DEPARTMENT OF ECONOMICS
JOHANNES KEPLER UNIVERSITY OF LINZ

Opportunism Problems of Colluding Manufacturers

by

Jana GIESELMANN
Matthias HUNOLD
Johannes MUTHERS
Alexander RASCH

Working Paper No. 2118
December 2021
(Revised Nov. 2022)

Johannes Kepler University of Linz
Department of Economics
Altenberger Strasse 69
A-4040 Linz - Auhof, Austria
www.econ.jku.at

johannes.muthers@jku.at

Opportunism Problems of Colluding Manufacturers*

Jana Gieselmann† Matthias Hunold‡ Johannes Muthers§ Alexander Rasch¶

November 2, 2022

Abstract

With two exclusive manufacturer-retailer pairs and secret contracting, we show that manufacturer collusion can be infeasible. Crucially, we establish that the stability of collusion depends on retailers' beliefs about their competitor's wholesale price offers. We demonstrate that beliefs that depend on the history of wholesale prices can enable collusion. With a new set of beliefs that match the manufacturers' trigger strategies, collusion is even renegotiation-proof under a novel condition of opportunism-proofness. We show that these beliefs, however, are too inflexible to allow for the formation of manufacturer collusion, and we introduce *adaptive* beliefs that allow for successful cartel formation.

JEL classification: C73, D43, L13, L41, L81.

Keywords: opportunism, credible punishment, cartel formation, manufacturer collusion, vertical relations, renegotiation-proof, secret contracting.

---

*We thank Carmen García Galindo, Joe Harrington, Paul Heidhues, Jeanine Miklós-Thal, Hans-Theo Normann, Anja Prummer, Christopher Snyder, and participants at the EARIE 2022 in Vienna, the Jornadas Industrial 2022 at Las Palmas de Gran Canaria, the annual meeting of the Austrian Economic Association 2022 in Vienna, and at seminars in Duesseldorf and Linz for very helpful comments and suggestions. Jana Gieselmann gratefully acknowledges financial support from the Graduate Programme Competition Economics (GRK 1974) of the German Research Foundation (DFG).

†Duesseldorf Institute for Competition Economics (DICE), University of Duesseldorf; Universitaetsstr. 1, 40225 Duesseldorf, Germany; e-mail: gieselmann@dice.hhu.de.

‡University of Siegen, Unteres Schloss 3, 57068 Siegen, Germany; e-mail: matthias.hunold@uni-siegen.de.

§Corresponding author. Johannes Kepler University Linz, Altenberger Straße 69, 4040 Linz, Austria; e-mail: johannes.muthers@jku.at

¶Duesseldorf Institute for Competition Economics (DICE), University of Duesseldorf; Universitaetsstr. 1, 40225 Duesseldorf, Germany; e-mail: rasch@dice.hhu.de; and Leibniz Centre for European Economic Research (ZEW).

1 Introduction

Manufacturers that secretly start an illegal cartel need to increase wholesale prices for initially often unsuspecting downstream retailers. These retailers may, however, refuse to accept higher prices for the fear of being outcompeted by fellow downstream firms that (continue to) receive better offers. While discussed and documented in practice,[1] this obstacle to collusion is hitherto unmodelled. Indeed, two interrelated economic issues arise in this context. First, if downstream firms are unaware, how do their beliefs about wholesale offers to competitors develop over time as they get initially (potentially) surprising offers? Second, could an upstream firm attempt to exploit the anticipation of possible collusion to increase prices – either in a collusive or a non-collusive environment. Indeed, it is well known that even an upstream monopolist can face difficulty to exploit its monopoly power due to an opportunism problem, yet the implications hereof for collusion have not been studied so far.

Formally, we consider an infinitely repeated pricing game that features private contracting within each of two exclusive manufacturer-retailer pairs. From the literature we know that the market outcome in a static game with a monopolistic manufacturer depends on the beliefs that retailers have about the rival retailer's contract. Similarly, in the presence of a cartel, the market outcome depends on retailers' beliefs in the supergame. The equilibrium concept of perfect Bayesian equilibria requires that retailers' beliefs are correct on the equilibrium path. Following a deviation, however, the market outcome in the current period depends on the retailers' beliefs. In line with the literature, we focus on *within-period* beliefs that are either passive or symmetric and study different ways in which current beliefs depend on behavior in previous periods.

Similarly, in a repeated game in which manufacturers may collude, retailers' beliefs are key for their ability to do so. We distinguish how retailers' beliefs may react dynamically to the observed actions in previous periods. First, we consider beliefs that do not react to past behavior. For these beliefs, we find that manufacturers employing grim-trigger strategies are incapable of sustaining *any* price above the competitive price. In this sense, the opportunism problem of a monopoly manufacturer carries over to render self-sustaining collusion infeasible. A colluding manufacturer is unable to commit that the other manufacturer will set the same contract, such that unsuspecting retailers fear that their rival may receive a better offer by their manufacturer. Hence, in contrast to a monopolist, it is not the lack of own commitment that drives the result, but the lack of commitment by the other manufacturer. We then turn to beliefs that adapt to observed past behavior of the manufacturer. First, we consider beliefs that (correctly) anticipate the manufacturers' trigger strategies. Implicitly thus, the retailers

[1] There is evidence from cartel cases in which manufacturers failed to achieve a retail price increase absent explicit communication by the manufacturers with the retailers. For example, in the German coffee cartel, the coffee roasters only sustained higher wholesale prices after coordinating a retail price increase with retailers (Holler and Rickert (2021)). Further cases include Anheuser Busch (beer), Haribo (gummi bears), Ritter (chocolate), and Melitta (coffee); last access 2020/02/03. The same issue to convince retailers to accept higher wholesale prices appears to be the underlying problem in a number of so-called hub-and-spoke cartels. See, for example, Harrington (2018) for a description of the cheese cartel in the UK.

have an understanding not only of the fact that manufacturers collude, but also of how they collude. For such beliefs capturing perfect anticipation of the collusive behavior, collusion at industry-profit-maximizing prices may be stable with respect to unilateral deviations. At the same time, however, collusion may not be *opportunism-proof* because the manufacturers may have a joint incentive to lower prices, similarly to the monopolist in Hart et al. (1990). We formally define opportunism-proofness and show that this condition is more restrictive than the stability condition.

Opportunism-proofness relates to the concept of renegotiation-proofness introduced by Farrell and Maskin (1989) in the sense that opportunism-proofness is one of the conditions for renegotiation-proofness. We apply their concept only to the coalition of manufacturers because we focus on manufacturer collusion absent communication with retailers. We define opportunism in this context as a joint deviation from the collusive agreement to reflect that manufacturers may renegotiate the collusive wholesale price if a joint deviation is profitable for given beliefs of the retailers. Collusion with trigger strategies is usually not renegotiation-proof (see, for example, Bernheim and Ray, 1989 and Farrell and Maskin, 1989): firms would have an incentive to jointly deviate in the punishment phases by renegotiating higher prices – undermining the credibility of the necessary threat to punish. Although a few solutions for certain settings are known, such as asymmetric punishment, the problem of renegotiation seems to be largely ignored in the applied literature.[2] We find that if the colluding firms sell via retailers using private contracts, renegotiation-proof equilibria with the usual symmetric punishment can exist, and we offer different dynamic beliefs for which they do. These beliefs determine the response of retailers to out-of-equilibrium offers and — similar to strategic delegation — implicitly commit to how a manufacturer responds to a past deviation of a rival. Interestingly, albeit punishment with trigger strategies is usually not renegotiation-proof absent retailers, retailer beliefs that anticipate punishment can render punishment credible in the first place. Hence, if the stricter opportunism-proofness conditions holds, with beliefs that anticipate a collusive strategy, collusion can become a renegotiation-proof equilibrium of the game for patient enough manufacturers.

Case descriptions reveal that manufacturer cartels occasionally struggle to establish collusion because retailers do not accept the increased wholesale prices.[3] This raises the question under which condition retailers' beliefs allow for the formation of collusion. Formally, we introduce a definition of *formability* of collusion that requires the existence of a potential path from a non-collusive history to collusion. For example, beliefs that anticipate collusion with grim-trigger strategies would never allow for collusion after observing a single period of competition. Hence, pure grim-trigger beliefs do not support the formation of collusion.

Conceptually, we argue that with strategic uncertainty of retailers, in the sense that retailers are unaware or do not expect manufacturer collusion, the simple Nash logic where

[2]An exception is McCutcheon (1997). We differ from McCutcheon (1997) who builds on the model of costly renegotiation proposed by Blume (1994). By contrast, following Farrell and Maskin (1989) we assume that renegotiation is costless and establish that the collusive equilibrium can nevertheless be renegotiation-proof when considering renegotiation between the collusive players.

[3]See footnote 1.

retailers know the manufacturer strategies, and hence can perfectly predict whether collusion is going on, is inadequate for studying the formation of collusion. As a solution, we propose an adaptive belief whereby retailers expect collusion or competition depending on the past behavior of the manufacturers. These beliefs are intuitive and can describe settings in which players cannot perfectly anticipate the actions of other players in parts of the game. We parameterize the adaption speed that specifies how fast a retailer that believes in competitive prices, but that receives collusive wholesale price offers, switches to believing in collusive prices. The adaption speed may range from a single period to many periods. We find that collusion can be formable with these adaptive beliefs, but such adaptive beliefs perform poorly in the punishment phase. We find that a faster speed of adaption makes collusion easier to form but harder to sustain; collusion becomes less opportunism-proof.

Finally, we consider symmetric beliefs that immediately adapt to the actions of the manufacturers in each period. With symmetric beliefs, stable collusion is also opportunism-proof. The punishment phase of trigger strategies, however, is not credible, such that there are no renegotiation-proof equilibria with symmetric beliefs and trigger strategies.

In summary, we find that the opportunism problem previously discussed in the monopoly context has rich implications for the theory of cartel stability and formation. In case of adverse beliefs (for example, history-independent passive beliefs), collusion can be entirely impossible due to opportunism of manufacturers,: retailers fear that other retailers obtain more competitive offers. Colluding manufacturers might thus have an incentive to influence retailers' beliefs towards a high price level. In practice, this may show when manufacturers publicly communicate non-binding price increase announcements, which may also influence the beliefs of rival retailers. A trade-off can arise between this incentive of communicating supra-competitive supply conditions to retailers while contracting, on the one hand, and hiding collusion from competition authorities, on the other hand.

The remainder of the paper is structured as follows. Section 2 relates our model to the relevant literature. We set up the model in Section 3 and subsequently analyze beliefs that are independent of the history of the game in Section 4. In Section 5, we study trigger beliefs and adaptive beliefs. We review symmetric beliefs in Section 6. Section 7 concludes.

