tilde{w}_2^N = k - \frac{\theta^2}{2}$, and

$$\begin{aligned}\Pi_1 =& \beta\theta^2\frac{8-7\beta+\beta^2}{2\left(2-\beta\right)^2} + \beta\tilde{w}_2^N - w_2^N\\ =& \beta\theta^2\frac{8-6\beta+\beta^2}{2\left(2-\beta\right)^2} - \beta\frac{\theta^2}{2} + \beta k,\end{aligned}$$

as well as

$$\begin{aligned}\hat{U}_0^N =& \frac{\beta^2\theta^2}{2}\frac{(1-\beta)}{(2-\beta)^2} - \beta\tilde{w}_2^N + w_2^N \\ =& \beta\theta^2\frac{2\,(1-\beta)}{(2-\beta)^2} - \beta k.\end{aligned}$$

■

As with an exogenous bonus, the agent is harmed by a higher termination cost $k$ because those allow the Principal to credibly promise a higher second-period wage and consequently to reduce $w_1^N$. In addition, the agent overestimates second-period incentives and his second-period effort which, for a given expected second-period wage $\tilde{w}_2^N$, allows the Principal to reduce $w_1^N$ even further. If $k$ is sufficiently high such that the Principal can already commit to the actually paid second-period wage, the wrong anticipation of a high second-period rent lets the agent's long-run utility completely vanish.

## D.2 Endogenous Bonus and Infinite Time Horizon

Before deriving an equilibrium in which the Principal endogenously commits to a bonus exceeding the myopic optimum, note that it can be shown that, for any number of periods in a *finite* time horizon, the Principal would set the bonus $b = \theta/(2-\beta)$ in every period, resulting in effort $e = \beta\delta\theta/\,(2-\beta)$. These are the same values as in the main part, incorporating the additional exponential discounting. This holds for the naive agent who does not anticipate any future rent anyway, but also for the sophisticated agent. There, any promise by the Principal to pay a higher bonus in future periods would not be credible because of a standard backwards induction argument: In the last period, there is a unique equilibrium with $b = \theta/(2-\beta)$, thus also in the second to last period, and so on.

With an infinite time horizon, an equilibrium with $b = \theta/(2-\beta)$ naturally exists as well. However, we now show that there exists another equilibrium in which the Principal can increase her long-run utility by promising a higher bonus in all periods $t \geq 2$. Such an equilibrium could be sustained by the following strategy: The Principal offers $b > \theta/(2-\beta)$ in every period $t \geq 2$, together with a wage that completely extracts the agent's rent from today's perspective. If the Principal deviates in any period $t$, she offers the spot bonus $\theta/\,(2-\beta)$ forever thereafter, which is incorporated by today's wage.

**Proposition 10** *With an infinite time horizon and an endogenous bonus, an equilibrium with the following characteristics exists:*

- *For the sophisticated agent, the Principal offers the myopic bonus $b_1^S = \theta/(2-\beta)$ in the first period, and a bonus $b^S > b_1^S$ in all subsequent periods. The agent's long-run utility is higher than if the myopic bonus is paid in every period.*
- *For the naive agent, the Principal offers the myopic bonus $b^N = \theta/(2-\beta)$ in every period. The naive agent's long-run utility can be larger or smaller than the sophisticated agent's long-run utility.*

**Proof:**

We start with the sophisticated agent and first derive a bonus $b_1^S$ for the first and a bonus $b^S$ for all future periods (together with optimal wages $w_1^S$ and $w^S$) which maximize the Principal's first-period profit stream. Then, we show that promising these profit-maximizing bonuses is credible if and only if $\delta$ is sufficiently large.

In any future period, effort equals $e^S = \beta\delta b^S$, thus the wage $w^S$ solves

$$\begin{aligned} & w^S - \frac{\left(e^S\right)^2}{2} + \beta\delta\left(e^S b^S + \frac{w^S - \frac{\left(e^S\right)^2}{2} + \delta e^S b^S}{1-\delta}\right) = 0 \\ \Rightarrow & w^S = -\frac{\left(\beta\delta b^S\right)^2}{2}\left(\frac{1+\delta(1-\beta)}{1-\delta(1-\beta)}\right) \end{aligned}$$

The agent's first period utility equals

$$\begin{aligned} U_1^S =& w_1^S - \frac{\left(e_1^S\right)^2}{2} + \beta\delta\left(e_1^S b_1^S + \frac{w^S - \frac{\left(e^S\right)^2}{2} + \delta e^S b^S}{1-\delta}\right) \\ =& w_1^S + \frac{\left(\beta\delta b_1^S\right)^2}{2} + \delta\frac{\left(\beta\delta b^S\right)^2(1-\beta)}{(1-\delta(1-\beta))} = 0 \end{aligned}$$

For later use, note that the Principal's long-run utility is

$$\begin{aligned} \hat{U}^S =& w_1^S - \frac{\left(e_1^S\right)^2}{2} + \delta\left(e_1^S b_1^S + \frac{w^S - \frac{\left(e^S\right)^2}{2} + \delta e^S b^S}{1-\delta}\right) \\ =& \beta\left(\delta b_1^S\right)^2(1-\beta) + \left(\delta b^S\right)^2\frac{\delta\beta(1-\beta)^2}{(1-\delta(1-\beta))}, \end{aligned}$$

which is strictly positive and increasing in $b^S$ (as well as in $b_1^S$).