2 Related Literature

Our paper contributes to four aspects that have been analyzed in the literature: (i) manufacturer collusion, (ii) a monopolist's opportunism problem, (iii) downstream retailers' types of beliefs, and (iv) cartel formation

Manufacturer collusion. Nocke and White (2007) and Normann (2009) analyze tacit collusion among manufacturers in vertical relationships, but in contrast to us focus on whether vertical integration makes tacit collusion easier to sustain. Both articles consider perfect Bertrand competition among manufactures and compare an industry with no integration to one in which one pair of firms is vertically integrated. In Nocke and White (2007), manufacturers compete in two-part tariffs. The authors show that it is easier for manufacturers to sustain collusion in a scenario with vertical integration. Normann (2009) shows that this finding carries over to a situation in which manufacturers set linear prices, even though double marginalization leads to different collusive and deviation profits. Piccolo and Miklós-Thal (2012) find similar results for the case in which retailers have full bargaining power. Under public contracts, Schinkel et al. (2008) show that when manufacturers have full bargaining power but need to make sure that retailers do not sue for private damages, upstream collusion requires low wholesale prices and possibly negative franchise fees. Mailath et al. (2017) show that in repeated extensive-form games, such as games with upstream and downstream firms, simple penal codes may not suffice to characterize the set of subgame-perfect outcomes. Instead, punishment profiles may be necessary whereby the continuation play after a deviation is tailored not only to the identity of the deviator but also to the nature of the deviation.Piccolo and Reisinger (2011) analyze the impact of exclusive territories granted to retailers on manufacturers' possibility to collude. Under observable contracts, establishing exclusive territories has two opposing effects on collusive stability. Exclusive territories soften punishment, but they also reduce deviation profits. The second effect is due to the fact that when a manufacturer deviates, retailers of competing products adjust their prices, whereas retailers of the same product do not. Because the effect on deviation tends to dominate, exclusive territories tend to facilitate tacit collusion.

Our contribution to this literature is twofold: First, we study the opportunism problem that colluding manufacturers face with secret contracting. Second, we demonstrate the relevance of retailers' beliefs about wholesale prices for establishing and maintaining collusion.[4]

Opportunism problem. We relate to the classic opportunism problem of a monopolist in a vertical structure with secret contracting (Hart et al., 1990; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994), as the colluding manufacturers are similar to a monopolist. In such a scenario, the upstream firm that deals with multiple competing downstream firms through bilateral contracts may – as discussed further below – encounter the following problem: The upstream firm is interested in maintaining high prices and profits, but it cannot commit to refraining from opportunistic moves. Indeed, the upstream firm has an incentive to increase bilateral surplus with one downstream firm, which is anticipated by the other downstream firm(s). The existence of this opportunism problem has been evoked as an explanation for vertical mergers and various vertical restraints as measures aimed at restoring the upstream firm's market power (O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Vergé, 2004). The restraints include exclusive dealing, non-discrimination clauses,

[4]The literature also contains explanations of how resale price maintenance (RPM) can facilitate manufacturer collusion but abstracts from the relevance of retailer beliefs. Jullien and Rey (2007) study how (RPM) affects collusion when only the retailers observe local shocks on demand. They assume that colluding manufacturers reveal all wholesale prices to all retailers. Hunold and Muthers (2020) show that, absent any uncertainty and asymmetric information, RPM can still facilitate manufacturer collusion when there is retail bargaining power. They focus on subgame perfect Nash equilibria.

and industry-wide RPM. Do and Miklos-Thal (2021) explore shortcomings of the seminal papers by considering a version of sequential (re)contracting between upstream and downstream firms. The opportunism problem that we study in detail has been neglected in the collusion literature so far, and we establish that it has important non-trivial consequences.

Beliefs. The literature on secret contracting between manufacturers and retailers, which dates back to Hart et al. (1990), McAfee and Schwartz (1994) and Segal (1999), emphasizes the relevance of the retailers' beliefs. The literature mostly focuses on so-called passive, symmetric, or wary beliefs in static settings. Whereas passive beliefs suppose that the agents treat unexpected offers as mistakes, symmetric beliefs could correspond to a rule of thumb, where agents conjecture that identical principals make identical offers (Pagnozzi and Piccolo, 2011). Wary beliefs – according to which a retailer anticipate that rivals get the offer that maximize the manufacturer's profits – are often used when passive beliefs are implausible or induce non-existence of equilibria (Rey and Vergé, 2004; Rey and Tirole, 2007; Miklós-Thal and Shaffer, 2016). Empirical evidence on passive and symmetric beliefs was tested by Zhang (2021) and by Martin et al. (2001) in an experiment. Moreover, Aoyagi et al. (2021) study beliefs in finitely and infinitely repeated games experimentally and find that the same history of play can lead to different beliefs, and the same belief can lead to different action choices. We contribute to this literature by providing an in-depth analysis of how retailers' beliefs affect manufacturer collusion in a dynamic setting with secret contracting. In our analysis, we use different beliefs, including history-dependent passive beliefs that reflect trigger-strategies and beliefs that adapt to changing manufacturer strategies.

Cartel formation. The literature on cartel membership formation has focused on a variety of different aspects. In this literature, the question of how to initiate cartels typically focuses, among other things, on contracts (Selten, 1973; d'Aspremont et al., 1983), on stochastic opportunities to form a cartel (Harrington and Chang, 2009), on the heterogeneity with regard to capacities and umbrella pricing (Bos and Harrington, 2010), on signals and beliefs of producers (Harrington and Zhao, 2012; Harrington, 2017), on quality differentiation (Bos et al., 2020), and the ability to overcome strategic uncertainty absent communication (Blume and Heidhues, 2008). Selten (1973)) analyzes the case of quantity competition and assumes that cartels can be enforced via contracts, and that a cartel acts as a Stackelberg leader. He shows that a cartel is stable in the sense that outsiders do not want to be part of the cartel, and insiders cannot profitably leave the cartel as long as the number of cartel members is relatively small. d'Aspremont et al. (1983) obtain a similar result for the case of price leadership.

Harrington and Chang (2009) consider a set of heterogeneous industries in which stochastic opportunities to form a cartel arise to explain the birth and death of cartels and to inform antitrust authorities about the extent of cartels that have not been discovered. Bos and Harrington (2010) endogenize the composition of a cartel in an industry in which heterogeneous firms differ in their capacities. They show that non-all-inclusive cartels set umbrella prices, and that mergers involving moderate-sized firms may result in the most severe coordinated effects. Harrington and Zhao (2012) analyze whether different types of players (patient and impatient) manage to cooperate via grim-trigger strategies when players signal and coordinate through their actions. The authors show that there is always a positive chance of cooperation, but cooperation may fail altogether. Moreover, the longer cooperation does not occur, the less likely it is to occur in the next period. Harrington (2017) focuses on mutual beliefs to coordinate prices. In the context of price leadership, firms are assumed to commonly believe that price increases will be at least matched. The firms, however, lack any shared understanding about who will lead, when they will, and at what prices. Sufficient conditions are derived, which ensure that supra-competitive prices emerge, but price is bounded below the maximal equilibrium price.

In contrast to the literature on cartel membership formation, we address the question whether firms can transit to a collusive equilibrium once they have reached – possibly explicitly – a common understanding to collude in a currently non-collusive industry. We thereby focus on the process of collusion that firms need to go through until they may reach a stable collusive equilibrium. At the core of this analysis is the adaption of the retailers' beliefs in the transition process.

3 Model

We study manufacturer collusion in an infinitely repeated stage game with time $t = 0, \ldots, \infty$. There are two symmetric manufacturers, M_A and M_B, that compete by selling imperfect substitutes to their exclusive retailers R_A and R_B. Each manufacturer makes an exclusive and secret two-part tariff offer with a unit wholesale price w_i and a franchise fee F_i to its retailer, with $i \in \{A, B\}$. The retailers compete in retail prices. There is horizontal product differentiation between the two retailer-product combinations.

Timing and information. In each period, the following stage game unfolds:

1. Each manufacturer makes a private contract offer to its retailer.
2. Retailers decide whether to accept offers. Post contract acceptance, the fixed fees are sunk.
3. The retailers simultaneously and non-cooperatively set their retail prices p_i.

Because the manufacturers' contract offers are secret, a retailer cannot observe the contract offered to the rival. When competing in the downstream market, the retailers are thus unable to observe each others' input prices.

At the end of each period, all actions are revealed to all players. All players thus know the complete history of the game at this point. This is a technical assumption to simplify the equilibrium analysis; however, it is also reasonable that this information becomes known in the industry over time.

Manufacturers are long-lived and discount next-period profits with the common discount factor $\delta > 0$. The retailers are short-lived (or, alternatively, have a discount factor of zero), such that they do not take future profits into account. This assumption ensures that retailers cannot collude.[5] The supergame is a game of complete information but unobservable actions. Our analysis focuses on manufacturers that use trigger strategies to collude, although many of the economic insights occur also in the subcase of grim-trigger strategies.

Assumptions on costs, demand, and profits. All (fundamental) costs are zero. We assume that the outside option (opportunity costs) of each retailer is equal to zero. We consider general demand functions that fulfill standard properties in the relevant range, which we summarize in the following assumption:

Assumption 1. *Demand $D_i(p_i, p_{-i})$ (with $i \in \{A, B\}$)*

- *decreases in the own price p_i ($\partial D_i(p_i, p_{-i})/\partial p_i < 0$),*
- *increases in the other product's price p_{-i} ($\partial D_i(p_i, p_{-i})/\partial p_{-i} > 0$), and*
- *decreases when all prices increase ($\partial D_i(p_i, p_{-i})/\partial p_i + \partial D_i(p_i, p_{-i})/\partial p_{-i} < 0$).*

To ensure that there exists a unique and stable equilibrium in the downstream market, we assume that the Hessian matrix of $D_i(p_i, p_{-i})$ has a negative and dominant main diagonal. This results in well-behaved retail profits that are twice differentiable and concave. Note that this also implies that the retailers' reactions behave normally, such that $\partial p_i \left(w_{it}, p^e_{-i}\right)/\partial w_{it} > 0$ and, consequently, $\partial D_i \left(p_i \left(w_{it}, p^e_{-i}\right), p^e_{-i}\right)/\partial p_i < 0$ hold.[6]

Our assumptions on retailer profits mostly carry over to manufacturer profits because manufacturers internalize retailer profits using two-part tariffs. In some cases, however, manufacturers' true actions and retailers' beliefs differ in such a way off the equilibrium path that the behavior of the manufacturers' profit is not identical to that of the retailers' profit. In those cases, we assume that manufacturers' profits are well behaved such that optimal behavior can be derived from the respective first-order conditions. We comment on these cases below.

Equilibrium concept. We consider (pure-strategy) perfect Bayesian equilibria (PBE) and focus on symmetric equilibria.[7] Formally, we define a perfect Bayesian equilibrium in our

[5] We follow the literature that assumes that retailers are short-lived or only live for one period (see, for example, Piccolo and Reisinger, 2011 and Jullien and Rey, 2007). Furthermore, this assumption circumvents other potential problems, such as multiple equilibria and hub-and-spoke collusion, where retailers are a part of the collusive strategy by manufacturers.

[6] This can be shown by applying the implicit function theorem on the retailer's first-order condition for optimal pricing.

[7] The game defined includes two groups of players, manufacturers and retailers. Within each group, players are symmetric. The equilibria analyzed are strongly symmetric in the sense that the (symmetric)

setting as follows: A manufacturer's strategy is optimal given the retailers' strategies and given the rival's strategy. Furthermore, retailers maximize their profits in every period given their beliefs, and these beliefs are consistent with equilibrium strategies. Note that when observing an out-of equilibrium offer in any continuation game, beliefs regarding the action of the rival are determined according to our definition of out-of equilibrium beliefs. On path, of course, the belief function of retailer i about the price of the other retailer p^e_{-it} is derived from the manufacturer's strategy using Bayes' rule where possible. We impose an additional condition for most of the analysis that is consistent with our use of passive beliefs.

Condition 1. The belief of retailer i about its rival retailer $-i$ at the beginning of period $t+1$ depends only on the history up to date t ($\mathcal{H}_t$), but not on the action of manufacturer i at date t ("no-signaling-what-you-don't-know").

This condition captures the idea that the deviation of a manufacturer should not signal (private) information that the manufacturer does not have. Hence, a retailer's belief should not change.[8] A retailer forms a belief about the contract offer made to the rival, and about how the rival reacts to its contract offer when accepting the contract in the second stage.

Without loss of generality, we focus on retailers that form beliefs about the resulting retail price of the rival, which is the payoff-relevant information that retailers are lacking.

The condition is essentially equivalent to the concept of passive beliefs, which means that a retailer's belief is constant in its own contract offer. In general, passive beliefs indicate that retailers do not anticipate or are not suspicious of communication among manufacturers, that is, retailers expect manufacturers to act independently off the equilibrium path. Therefore, Condition 1 holds for the equilibria throughout Sections 4 to 5. The underlying idea is that unilateral deviations should not trigger a change in the retailer's assessment of other players, which is in the spirit of sequential equilibrium, such that (out-of equilibrium) beliefs are naturally passive because deviations are the result of a random mistake or "tremble".[9]

In Section 6, however, Condition 1 is violated in the case of symmetric retailer beliefs that capture the idea that retailers may anticipate communication between manufacturers.

Our game is reduced-form, but reflects a game that could be richer, that is, by including a cheap-talk stage. If retailers consider a common possibility that manufacturers may communicate (via cheap talk), manufacturers' actions might be correlated when a deviation occurs.

manufacturers use a common continuation strategy on and off the equilibrium path (Athey et al., 2004; Jullien and Rey, 2007). Retailers choose their action according to the symmetric equilibrium in the downstream market.

[8]Condition 1 is derived from the definition of the PBE that Fudenberg and Tirole (1991a,b) provide. This definition is suited for games of incomplete information with independent types. Because in our game of complete information, the private information of the retailers is generated by manufacturers' actions, the concept can only be applied analogously. Similarly to Pagnozzi and Piccolo (2011), we adapt condition B(iii) ("no-signaling-what-you-don't-know") on p. 332 in Fudenberg and Tirole (1991a) to our game. Originally, the condition means that in a signaling game, the actions by other players with independent types have no effect on beliefs about a player's type if this player acts the same. In our setting, because retailers act simultaneously, one retailer cannot observe the other retailer's action such that the condition translates into: A retailer cannot infer anything about a rival's current pricing from the current action of its own manufacturer.

[9]Note that we cannot apply sequential equilibrium due to a continuous action space.

Given this interpretation, Condition 1 does not naturally capture the spirit of sequential equilibrium in this "extended game".[10]

We adapt the concept of *renegotiation-proofness* as developed by Farrell and Maskin (1989) to select among equilibria. According to their definition, an equilibrium is renegotiation-proof if there is no continuation equilibrium that Pareto-dominates this equilibrium. We use renegotiation-proofness to ensure that manufacturers do not want to jointly deviate from their 'collusive agreement'. Hence, we look for Pareto-dominating continuation equilibria that can be reached by joint deviations of manufacturers, and that are Pareto-improvements for the coalition of manufactures. Thus, we exclude changes in retailer strategies or beliefs in the renegotiation.[11] An equilibrium candidate is renegotiation-proof if additional conditions concerning renegotiation opportunities on the collusive and on the non-collusive path hold. Renegotiation takes place implicitly in our model if *manufacturers* have a collective interest in revising their agreement. This assumption is in line with other theories on renegotiation-proofness (see, for example, Bernheim et al., 1987 and Bernheim and Ray, 1989).

Outlook. In the following sections, we consider Perfect Bayesian Equilibria (PBE) of the game and focus on symmetric equilibria with constant equilibrium paths. On the collusive equilibrium path, beliefs are correct and thus pinned down by the equilibrium strategy profile. Off the equilibrium path, the equilibrium concept of PBE does not have sufficient bite. We restrict off-equilibrium beliefs in different forms (for example, passive) and develop novel criteria, such as opportunism-proofness, to assess the plausibility of a given PBE.

4 History-Independent Passive Beliefs

In this section, we focus on retailer beliefs that are independent of the history of actions in the game. With these history-independent beliefs, we will show that an opportunism problem arises if manufacturers try to collude.

Definition 1. (History-Independent Passive Beliefs) The price expectation p^e_{-it} of retailer i in period t about the price of retailer $-i$ is independent of the history of the actions in the game up to period $t-1$ and independent of the offer (w_{it}, F_{it}) made by its supplier in period t.

[10]As pointed out by Pagnozzi and Piccolo (2011), symmetric beliefs violate the spirit of the PBE definition of Fudenberg and Tirole (1991a), even though none of the additional conditions B(i) to B(iv) (p. 332) is formally violated. These conditions have no bite in our setting because they apply only to situations in which the player forming the belief observes an action of the player holding private information, which is never the case with our game.

[11]This is in line with the original idea presented by Farrell and Maskin (1989) who consider subgame perfect Nash equilibria (SPNE) in a collusive game and treat the buyers as non-strategic. Similarly, renegotiation-proofness is usually applied to the parties of a *relational contract* only, in our case the collusive agreement, e.g. Buehler and Gärtner (2013); Goldlücke and Kranz (2013). Alternatively, one can understand our approach as applying the exact concept of Farrell and Maskin (1989) to the reduced form SPNE taking retailers' strategies and beliefs from the PBE as fixed.

The proposed belief refinement above defines out-of equilibrium beliefs of retailers about the retail price of their competitor to be independent of the history. As discussed before, the fact that beliefs are passive within a period arises from the definition of perfect Bayesian equilibrium and imposing Condition 1. The definition, however, does not impose a restriction on how the beliefs react to the past play of actions. Therefore, we differentiate between between passive beliefs that are independent of or depend on the history of the game.

With retailers holding history-independent beliefs as above, we first solve for the equilibrium of the game that results when both manufacturers maximize stage-game profits. In the last stage within each period, on the equilibrium path, each retailer has accepted the contract, but the rival's wholesale price remains secret. Each retailer i faces the own wholesale price w_{it} and holds a belief p^e_{-it} about the retail price of the rival. The retailers set the retail prices p_{it} simultaneously. Thus, the profit of retailer i is

$$\pi_{it}\left(p_{it}, p^e_{-it}\right) = \left(p_{it} - w_{it}\right) D_i\left(p_{it}, p^e_{-it}\right) - F_{it}.$$

Retailer i maximizes its profit with respect to p_{it}. The first-order condition is

$$D_i\left(p_{it}, p^e_{-it}\right) + \left(p_{it} - w_{it}\right) \frac{\partial D_i\left(p_{it}, p^e_{-it}\right)}{\partial p_{it}} = 0 \tag{1}$$

and defines retailer i's reaction function $p_i\left(w_{it}, p^e_{-it}\right)$. Anticipating the pricing outcome and resulting profits, each retailer decides whether to accept the wholesale tariff offered by its manufacturer in the second stage. Based on our Assumption 1, there exists a unique symmetric Nash equilibrium in the downstream market for offers (w_{it}, F_{it}) made by the manufacturers given retailers accept the offers.

In the first stage, the manufacturers offer their wholesale tariffs. Under manufacturer competition, each manufacturer i maximizes its profit, anticipating that its retailer sets a price of $p_i\left(w_{it}, p^e_{-it}\right)$:

$$\max_{w_{it}, F_{it}} \Pi_{it} = w_{it} \cdot D_i\left(p_i\left(w_{it}, p^e_{-it}\right), p_{-i}(w_{-it}, p^e_{it})\right) + F_{it},$$

subject to the retailer's participation constraint

$$\left(p_i\left(w_{it}, p^e_{-it}\right) - w_{it}\right) \cdot D_i\left(p_i\left(w_{it}, p^e_{-it}\right), p^e_{-it}\right) - F_{it} \geq 0.$$

The profit is determined by two parts. The variable part consists of the units sold times the unit wholesale price, and the fixed part consists of an up-front payment from the retailer to the manufacturer. The maximum fixed payment cannot be larger than the revenue that the retailer earns. This participation constraint binds in equilibrium.

Let us first characterize the equilibrium of the one-shot game. In equilibrium, the belief of the retailer about the retail price of the rival coincides with the expectation of the manufacturer about the retail prices. This implies $p_{-i}(w_{-it}, p^e_{it}) = p^e_{-it}$. Inserting the price and simplifying yields

$$\Pi_{it}\left(w_{it}, p^e_{-it}\right) = p_i\left(w_{it}, p^e_{-it}\right) \cdot D_i\left(p_{it}\left(w_{it}, p^e_{-it}\right), p^e_{-it}\right).$$

Maximizing with respect to w_{it} gives the first-order condition

$$\frac{\partial p_i\left(w_{it}, p^e_{-it}\right)}{\partial w_{it}} D_i\left(p_i\left(w_{it}, p^e_{-it}\right), p^e_{-it}\right) + \frac{\partial D_i\left(p_i\left(w_{it}, p^e_{-it}\right), p^e_{-it}\right)}{\partial p_i} \frac{\partial p_i\left(w_{it}, p^e_{-it}\right)}{\partial w_{it}} p_i\left(w_{it}, p^e_{-it}\right) = 0$$
$$\iff \frac{\partial p_i\left(w_{it}, p^e_{-it}\right)}{\partial w_{it}} \frac{\partial D_i\left(p_i\left(w_{it}, p^e_{-it}\right), p^e_{-it}\right)}{\partial p_i} w_{it} = 0. \tag{2}$$

Because the first term on the left-hand side in the previous line is assumed to be strictly positive, and the second term is assumed to be strictly negative (Assumption 1), the only solution to the first-order condition is $w_{it} = 0$. This yields the following result:

Lemma 1. *With history-independent passive beliefs, the wholesale prices in the equilibrium with competing manufacturers are equal to zero.*

This history-independent benchmark defines the competitive level of wholesale prices which is at $w_i = 0$, the corresponding fxed fees and the resulting retail prices. Denote the competitive wholesale price by w^P and the resulting retail price by $p^P = p_i(0, p^P)$. The fixed fee equal to the retailer's profit, $F_i = \pi_i(p^P, p^P)$, corresponding to a non-colluding industry. In the next steps, we analyze the equilibrium of the repeated game if retailers hold history-independent beliefs. To show that firms are unable to collude on a price different from the competitive wholesale price of zero, we take the whole dynamic game into account. Manufacturers may collude using any dynamic strategy (for example, grim-trigger strategies). Each strategy, however, is a mapping from the previous history to an action chosen in period t.

Proposition 1. *Suppose retailers have history-independent passive beliefs. Then, there exists a unique symmetric perfect Bayesian equilibrium in which the manufacturers set wholesale prices equal to zero and extract all profits of retailers via the fixed fee. Collusion cannot effectively increase prices above the competitive level, that is, manufacturers face an opportunism problem.*

Proof. See Appendix A. □

Due to the nature of the history-independent passive beliefs, manufacturers face an opportunism problem similar to a monopolist. A manufacturer, however, does not face a commitment problem for its own actions, but lacks commitment with regard to the other manufacturer. Retailers fear that their rival receives a better offer from its manufacturer instead of fearing to be exploited by their own manufacturer. As a consequence, each manufacturer is unable to charge a higher wholesale price than the other manufacturer – both statically and dynamically. Perhaps surprisingly, the opportunism problem arises even for colluding manufacturers that can use grim-trigger strategies that usually support a large set of equilibria for patient firms. Here, punishment is limited by the assumption that retailers do not update their beliefs after a deviation. This allows deviating manufacturers to maintain a high profit level and destroys the commitment power of the grim-trigger strategies. By contrast, if retailers' beliefs react to deviations, punishment might become more effective. We consider this case in the next sections in which we introduce history-dependent beliefs.