In the next step, we compute the levels of $b_1^S$ and $b^S$ that maximize the Principal's profits,

$$\begin{aligned}\Pi_1 =& -w_1^S + e_1^S\delta\left(\theta - b_1^S\right) + \delta\frac{e\delta\left(\theta - b^S\right) - w^S}{1-\delta}\\ =&\beta\delta^2 b_1^S\left(\theta - b_1^S\frac{2-\beta}{2}\right) + \delta\frac{\beta\delta^2 b^S\left(\theta - b^S\right) + \frac{\left(\beta\delta b^S\right)^2[2(1-\beta)+1-\delta(1-\beta)]}{2(1-\delta(1-\beta))}}{1-\delta}.\end{aligned}$$

First-order conditions yield

$$\begin{aligned}b_1^S =& \frac{\theta}{(2-\beta)}\\ b^S =& \frac{\theta\left(1-\delta\left(1-\beta\right)\right)}{(2-\beta)\left(1-\delta\left(1-\beta\right)\right) - 2\beta\left(1-\beta\right)},\end{aligned}$$

with $b^S > b_1^S$.

Promising $b^S$ is credible if a deviation to the myopic optimal $\tilde{b} = \theta/(2-\beta)$ is not optimal. Taking $e = \beta\delta b$ into account, this holds if

$$\frac{\beta\delta^2 b^S\left(\theta - b^S\right) - w^S}{1-\delta} \geq \frac{\beta\delta^2\tilde{b}\left(\theta - \tilde{b}\right) - \tilde{w}}{1-\delta}, \tag{4}$$

where $\tilde{w}$ is set to keep the agent's utility at zero.

It can be shown that condition (4) holds for $\delta$ sufficiently close to 1, and is violated if $\delta$ is small. If it is violated, $b^S$ is determined by the binding constraint (4). In any case, $b^S \geq \frac{\theta(1-\delta(1-\beta))}{2(1-\delta(1-\beta))-\beta(1+\delta(1-\beta))}$, which is the bonus that maximizes $\beta\delta^2 b\left(\theta - b\right) - w$, and thus exceeds $b_1^S$.

For the naive agent, a contract that differs from the myopic optimum would only be possible if a deviation generated lower continuation profits. Since the Principal cannot do worse than the myopic optimum, her continuation profits after a deviation, assessed from the perspective of the naive agent, would involve the optimal contract for the time-consistent agent, with a bonus $\tilde{b}^N = \theta$ and a wage $\tilde{w}^N = -\left(\delta\theta\right)^2/2$. No other contract could generate higher profits and would be accepted by the naive agent's perceived future self, thus the optimal continuation contract (from the perspective of the naive agent) after every history would be the myopic optimum. But this implies that no other contract than the myopic optimum can be offered in any period.

Finally, plugging optimal bonuses into an agent's long-run utility yields

$$
\begin{aligned}
\hat{U}^N =& \frac{\beta\left(\frac{\delta\theta}{(2-\beta)}\right)^2(1-\beta)}{1-\delta} \\
\hat{U}^S =& \beta\left(\frac{\delta\theta}{(2-\beta)}\right)^2(1-\beta) + \left(\delta b^S\right)^2 \frac{\delta\beta(1-\beta)^2}{(1-\delta(1-\beta))} \\
\Rightarrow& \hat{U}^N \geq \hat{U}^S \\
\Leftrightarrow& 4\beta(1-\beta)^2 \\
&\geq (2-\beta)(1-\delta(1-\beta))(2-3\beta).
\end{aligned}
$$

This holds for $\beta$ sufficiently large (then, the right hand side is negative), and is violated if $\beta$ is small.

■

With the sophisticated agent, the Principal can use the infinite time horizon to credibly promise a bonus which exceeds the myopic optimum. This increases the agent's extra rent from his future time inconsistency from which both, Principal and agent, benefit in the long run. The promise is credible (in every period, the Principal would actually like to offer $b_1^S$ in the present and $b^S$ in the future) if a deviation is not too tempting. In the proof to Proposition (10), we show that a $b^S > \theta/(2-\beta)$ can always be implemented. Whether this also holds for the bonus maximizing the Principal's long-run profits depends on the discount factor $\delta$. Only if it is high enough, a permanent reversion to the myopic optimum in the future is sufficiently unattractive for a deviation today being deterred.