5 History-Dependent Passive Beliefs

Consider a dynamic version of passive beliefs that depend on the history of the game. We focus on the history of wholesale prices and disregard the history of retail prices, such that the retailers' beliefs cannot directly depend on the competing retailers' past actions. Otherwise, such beliefs could support retail collusion, which we want to abstract from.[12] Thus, the relevant history of the game in period t is $\mathcal{H}_t \equiv [(w_{A0}, w_{B0})\,; \ldots ; (w_{At-1}, w_{Bt-1})]$, that is, the set of the pairs of wholesale prices that the manufacturers have set in all periods up to period $t-1$.

In contrast to constant passive beliefs, the question is whether it is possible for the manufacturers to support the optimal collusive price in equilibrium. We define history-dependent passive beliefs as follows:

Definition 2. (History-Dependent Passive Beliefs) The beliefs are passive within each period t, that is, the offer about w_{it} does not affect the belief p^e_{-it} about the competitor's retail price. The beliefs are dynamic: For each retailer i, the belief is a function $p^e_{-it}(\mathcal{H}_t)$ that depends on the history of past wholesale prices.

There exist multiple variants of history-dependent passive beliefs. We derive beliefs that are in line with the strategies of the manufacturers (such as grim-trigger strategies). Given our definition of beliefs, we consider additional conditions that characterize the PBE. The conditions address different characteristics of the collusive equilibrium beyond stability. First, we consider a condition that excludes the scope for opportunistic manufacturer behavior in equilibrium.

Definition 3. (Opportunism-Proofness) A collusive equilibrium strategy for given beliefs (a Perfect Bayesian Equilibrium) is said to be opportunism-proof if the manufacturers, on the equilibrium path, do not benefit if they jointly reduce their wholesale prices.

Opportunism-proofness is not a necessary condition for a PBE to exist. It is an additional condition that, in our opinion, increases the plausibility that an equilibrium can be

[12]Suppose that each retailer believes that the price of the competitor is the monopoly price unless they have observed a different price in the past in which case they would believe that the price is the competitive price. Such beliefs could support a collusive action profile even with otherwise history-independent strategies.

sustained in a collusive environment. Without opportunism-proofness, the implicit "collusive agreement" between the manufacturers is not renegotiation-proof.

In addition to opportunism-proofness, we present a condition that concerns punishment.

Definition 4. (Credible Punishment) A collusive strategy for given beliefs has the property of credible punishment if there is no punishment period in which the manufacturers would benefit if they jointly changed their wholesale prices.

Whereas opportunism-proofness considers renegotiation-proofness of the collusive strategy on the collusive path, the criterion of credible punishment does so for the punishment phase. Together, opportunism-proofness and credible punishment ensure *weak* renegotiation-proofness as defined in Farrell and Maskin (1989). *Strong* renegotiation-proofness requires that not only all the continuation game equilibria do not invite joint deviation, but also that there is no other renegotiation-proof strategy profile that Pareto dominates the candidate equilibrium of the whole game.

For reference, consider a standard model of horizontal collusion absent a vertical dimension, such as our model when each manufacturer-retailer pair is vertically integrated. In this industry structure, punishment is typically not credible because the firms would prefer to renegotiate to return to the collusive pricing scheme. This property can prevent the existence of renegotiation-proof equilibria in the case of standard horizontal collusion. By contrast, as we will prove later, manufacturer collusion with trigger strategies in a setting with retailers and secret contracting can have the property of credible punishment, depending on the retailers' beliefs.

We also define when collusion is formable in the sense that collusion can be started and maintained:

Definition 5. (Formable) Consider a PBE with a collusive strategy profile and given beliefs resulting in discounted collusive payoffs on the equilibrium path denoted by V^C. Collusion is formable if there exists a strategy profile, such that for given beliefs:

1. Starting in some period s with an arbitrary history $\mathcal{H}_s$, the action profile consists of mutually best responses in $t \geq s$.
2. The continuation game starting in period $s + x$, with $x < \infty$, constitutes a PBE with payoff V^C.

This definition is stronger than the condition for sustainability of collusion because the latter is necessary but not sufficient for formation. The definition implies a property that beliefs 'forgive' any non-collusive behavior after some periods; this includes that beliefs do not react to opportunistic behavior. The number of transition periods x can be very long, such that our definition is not restrictive with regard to the speed of formation.

With formability we address the question whether firms can transit to a collusive equilibrium once they have reached a common understanding to collude in a currently non-collusive industry. This reflects the process of collusion that firms need to go through until they may reach a stable collusive equilibrium. We take a “meeting of the minds”, that is, how firms reach a common understanding to collude among themselves, as the starting point. Formability requires that there is a feasible path towards collusion that is incentive compatible (self-enforcing). The underlying complication for this is that the environment may first need to adapt over time to make collusion profitable. Within our model, this is reflected in the retailers' beliefs that may need time to adapt to collusion among manufacturers out of a situation of manufacturer competition.

In a sense, the property of formability is similar to the property of stability of static Nash equilibria because both properties consider wether there exists a path of mutual best responses that lead to the proposed equilibrium.

For example, consider a standard model of horizontal collusion with grim-trigger strategies and without private information. Suppose that firms collude at the industry-profit-maximizing level, and that the usual stability condition is met. Such a collusion is formable because switching to grim-trigger strategies is an equilibrium of the continuation game independent of the history of actions in the game. Collusion is then also opportunism-proof because joint profits are maximized, whereas punishment is not credible because jointly reverting to collusion yields larger profits than continuing punishment.

5.1 Collusive Strategy

In the following, we will consider collusion at industry-profit-maximizing prices. Thus, let us derive the conditions for the industry profit maximum. First, we define the optimal collusive price provided that the retailers understand the grim-trigger strategy. The optimal collusive wholesale price is determined by the price that maximizes the joint profit of the manufacturers given that the retailers' belief is identical to that price. Because we focus on manufacturers colluding on the industry profit maximum, collusive profits are Pareto-optimal if there is no joint deviation that is more profitable (see Definition 3). If weak renegotiation-proofness is fulfilled, then the equilibrium is also strongly renegotiation-proof because the profit on the equilibrium path would be Pareto-efficient for the colluding manufacturers.

Optimal Collusive Price: Integrated Industry Solution

We consider collusion at the prices that maximize the integrated industry profit

$$p^M = \max_{p_i} p_i \cdot D_i \left(p_i, p_{-i}\right) + p_{-i} \cdot D_{-i} \left(p_{-i}, p_i\right) \tag{3}$$

and define w^M implicitly through $p^M = p_i \left(w^M, w^M\right)$. Denote the maximal industry profit by

$$\Pi^M := p^M \cdot D_i \left(p^M, p^M\right) + p^M \cdot D_{-i} \left(p^M, p^M\right). \tag{4}$$

The optimal collusive wholesale price w^M and the respective belief about the retail price, p^M, are then determined by equation (3) as well. Thus, whenever the manufacturers collude at the industry-profit-maximizing wholesale prices w^M, each manufacturer earns a profit of

$$\Pi^C := \frac{\Pi^M}{2}.$$

5.2 Grim-Trigger Beliefs

We construct history-dependent passive beliefs (Definition 2) that have a grim-trigger property. For this, consider that manufacturers play grim-trigger strategies as described above.

We focus on industry-profit-maximizing collusion, such that $w^C = w^M$. Any deviation by one manufacturer causes the other manufacturer to set the punitive price w^P forever. Because we are solving for perfect Bayesian equilibria, the punishment with w^P must be individually rational. This implies that $w^P = 0$ must hold in equilibrium, which follows from the logic presented in the proof of Proposition 1: Otherwise, each manufacturer would have an incentive deviate from a $w^P \neq 0$ in a punishment period in which future beliefs and actions are fixed and unaffected by current actions. In punishment periods, manufacturers thus behave as if they maximize short-term profits.

Histories and beliefs. If manufacturers play grim-trigger strategies, actions are only conditional on two kinds of histories: the collusive history $\mathcal{H}^{\mathcal{C}}$, where both manufacturers have only played w^C, and the deviation histories $\mathcal{H}^{\mathcal{D}}$ (any history other than $\mathcal{H}^{\mathcal{C}}$). Define the grim-trigger strategy as follows: Manufacturers set the collusive wholesale price w^Cin the first period. Then, in the t^{th} period, if both manufacturers have set the collusive price in each of the $t-1$ previous periods (history $\mathcal{H}^{\mathcal{C}}$), they set the collusive wholesale price w^C, otherwise after histories $\mathcal{H}^{\mathcal{D}}$ manufacturers set a punishment price $w^P \neq w^C$ forever. Grim-trigger beliefs match these strategiesby assigning two different beliefs to these histories. Grim-trigger beliefs $p_i^e(\mathcal{H})$ are thus history-dependent (Definition 2) and differentiate between the two histories $\mathcal{H}^{\mathcal{C}}$ and $\mathcal{H}^{\mathcal{D}}$.

Definition 6. (Grim-trigger beliefs)

In the first period, retailers believe that the rival sets a retail price of p^C.

As long as both manufacturers play w^C, that is, the collusive history $\mathcal{H}^C$ prevails, each retailer believes that the other retailer sets p^C in the current period.

Once one manufacturer has deviated, the history is $\mathcal{H}^{\mathcal{D}}$, and both retailers believe that the other retailer sets the competitive price p^P. This corresponds to a situation in which both retailers have common knowledge that the wholesale prices are $w^P = 0$ in the current period.

Given the passive nature of beliefs in a period (Definition 2), the beliefs are not correct in deviation periods. Neither a retailer that is offered the equilibrium contract updates its belief, nor updates the retailer that receives a deviating offer.

Because a deviation does not occur on the equilibrium path, the beliefs are nevertheless correct on the equilibrium path. Off the equilibrium path, the retailers' beliefs anticipate that manufacturers play grim-trigger strategies in that they also punish a deviation by one manufacturer in period t in all future periods.

Equilibrium. To determine an equilibrium of the dynamic game, we must consider deviations from the collusive strategy. In equilibrium, each manufacturer realizes per-period profits of $\Pi^C = \Pi^M/2$. In a deviation period, both retailers believe that the wholesale price is w^C and anticipate that the other retailer sets p^C. This results in a belief $p^e_{it} := p_i\left(w_{it}, p^C\right)$. Suppose that manufacturer i maximizes its deviating profit in period t in view of history $\mathcal{H}^C$. When there is a deviation, that is, $w_{it} \neq w^C$ holds, grim-trigger beliefs imply that the level of w_{it} has no impact on future beliefs. The deviation profit is given by

$$\begin{aligned}&w_{it} \cdot D_i\left(p_i\left(w_{it}, p^C\right), p^C\right) + \left[p_i\left(w_{it}, p^C\right) - w_{it}\right] \cdot D_i\left(p_i\left(w_{it}, p^C\right), p^C\right) \\ &= p_i\left(w_{it}, p^C\right) \cdot D_i\left(p_i\left(w_{it}, p^C\right), p^C\right).\end{aligned} \tag{5}$$

Maximizing with respect to w_{it} yields the following first-order condition

$$\begin{aligned}&\frac{\partial p_i\left(w_{it}, p^C\right)}{\partial w_{it}} D_i\left(p_i\left(w_{it}, p^C\right), p^C\right) + \frac{\partial D_i\left(p_i\left(w_{it}, p^C\right), p^C\right)}{\partial p_i} \frac{\partial p_i\left(w_{it}, p^C\right)}{\partial w_{it}} p_i(w_{it}, p^C) = 0 \\ &\iff \frac{\partial p_{it}\left(w_{it}, p^C\right)}{\partial w_{it}} \frac{\partial D_{it}\left(p_{it}\left(w_{it}, p^C\right), p^C\right)}{\partial p_{it}} w_{it} = 0.\end{aligned} \tag{6}$$

The last step follows from the first-order condition of the retailers, Equation 1, and inserting it in the first line above yields the second line. Because the first two factors in equation (6) are non-zero. The manufacturer optimally deviates to $w^D = 0$. This results in

$$\Pi^D := p_i(0, p^C) \cdot D_i\left(p_i(0, p^C), p^C\right).$$

After any deviation by a manufacturer, the beliefs revert to $p^e_{-i} = p^P$, that is, the belief in the punishment period, forever. This results in profits of

$$\Pi^P := p^P \cdot D_i(p^P, p^P). \tag{7}$$

where $p^P = p_{it}(0, p^P)$ is the competitive price.

Collusion is sustainable when no manufacturer wants to deviate from the grim-trigger strategy. Using the one-shot deviation principle, the relevant incentive constraint for stability is

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \frac{\delta \Pi^P}{1-\delta}. \tag{8}$$

The left-hand side contains the present value on the equilibrium path and the right-hand side the present value of a deviation. We can rewrite the incentive constraint for manufacturer i

as follows:

$$\frac{p^C \cdot D_i(p^C, p^C)}{(1-\delta)} \geq p_i(0, p^C) \cdot D_i\left(p_i(0, p^C), p^C\right) + \frac{\delta}{1-\delta}\left(p^P \cdot D_i(p^P, p^P)\right). \tag{9}$$

This inequality is equivalent to the incentive constraint for standard horizontal collusion (when manufacturers and retailers are pairwise integrated), where the following order holds: $\Pi^D > \Pi^C = \Pi^M/2 > \Pi^P$.

Let us check whether the equilibrium is opportunism-proof (see Definition 3). Jointly, the manufacturers may have an incentive to reduce their wholesale prices for given beliefs. Suppose $\mathcal{H}_C$ is the history of the game, such that each retailer believes that its competitor sets p^C and anticipates to set $p_i\left(w_{it}, p^C\right)$. First, we show that jointly deviating manufacturers set a wholesale price of $w_{it} = w_{-it} < 0$ to maximize spot profits. To see this, let us inspect the profit per manufacturer in the case of a joint deviation:

$$\begin{aligned}\Pi^{JD} :=& \frac{1}{2}\max_{w_{it}, w_{-it}} w_{it} \cdot D_i\left(p_i(w_{it}, p^C), p_{-i}(w_{-it}, p^C)\right) + \left[p_i(w_{it}, p^C) - w_{it}\right] \cdot D_i\left(p_i(w_{it}, p^C), p^C\right) \\ &+ w_{-it} \cdot D_{-i}\left(p_{-i}(w_{-it}, p^C), p_i(w_{it}, p^C)\right) + \left[p_{-i}(w_{-it}, p^C) - w_{-it}\right] \cdot D_{-i}\left(p_{-i}(w_{-it}, p^C), p^C\right).\end{aligned}$$

We assume that this profit is quasi-concave, such that we can use first-order conditions.[13] We rewrite the first-order conditions in Equation (1) by applying symmetry. This is possible because we assume that the manufacturer profits are well behaved in the sense that the optimal joint action of the manufacturers is symmetric. This yields

$$\left[\underbrace{D_i\left(p_i(w, p^C), p_{-i}(w, p^C)\right) - D_i\left(p_i(w, p^C), p^C\right)}_{<0}\right] + w\underbrace{\frac{\partial p_i}{\partial w_{it}}}_{>0}\left[\underbrace{\frac{\partial D_i}{\partial p_i} + \frac{\partial D_{-i}}{\partial p_i}}_{<0}\right] = 0, \forall i. \tag{10}$$

Equation (10) only holds for $w < 0$. Hence, manufacturers optimally deviate to $w < 0$ jointly. A manufacturer makes a higher profit in the case of a joint deviation than when deviating unilaterally: $\Pi^{JD} > \Pi^D$. To explain the last inequality, note that the manufacturers could replicate the profit of Π^D for each of them by setting $w = 0$. The manufacturers, however, optimally set a price w below zero because this yields strictly larger profits. Setting a price below zero has a negative externality on the rival retailer they do not internalize; it enables them to profitably exploit the incorrect beliefs. Following a deviation, the manufacturers make profits Π^P in future periods due to the grim-trigger beliefs. This results in the following opportunism-proofness condition

$$\frac{\Pi^C}{1-\delta} \geq \Pi^{JD} + \frac{\delta \Pi^P}{1-\delta}. \tag{11}$$

Comparing equation (11) with equation (8), it can be seen that the only difference is the

[13]Note that this holds for linear demand. In general, quasi-concavity depends on higher-order derivatives of demand at different loci of demand.

deviation profit on the right-hand side. Because $\Pi^{JD} > \Pi^{D}$, the condition for opportunism-proofness is harder to satisfy than the stability condition above.

Proposition 2. *With grim-trigger beliefs, there exists an equilibrium in which manufacturers are able to sustain collusion on the industry-profit-maximizing wholesale price using grim trigger-strategies if the discount factor is high enough to satisfy equation* (9). *This condition is equivalent to the incentive constraint when manufacturers and retailers are pairwise integrated, that is, under horizontal collusion. Under the stronger condition* (11), *collusion is also opportunism-proof. The punishment is credible, but collusion is not formable.*

Proof. See Appendix A. □

As derived above, the opportunism problem gives rise to the incentive condition (11) that is harder to satisfy (that is, only for more patient firms) than condition (8) for stable collusion. Hence, opportunism can make cartels less sustainable. Similar to the literature on the opportunism problem in single-shot games, this result depends on the beliefs. Due to their passive beliefs, the retailers do not react immediately to opportunism, which allows the manufacturers to 'trick' the retailers because the price that retailers expect will turn out to be incorrect if manufacturers jointly deviate. This joint deviation differs from a unilateral deviation of one manufacturer-retailer pair from a candidate equilibrium in which instead the belief of the deviating retailer is correct. In the latter case, it is optimal for the deviating manufacturer to set the wholesale price equal to the true costs. In the former case of a joint deviation, each retailer wrongly believes that the other retailer will buy at a high wholesale price and will thus sell at a high price, such that demand is high. It is, hence, profitable for the manufacturer to demand a high fixed fee in return for a marginal wholesale price below costs because the retailer believes that it can sell a large quantity. A marginal wholesale price below costs, however, only becomes profitable for a manufacturer when the retailer has a wrong belief. It could thus signal the retailer that its belief is wrong because, if it were correct, the manufacturer's offer would be dominated by an offer with a wholesale price of $w = 0$. The stricter condition for opportunism-proofness is thus an implication of the passive nature of the beliefs. We later consider symmetric beliefs, where retailers' beliefs instantly react to any change in wholesale prices.

If marginal wholesale prices below marginal costs are impossible (for instance, due to competition law), the incentive constraint for opportunism and for stability are identical, implying that any stable cartel is also opportunism-proof.

In any case, the opportunism problem is mitigated under grim-trigger beliefs compared to the one-shot game, or history-independent beliefs, because retailers with grim-trigger beliefs react to the deviation from the expected cartel wholesale prices by adapting their beliefs from the next period on to the level of competitive prices. This effectively punishes manufacturer opportunism, such that joint deviations are not profitable when manufacturers are sufficiently patient.

Interestingly however, we find that under grim-trigger beliefs, collusion is *not* formable. Because these beliefs are unforgiving, they are beneficial in supporting the collusive equilibrium, but do not allow for new cycles of collusion, even after a long time.

The characterized equilibrium is also renegotiation-proof in the sense of (Farrell and Maskin, 1989). Because the opportunism-proofness condition (11) ensures that manufacturers have no incentives to deviate jointly from the collusive agreement, and credible punishment is fulfilled as well, the equilibrium is *weakly* renegotiation-proof. Because the equilibrium features collusion on the industry-profit-maximizing prices that are Pareto-optimal for the manufacturers, the equilibrium is also *strongly* renegotiation-proof.

Corollary 1. *The equilibrium with grim-tigger beliefs described in Proposition 2 is strongly renegotiation-proof if the opportunism-proofness condition* (11) *is satisfied.*

Note that this corollary extends naturally to general trigger-strategies and more general beliefs as employed in the next section. Recall that the concept of renegotiation-proofness is applied to the coalition of manufacturers; but the result may be more general if retailers are allowed to be part of the negotiation, as long as beliefs are not affected.

5.3 Trigger beliefs

In this section, we define trigger beliefs and derive the conditions for the sustainability and for other properties of collusion. First, consider that the manufacturers play trigger-strategies similar to the grim-trigger strategy above, but a deviation is forgiven after κ periods.

We define trigger-strategies as in Green and Porter (1984):

Formally, let $y = (w^C, w^C)$ be a profile of collusive wholesale prices and let $z = (w^P, w^P)$ be a punishment wholesale price profile. Let κ denote a time length measured in periods. Define period t to be collusive if

(a) $t = 0$, or

(b) $t-1$ was collusive and $w_{it-1} = w^C$ for all i, or

(c) $t-\kappa$ was collusive and $w_{it-\kappa} \neq w^C$ for some i.

Define t to be reversionary otherwise. Manufacturer i sets

$$w_{it} = \begin{cases} y_i & \text{if } t \text{ is collusive,} \\ z_i & \text{if } t \text{ is reversionary.} \end{cases}$$

Assume that the collusive price equals the monopoly price ($w^C = w^M$), as defined in Subsection 5.1. Any deviation by one manufacturer causes the other manufacturer to carry out a punitive action of $w^P = 0$. Again, $w^P = 0$ must hold because a deviation during the reversionary periods is not punished due to the fact that the future actions and beliefs are fixed. Hence, the punishment action must be the same as the short-term optimal action, that is $w^P = 0$ as we demonstrated before. This maximizes the manufacturer profits because it aligns the incentives of retailer and manufacturer.

Definition 7. (Trigger beliefs)

Choose a collusive price level p^C and a punishment price level p^P. Formally, the retailers' beliefs correspond to the manufacturer strategies for collusive and reversionary periods as defined above:

$$p^e_{-it} = \begin{cases} p^C & \text{if } t \text{ is collusive} \\ p^P & \text{if } t \text{ is reversionary.} \end{cases}$$

When both manufacturers play trigger-strategies with actions w^C and w^P, the retailers' corresponding trigger-beliefs are correct on the collusive equilibrium path. They are also correct in the punishment phase.

To analyze the equilibrium of the dynamic game with trigger beliefs, it is necessary to consider the conditions that are needed to sustain collusion. After any deviation by a manufacturer, the beliefs revert to $p^e_{-i} = p^P$ for κ periods, which results in profits of $\Pi^P = p^P \cdot D_i(p^P, p^P)$. After κ periods, however, the retailers believe in collusion p^C again. Collusion is sustainable when no manufacturer wants to deviate from the trigger strategy given by the incentive condition:

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \delta \left(\frac{1-\delta^\kappa}{1-\delta} \Pi^P + \frac{\delta^\kappa}{1-\delta} \Pi^C \right). \tag{12}$$

This condition is more difficult to fulfill than the incentive condition (9) for grim-trigger strategies and beliefs. The punishment in condition (12) is less harsh and ends after κ periods, such that the expression on the right-hand side is larger than in the condition with grim-trigger strategies. Recall that manufacturers make the same profits as pairwise integrated manufacturer-retailer pairs (Proposition 2). Note that the individual profits, Π^C, Π^D, and Π^P, are still identical to an integrated firm's profits. This implies that the stability condition (12) is the same for vertically separated and vertically integrated manufacturer-retailer pairs whenever they play trigger strategies of length κ.

To check whether the equilibrium is opportunism-proof, we must consider a revised version of condition (11) that applies to trigger beliefs. Because trigger beliefs are forgiving after κ periods, a joint deviation of both manufacturers is not "punished by the beliefs" forever.[14] Hence, the conditions can be written as

$$\frac{\Pi^C}{1-\delta} \geq \Pi^{JD} + \delta \left(\frac{1-\delta^\kappa}{1-\delta} \Pi^P + \frac{\delta^\kappa}{1-\delta} \Pi^C \right). \tag{13}$$

Again, the opportunism-proofness condition (13) resembles the stability condition (12), except that $\Pi^{JD} > \Pi^D$, which makes the condition harder to meet. If condition (13) holds,

[14] By contrast, the grim-trigger beliefs switch forever to the competitive price level in response to a deviation. This effectively punishes the manufacturers that face pessimistic retailers from then on.

collusion is robust against joint deviations by the manufacturers, that is, opportunistic behavior.

The punishment is credible because in a punishment phase, both retailers believe $w^P = 0$ to which the best response is $w = 0$ as well. Beliefs are constant in the wholesale price played in the punishment phase. Thus, the argument in Subsection 5.2 applies, such that focusing on short-term best responses is valid.

To see that collusion is not formable with trigger beliefs, consider an argument by contradiction. Assume that in period $t = 0$, no collusion is played. It follows that it is impossible for retailers to hold collusive beliefs at any future point of the game. We summarize the above results in the following proposition:

Proposition 3. *With trigger beliefs, there exists an equilibrium in which manufacturers are able to sustain collusion on the industry-profit-maximizing wholesale price (with trigger strategies) if the discount factor is high enough to satisfy condition* (12)*. This condition is equivalent to the incentive constraint when manufacturers and retailers are pairwise integrated. Furthermore, the condition requires more patience than under grim-trigger beliefs and strategies. Only under the stricter condition* (13)*, collusion is also opportunism-proof. Punishment is credible, and collusion is not formable.*

Qualitatively, our insights for grim-trigger strategies carry over to more general trigger strategies. Trigger strategies with limited punishment imply stricter conditions for stability and opportunism-proofness compared to grim-trigger strategies. Nevertheless, they may be attractive for very relevant reasons that we do not model, including cost and demand shocks as well as other uncertainty that could result in unwarranted punishment, which – in case of grim-trigger strategies – would be very costly. The derived equilibrium with trigger beliefs is Pareto-efficient and features credible punishment. If, in addition, the manufacturers are patient enough for the condition of opportunism-proofness to hold, the equilibrium is strongly renegotiation-proof.

Corollary 2. *The equilibrium with trigger beliefs described in Proposition 3 is strongly renegotiation-proof if the opportunism-proofness condition* (13) *is satisfied.*

5.4 Adaptive beliefs

After analyzing beliefs that follow (grim-)trigger strategies, we provide a simple example of history-dependent passive beliefs that allow for collusion that is stable, formable, and opportunism-proof. Formability intuitively requires that beliefs can switch between competitive and collusive phases. The following beliefs allow this by adapting to the behavior of manufacturers. These adaptive beliefs also can be used to analyze equilibria with (grim-)trigger strategies. We define the beliefs in a way that they only depend on the actions of the manufacturers in the last T periods and not on the full history of the game.

Definition 8. (Adaptive Beliefs)

Beliefs are passive within each period t, that is, w_{it} does not affect the belief p^e_{-it}. Beliefs are dynamic: p^e_{-it} can depend on the history of past wholesale prices. There are three relevant histories:

1. In period t, the manufacturers offer contracts that are identical to the ex ante beliefs in period t. Then, both retailers retain the same belief in period $t+1$.

2. In period t, both manufacturers play the same $w \in W^*$ that differs from the ex ante beliefs of the retailers in period t. The same holds for all previous periods up to $t-(T-1)$, with $T \in \{1, 2, ...\}$ being a parameter measuring the adaptation length in periods. In $t+1$, both retailers hold the new (passive) belief p^*. The set W^* contains wholesale prices that the retailers accept as possible equilibria (for example, the collusive wholesale price w^C).

3. In period t, at least one of the manufacturers does not play a price consistent with a retailer's ex ante belief, and it is not the case that both manufacturers play a price $w \in W^*$. In $t+1$, both retailers hold the new belief p^P, where p^P is the wholesale price of the perfect Bayesian Nash equilibrium of the stage game (Nash reversal).

An important feature of these beliefs is that they allow for the formation of collusion. Keeping in mind our definition of formability, suppose that, starting from any history in period s, the manufacturers start to play the collusive wholesale price w^C. If the manufacturers want to form collusion, the worst history in terms of our beliefs is that there was no collusion in $s-1$. This implies that, for the next T periods, the retailers' beliefs are fixed at p^P, such that the manufacturers can extract lower transfers. Denote the resulting manufacturer profit by Π^F. In each of these formation periods, the retailers' beliefs remain at p^P, but the manufacturers have to play w^C to establish collusion and to change the retailers' beliefs in the future. In such a period, each manufacturer could deviate to a lower wholesale price and realize a profit of $\Pi^{F,D}$.

To analyze the equilibria with adaptive beliefs, consider the constraints implied by opportunism-proofness, credible punishment, and formability of collusion. In general, collusion can be sustained if manufacturers are sufficiently patient.

In the following, we use the example of grim-trigger strategies to describe the necessary incentive constraints that arise from considering adaptive beliefs. We also solve for trigger strategies and present the results in the proposition, relegating the exposition to the proof.

Stability. The stability condition for collusion, once established, is

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \frac{\delta \Pi^P}{1-\delta}. \tag{14}$$

In this case, retailers with adaptive beliefs already have the belief p^C and revert to the belief p^P after a deviation. Hence, with grim-trigger strategies, the condition is the same as the incentive condition (8). As a consequence, collusion is stable if manufacturers are sufficiently patient (δ is large enough). Again, the condition is identical to the stability condition under horizontal collusion.

Formability. Suppose that the retailers' belief is p^P, that is, the belief prevailing in periods of punishment and manufacturer competition. Hence, in the following inequalities, Π^P denotes the profit that manufacturers earn when they compete and play the perfect Bayesian equilibrium of the stage game. Formability is given if there is a transition path from competition to collusion, where – on the path – the manufacturers' actions are mutually best responses.

Suppose that the manufacturers decide to start colluding, while the retailers have a belief p^P. Both manufacturers start to play w^C with the usual grim-trigger strategies that punish any deviation. During formation, the retailers believe in competition, but the manufacturers set collusive prices. The retailers' beliefs are incorrect in transition periods (they adjust only after T periods). In the transition periods, the manufacturer profits are lower than under competition, $\Pi^F < \Pi^P$. The reason is that the beliefs are identical in both cases, and whereas the manufacturers play their unique best response to the belief in periods of competition, which results in Π^P, they play a worse action with respect to stage-game profits as response to the same belief in transition periods, resulting in a profit of Π^F.

With grim-trigger strategies, the following condition implies that manufacturers have an incentive to jointly start to collude in a competitive period, such that they eventually arrive at the collusive equilibrium path:

$$\frac{\Pi^P}{1-\delta} \leq \frac{1-\delta^T}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C. \tag{15}$$

The left-hand side contains the present value of perpetual competition. The first term on the right-hand-side is the discounted profit of the T periods in which retailers' belief is p^P, while manufacturers actually set w^C; the second term is the discounted profit of perpetual collusion starting after T formation periods.

Because manufacturer profits are lower during the formation period than under competition, each manufacturer may have an incentive to deviate during formation. Deviating during the formation phase yields a period profit of $\Pi^{F,D}$, but triggers a punitive action forever. Consider the incentives to stick to w^C in the formation phase. Under the following condition, no manufacturer wants to deviate unilaterally in the formation phase, which implies that actions in the transition are mutually best responses:

$$\frac{1-\delta^T}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C \geq \Pi^{F,D} + \frac{\delta\Pi^P}{1-\delta}. \tag{16}$$

Comparing the inequalities (15) and (16) shows that the latter is stricter if and only if $\Pi^{F,D} \geq \Pi^P$. This is always the case as we demonstrate in the proof of Proposition 4. Hence, the deviation condition (16) is not only necessary but also sufficient for formability. Note that formability decreases in T, that is, it holds for a smaller set of discount factors because the left-hand side decreases in T. This holds because an increase in T, which only affects the left-hand side of condition (16), shifts the weight Π^C to the smaller term Π^F.

Opportunism-proofness. Collusion is opportunism-proof if the manufacturers have no incentive to deviate jointly from the collusive price. Suppose the manufacturers jointly behave opportunistically in the present period. They can earn an opportunism profit of Π^{JD} by lowering w_i and increasing F_i for each retailer. As a result, the retailers believe in competition in the next period. Confronted with competitive beliefs, the reformation phase starts so that the manufacturers need to play w^C for T periods to convince retailers of collusive prices again. This yields the condition of opportunism-proofness

$$\frac{\Pi^C}{1-\delta} \geq \Pi^{JD} + \delta \left(\frac{\left(1-\delta^T\right)}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C \right). \tag{17}$$

For grim-trigger strategies, the stability condition is the same as for vertically integrated collusion. Collusion is formable and opportunism-proof if manufacturers are sufficiently patient, that is, if the condition (16) for formability and the condition (17) for opportunism-proofness hold. Increasing the adaptation length T of the beliefs makes collusion less formable but more opportunism-proof, that is, relaxes condition (17), but tightens condition (16). Opportunsim-proofness is harder to satisfy than stability whenever collusion is formable. To see this, compare the right-hand side of Condition (17) with the stability condition (14) and note that $\Pi^{JD} > \Pi^D$.

Punishment is not credible whenever collusion is formable. To see this, note that whenever the formability condition holds, and the manufacturers are supposed to punish, they are better off forming collusion again. This implies that formation dominates competitive pricing in the punishment phase.

Note that grim-trigger strategies are a special case of trigger strategies with $\kappa \to \infty$. For trigger strategies, we can generalize the result as follows:

Proposition 4. *With adaptive beliefs and trigger strategies, the stability condition is the same as for vertically integrated collusion with the equivalent trigger strategies. Stability increases in the number of punishment periods κ. Collusion is formable and opportunism-proof if δ is sufficiently large. Increasing the adaptation periods T of the retailer beliefs makes collusion less formable but more opportunism-proof. Punishment is not credible for any $\kappa > 0$ whenever collusion is formable, but collusion may be sustained even without punishment, that is, for $\kappa = 0$.*

Proof. See Appendix A. □

The take-away from our analysis of adaptive beliefs is that there exist beliefs that make collusion formable, sustainable, and opportunism-proof. Formability intuitively requires that beliefs can adapt to collusion after a 'history' of competition or punishment. The adaptability can have a cost, the adaptable beliefs do make punishment non-credible because the credibility of punishment with trigger beliefs was supported by the belief alone. An interesting observation is that for adaptive beliefs, there is a trade-off between opportunism-proofness and formability: The longer beliefs take to adapt the harder it is to start collusion, while opportunistic behavior that counts on restarting collusion becomes less of a problem.

As a polar case, we find that the strategy to always collude, the degenerate trigger strategy with a punishment length of zero, can support a collusive equilibrium. If manufacturers play "always collusion", the incentive constraint for stability is

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \delta\left(\frac{1-\delta^T}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C\right). \tag{18}$$

Note that the stability is supported by the beliefs because deviation requires a new formation of collusion of length T. While this strategy is the least stable strategy, it is the only strategy that features formability and, in addition, features credible punishment.

Corollary 3. *For $\kappa > 0$, there exists a strongly renegotiation-proof equilibrium that is not formable if T and δ are sufficiently large. For $\kappa > 0$ and if formability is given, there is no renegotiation-proof equilibrium with trigger strategies. For $\kappa = 0$, a strongly renegotiation-proof equilibrium exists if the opportunism-proofness condition* (17) *holds (implying also stability).*

Proof. The equilibrium path is always Pareto-efficient by assumption, such that any weakly renegotiation-proof equilibrium is also strongly renegotiation-proof.

To find the critical value of T, such that collusion is not formable, but opportunism-proof for $\kappa > 0$, we must compare the relevant conditions. If collusion is not formable, punishment will be credible. The equilibrium is thus weakly renegotiation-proof for certain values of T. As shown in the proof of Proposition 4, the relevant condition for formation is the condition whereby a deviation from formation is not profitable:

$$\delta^T\left(\Pi^C - \Pi^F\right) \geq \left(\frac{1-\delta}{1-\delta^{\kappa+1}}\right)\Pi^{F,D} - \Pi^F + \delta\left(\frac{1-\delta^{\kappa}}{1-\delta^{\kappa+1}}\right)\Pi^P > 0.$$

Because $\delta < 1$, this condition is violated for sufficiently large T, such that δ^T and thus the left-hand side of the formation condition becomes arbitrarily small.

The condition for opportunism-proofness 17can be written as

$$\delta^T\left(\Pi^C - \Pi^F\right) \leq \frac{1}{\delta}\left[\Pi^C - (1-\delta)\,\Pi^{JD} - \delta\Pi^F\right].$$

Because for $\delta < 1$, the left-hand side of the opportunism-proofness condition also becomes arbitrarily small as T increases, there exists a T, such that the opportunism condition holds, whenever the right-hand side is non-negative (which holds for sufficiently large δ), while the formation condition is violated.

□

6 Symmetric Beliefs

After considering passive beliefs, we turn to the analysis of the case that retailers have symmetric beliefs defined as follows:

Definition 9. (Symmetric Beliefs) The price expectation p^e_{-it} of retailer i in period t about the price of retailer $-i$ is $p_i\left(w_{it}, p(w_{it})\right)$.

In other words, when a retailer receives an unexpected offer deviating from the candidate equilibrium, the retailer revises its belief and believes that its rival has received the same offer by its manufacturer. Symmetric beliefs are history-independent because they only rely on the information contained in the current wholesale price offer. We assume that manufacturers play (grim-)trigger as in the previous sections. The case of symmetric beliefs is also analyzed in Liu and Thomes (2020)who consider the linear demand case and, hence, offer closed-form solutions for the critical discount factor.

We focus again on industry-profit-maximizing collusion that naturally arises when manufacturers jointly maximize their profits given symmetric beliefs. Denote the price expectation of retailer i with symmetric beliefs by p^e_{it} as above. This allows the manufacturer to essentially choose the symmetric price level, such that the joint maximization problem of the manufacturerscan be rewritten as

$$\Pi^C := \frac{1}{2}\max_p p \cdot D_i\left(p, p\right) + p \cdot D_{-i}\left(p, p\right) = \Pi^M.$$

Hence, the joint-profit-maximizing wholesale price of the manufacturers is equal to the industry-profit-maximizing price. Moreover, this implies that any joint deviation by the manufacturers will always be the industry-profit-maximizing price if retailers hold symmetric beliefs. In contrast to (grim-)trigger beliefs, where manufacturers could jointly optimally deviate to a wholesale price below zero, such a deviation is not optimal because only the collusive price maximizes the manufacturers' profits. Because the manufacturers collude at the Pareto-efficient level, collusion is opportunism-proof. Additionally, punishment is not credible either because the manufacturers would prefer to revert back to collusion in every punishment period.

Collusion is formable because symmetric beliefs instantly adapt to the new wholesale price in every period. Manufacturers only need to agree on the collusive price and set it in any period. In a period in which the manufacturers set wholesale prices of w^C, the retailers' expectations are immediately equal to $p_i\left(w^C, p(w^C)\right) = p_i(w^C, p^C) = p^C$, which corresponds to the expectation of collusion. Forming collusion immediately leads to stable collusion as long as the stability condition is fulfilled.

From Pagnozzi and Piccolo (2011), we know that symmetric beliefs affect competition between vertically separated manufacturers. Competition is less fierce due to a so-called belief effect, which increases the competitive wholesale price above marginal costs. If punishment, however, relies on the competitive wholesale prices and profits, the stability of collusion is affected. Manufacturers must be more patient to satisfy the condition of stable collusion.

Symmetric beliefs violate the "no-signaling-what-you-don't-know" Condition 1.[15] We thus look for a perfect Bayesian equilibrium in the context of symmetric beliefs:

Proposition 5. *With symmetric beliefs, collusion with (grim-)trigger strategies is stable if condition 24 holds. Collusion is also formable if the stability condition holds. Collusion is always opportunism-proof, but punishment is never credible.*

Proof. See Appendix A. □

Symmetric beliefs solve the opportunism problem by making it impossible to lower the wholesale prices without negatively affecting the belief. Because the symmetric belief follows the manufacturers' actions, it does not restrict and commit the manufacturers as, for example, the trigger beliefs do. This implies that formation, in the sense of needing to convince retailers, is not an issue with symmetric beliefs. As a downside for collusion, symmetric beliefs do not support the credibility of punishment because they make the punishment phase prone to renegotiation incentives. This yields the following result:

Corollary 4. *The equilibrium with symmetric beliefs described in Proposition 5 is not renegotiation-proof.*

7 Conclusion

When beliefs of retailers do not react to present and past behavior, we document that collusion is not feasible. Such beliefs may arise in industries that have long-standing competitive conduct. Thus, an implication could be that belief differences over industries may explain why some industries stay competitive, while other industries give birth to collusion over and over again. Because these beliefs give rise to a perfect Bayesian equilibrium of the infinitely repeated game and, consequently, are correct on the equilibrium path, they are self-fulfilling and may never be challenged.

We then analyze beliefs that react to observed past actions. These beliefs can support trigger-based collusive equilibria and even make the punishment credible in situations in which this would not be the case in vertically integrated industries. We show that opportunism can still be the most important challenge for the colluding firms, more so than the usual unilateral deviation incentives.

Beliefs that mirror the collusive strategy might not be flexible enough to allow for the creation of collusion. The intuition here is that such beliefs cannot handle the transition from a non-collusive state to working collusion because they would be too "pessimistic" about the future whenever they observe a non-collusive price in the history of the game.

The retailer beliefs studied in this paper have opposing effects on the ability of manufacturers to collude. Trigger beliefs and particularly grim-trigger beliefs feature credible

[15] Pagnozzi and Piccolo (2011) discuss this observation and also show that, with common cost shocks, symmetric beliefs can be consistent with a PBE.

punishment and opportunism-proofness because they make it difficult for manufacturers to obtain high wholesale prices again after a breakdown. These beliefs, however, do not allow for the formation of collusion.

We introduce a new belief that is adaptive to the manufacturers' pricing over time. From the manufacturers' point of view, the advantage of an adaptive retailer belief is that it facilitates the formation of collusion. Because this belief does not mirror the collusive strategy, the belief does not support credible punishment; it may still support a credible collusive strategy because it 'punishes' deviation because retailers believe in competitive conduct for a certain number of periods. Adaptive beliefs can also satisfy the conditions for stability and opportunism-proofness.

Our approach of explicitly characterizing belief types and discussing the equilibria consistent with such beliefs differs from the usual logic of Nash equilibrium where players know their rivals' strategy. The concept of PBE in our game of complete information but unobservable action gives some flexibility to beliefs that allow for misunderstandings with regard to the equilibrium path. This sheds more light on how an outcome can be reached and sustained depending on the type and dynamics of beliefs. Interestingly, the opportunism problem of colluding firms highlights that communication among manufacturers may be a source of misinterpretation by retailers. This contrasts the understanding that communication usually reduces strategic uncertainty as contributed by Blume (1994) as well as Blume and Heidhues (2008). We suggest that potential strategic misinterpretations and misunderstandings are important for the feasibility of collusion in vertically related markets.

Our setting would, furthermore, allow for a combination of adaptive beliefs and trigger beliefs that could improve manufacturer collusion. These retailer beliefs, which one could call *adaptive trigger beliefs*, would inherit the credibility of punishment from the trigger beliefs and combine it with formability of the adaptive beliefs. The idea is that the beliefs are adaptive in competitive periods that are not punishment periods, such that the formation of collusion is feasible. Once the formation process is complete, the adaptive beliefs switch to the trigger beliefs. Consequently, in punishment periods that are triggered by a deviation from the collusive prices, the belief is not adaptive. This lasts until the punishment phase is over; at this point, the beliefs switch from trigger beliefs to adaptive beliefs again. Together, these beliefs would be able to meet all the criteria, making collusion both renegotiation-proof and formable.

Because manufacturer cartels are ubiquitous, our results may help competition authorities to screen markets. Whenever supply contracts are not public or easily renegotiable, we find that the ability to form and sustain collusion critically depends on retailers' beliefs about the supply conditions of other retailers. This may make it easier to sustain collusion in markets in which the retailers are used to manufacturer collusion. Our findings suggest that it should be in the interest of colluding manufacturers to manage and influence their retailers' beliefs about the conditions in the wholesale market. One conjecture is thus that the opportunism problem may be one of the causes behind the widespread use of resale price maintenance and hub-and-spoke arrangements when manufacturers collude.[16] A more direct control of retail prices by manufacturers in form of resale-price maintenance may circumvent the problem of skeptical retailer beliefs. Similarly, coordinated downsizing of packages by manufacturers, as observed in the chocolate case in Germany, may be used to reduce strategic uncertainty for retailers. Consequently, coordinated behavior and communication of manufacturers vis à vis their retailers may deserve more antitrust scrutiny because such coordination can be essential for making manufacturer cartels work.

[16]See footnote 1.

References

Aoyagi, M., Frechette, G., and Yuksel, S., "Beliefs in Repeated Games." ISER Discussion Paper 1119r, Institute of Social and Economic Research, Osaka University (2021).

Athey, S., Bagwell, K., and Sanchirico, C., "Collusion and Price Rigidity." *The Review of Economic Studies*, Vol. 71 (2004), pp. 317–349.

Bernheim, B. D., Peleg, B., and Whinston, M. D., "Coalition-Proof Nash Equilibria i. Concepts." *Journal of Economic Theory*, Vol. 42 (1987), pp. 1–12.

Bernheim, B. D. and Ray, D., "Collective dynamic consistency in repeated games." *Games and Economic Behavior*, Vol. 1 (1989), pp. 295–326.

Blume, A., "Intraplay Communication in Repeated Games." *Games and Economic Behavior*, Vol. 6 (1994), pp. 181–211.

Blume, A. and Heidhues, P., "Modeling Tacit Collusion in Auctions." *Journal of Institutional and Theoretical Economics*, (2008), pp. 163–184.

Bos, I. and Harrington, J. E., "Endogenous cartel formation with heterogenous firms." *The RAND Journal of Economics*, Vol. 41 (2010), pp. 92–117.

Bos, I., Marini, M. A., and Saulle, R. D., "Cartel formation with quality differentiation." *Mathematical Social Sciences*, Vol. 106 (2020), pp. 36–50.

Buehler, S. and Gärtner, D. L., "Making sense of nonbinding retail-price recommendations." *American Economic Review*, Vol. 103 (2013), pp. 335–59.

d'Aspremont, C., Jacquemin, A., Gabszewicz, J., and Weymark, J., "On the Stability of Collusive Price Leadership." *Canadian Journal of Economics*, Vol. 16 (1983), pp. 17–25.

Do, J. and Miklos-Thal, J., "Opportunism in Vertical Contracting: A Dynamic Perspective." *available at SSRN*, (2021).

Farrell, J. and Maskin, E., "Renegotiation in repeated games." *Games and Economic Behavior*, Vol. 1 (1989), pp. 327–360.

Fudenberg, D. and Tirole, J., *Game Theory*, Vol. 1 of *MIT Press Books*. The MIT Press (1991a).

Fudenberg, D. and Tirole, J., "Perfect Bayesian equilibrium and sequential equilibrium." *Journal of Economic Theory*, Vol. 53 (1991b), pp. 236–260.

Goldlücke, S. and Kranz, S., "Renegotiation-proof relational contracts." *Games and Economic Behavior*, Vol. 80 (2013), pp. 157–178.

Green, E. and Porter, R., "Noncooperative Collusion under Imperfect Price Information." *Econometrica*, Vol. 52 (1984), pp. 87–100.

Harrington, J. E., "A theory of collusion with partial mutual understanding." *Research in Economics*, Vol. 71 (2017), pp. 140–158.

Harrington, J. E., "How Do Hub-and-Spoke Cartels Operate? Lessons from Nine Case Studies." *available at SSRN*, (2018).

Harrington, J. E. and Chang, M.-H., "Modeling the Birth and Death of Cartels with an Application to Evaluating Competition Policy." *Journal of the European Economic Association*, Vol. 7 (2009), pp. 1400–1435.

Harrington, J. E. and Zhao, W., "Signaling and tacit collusion in an infinitely repeated Prisoners' Dilemma." *Mathematical Social Sciences*, Vol. 64 (2012), pp. 277–289.

Hart, O., Tirole, J., Carlton, D. W., and Williamson, O. E., "Vertical integration and market foreclosure." *Brookings papers on economic activity. Microeconomics*, Vol. 1990 (1990), pp. 205–286.

Holler, E. and Rickert, D., "Upstream Collusion and Vertical Restraints: An Anatomy of the German Coffee Cartel." (2021).

Hunold, M. and Muthers, J., "Manufacturer Cartels and Resale Price Maintenance." Economics working papers 2020-06, Department of Economics, Johannes Kepler University Linz, Austria (2020).

Jullien, B. and Rey, P., "Resale price maintenance and collusion." *The RAND Journal of Economics*, Vol. 38 (2007), pp. 983–1001.

Liu, F. and Thomes, T. P., "Manufacturer Collusion-Vertical Integration vs. Delegation with Private Contracts." (2020).

Mailath, G. J., Nocke, V., and White, L., "When And How The Punishment Must Fit The Crime." *International Economic Review*, Vol. 58 (2017), pp. 315–330.

Martin, S., Normann, H.-T., and Snyder, C. M., "Vertical foreclosure in experimental markets." *RAND Journal of Economics*, (2001), pp. 466–496.

McAfee, R. and Schwartz, M., "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity." *American Economic Review*, Vol. 84 (1994), pp. 210–30.

McCutcheon, B., "Do meetings in smoke-filled rooms facilitate collusion?" *Journal of Political Economy*, Vol. 105 (1997), pp. 330–350.

Miklós-Thal, J. and Shaffer, G., “Naked Exclusion with Private Offers.” *ERN: Expectations in Economic Theory & Markets (Topic)*, (2016).

Nocke, V. and White, L., “Do vertical mergers facilitate upstream collusion?” *American Economic Review*, Vol. 97 (2007), pp. 1321–1339.

Normann, H.-T., “Vertical integration, raising rivals’ costs and upstream collusion.” *European Economic Review*, Vol. 53 (2009), pp. 461–480.

O’Brien, D. P. and Shaffer, G., “Vertical control with bilateral contracts.” *The RAND Journal of Economics*, (1992), pp. 299–308.

Pagnozzi, M. and Piccolo, S., “Vertical separation with private contracts.” *The Economic Journal*, Vol. 122 (2011), pp. 173–207.

Piccolo, S. and Miklós-Thal, J., “Colluding through suppliers.” *The RAND Journal of Economics*, Vol. 43 (2012), pp. 492–513.

Piccolo, S. and Reisinger, M., “Exclusive territories and manufacturers’ collusion.” *Management Science*, Vol. 57 (2011), pp. 1250–1266.

Rey, P. and Tirole, J., “A Primer on Foreclosure.” Vol. 3, chap. 33, pp. 2145–2220, Elsevier, 1 edn. (2007).

Rey, P. and Vergé, T., “Bilateral Control with Vertical Contracts.” *The RAND Journal of Economics*, Vol. 35 (2004), pp. 728–746.

Schinkel, P. M., Tuinstra, J., and Rüggeberg, J., “Illinois Walls: how barring indirect purchaser suits facilitates collusion.” *The RAND Journal of Economics*, Vol. 39 (2008), pp. 683–698.

Segal, I., “Contracting with externalities.” *The Quarterly Journal of Economics*, Vol. 114 (1999), pp. 337–388.

Selten, R., “A simple model of imperfect competition, where 4 are few and 6 are many.” *International Journal of Game Theory*, Vol. 2 (1973), pp. 141–201.

Zhang, D., “Testing Passive versus Symmetric Beliefs in Contracting with Externalities.” *International Economic Review*, Vol. 62 (2021), pp. 723–767.

Appendix A: Proofs

Proof of Proposition 1.

Proof. Step (i): Let us construct this equilibrium by showing that it is uniquely optimal for each manufacturer to set $(w_i = 0, F_{it} = \pi_i^*(w_i = 0, p^e_{-it}))$ in each period independent of the strategy of the other manufacturer

The participation constraint of the retailer is binding in equilibrium because otherwise the manufacturer could increase profits by raising the fixed fee without affecting p^e_{-it} by the assumption of passive beliefs. Hence, $F_{it} = \pi_i^*(w_i = 0, p^e_{-it}))$ holds.

On the equilibrium path, $p^e_{-it} = p^*_{-it}$, that is, retailers' beliefs are correct and thus identical to manufacturer's conjectures such that the in-period manufacturer profits can be simplified as in the stage game to:

$$\Pi_{it}\left(w_{it}, p^e_{-it}\right) = p_i\left(w_{it}, p^e_{-it}\right) \cdot D_i\left(p_{it}\left(w_{it}, p^e_{-it}\right), p^e_{-it}\right).$$

Note that this in-period profit is insulated from the actual actions of the other manufacturer, and w_{it} only affects the manufacturer profits through the price setting of the retailer. Because the manufacturers' profit in each period, on any equilibrium path, only depends on w_{it} and the belief p^e_{-it}, the discounted equilibrium profits of a manufacturer do not depend on the strategy of the other manufacturer either. Fixing p^e_{-it}, Equation (2) implies that $w_i = 0$ is optimal (independent of w_{-it} in each period). Hence, there is an equilibrium path with each manufacturer setting $w_i = 0$ in every subgame and a matching time-constant belief by retailers, where the time-constant retail price follows because the retail price equilibrium is unique for $w_i = w_{-i} = 0$.

Step (ii) Next, we exclude other equilibrium paths in which $w_{it} \neq 0$ by contradiction: Suppose that there is an equilibrium path with $w_{it} \neq 0$ in some periods. It follows from Equation 2 that each manufacturer can increase current period profits through setting $w_{it} = 0$ and $F_{it} = \pi(0, p^e_{-it})$, resulting in $\Pi_{it}\left(0, p^e_{-it}\right)$. Because with history-independent passive beliefs, this action does not affect the retailer's future acceptance decisions, we can replace any period in which $w_{it} \neq 0$ by setting $w_{it} = 0$ and $F_{it} = \pi(0, p^e_{-it})$, each time increasing the deviating manufacturer's profits. Doing so finitely often, as a result of continuity at infinity, shows a profitable deviation exists, which is a contradiction.

□

Proof of Proposition 2.

Proof. We established in the main text that manufacturers are able to *sustain* collusion if equation (9) is fulfilled. To show that the incentive constraint is the same if manufacturers and retailers are pairwise integrated, it is sufficient to prove that the profits are the same:

$\Pi_V^j = \Pi_I^j$ for $j = C, D, P$ (V: vertical, I: integrated). Because we define Π_V^C to be half of the integrated industry-maximizing profit (4), the profit from collusion in the integrated case Π_I^C is the same.

The deviation profit Π_V^D is given by the first line of equation (5) and simplifies further in the second line because the manufacturers' true actions and retailers' beliefs are aligned. The second line, however, is equal to the maximization problem of an integrated manufacturer that maximizes the profit with respect to the retail price. Lastly, the punishment profits are aligned as well, following the same argument as before. The punishment profits are given by equation (7). Manufacturers set $w^P = 0$, which results in the same retail prices and profits as in a vertically integrated industry.

To see that punishment is *credible*, note that with grim-trigger beliefs, any deviation leads to the belief p^P forever such that the current actions of the manufacturers have no effect on the belief of retailers. To establish this we check for a joint action by manufacturers that would yield a Pareto improvement for manufactures. Note that it is the best response is, in each period, for each manufacturer, to set $w = 0$ individually. Next we consider the profit maximization for manufacturers when they would optimize jointly during a punishment period:

$$\begin{aligned}\frac{1}{2}\max_{w_{it},w_{-i,t}} w_{it} \cdot D_i\left(p_i(w_{it}, p^P), p_{-i}(w_{-it}, p^P)\right) + \left[p_i(w_{it}, p^P) - w_{it}\right] \cdot D_i\left(p_i(w_{it}, p^P), p^P\right) \\ + w_{-it} \cdot D_{-i}\left(p_{-i}(w_{-it}, p^P), p_i(w_{it}, p^P)\right) + \left[p_{-i}(w_{-it}, p^P) - w_{-it}\right] \cdot D_{-i}\left(p_{-i}(w_{-it}, p^P), p^P\right).\end{aligned}$$

We again assume that this profit is quasi-concave such that we can use first-order conditions. Using the retailers' first-order conditions and applying symmetry, using that manufacturer profits are well-behaved by assumption such that the optimum is symmetric, the first-order conditions can be rewritten as

$$\left[\underbrace{D_i\left(p_i(w, p^P), p_{-i}(w, p^P)\right) - D_i\left(p_i(w, p^P), p^P\right)}_{\geq 0}\right] + w\underbrace{\frac{\partial p_i}{\partial w_{it}}}_{>0}\left[\underbrace{\frac{\partial D_i}{\partial p_i} + \frac{\partial D_{-i}}{\partial p_i}}_{<0}\right] = 0 \ \forall i. \qquad (19)$$

This holds for $w = 0$. Hence, playing $w = 0$ is the best manufacturers can do such that there is no continuation game that can be reached by manufacturers and that yields larger profits.

To see that collusion is *not formable*, consider any history with $w_i \neq w^C$ in $t = 0$. With this history, grim-trigger beliefs imply that the belief is p^P forever, which violates the second condition of the definition of formability that a collusive PBE can be obtained in some future period for any history. □

Proof of Proposition 4.

Proof. Similarly to the incentive constraints above the proposition for grim-trigger strategies, we can also describe the incentive constraints when manufacturers play more general trigger strategies. The difference between these strategies is the length of the punishment phase κ. Because neither the formation condition (15) nor the opportunism-proofness condition (17) rely on punishment between the colluding firms, the conditions remain the same.

In the remainder of the proof, we characterize the stability condition and the stability condition of the formation phase for trigger strategies. If firms play trigger strategies with punishment length κ, the incentive constraint for stability is

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \delta \left(\frac{1-\delta^\kappa}{1-\delta} \right) \Pi^P + \delta^{\kappa+1} \left(\frac{1-\delta^T}{1-\delta} \right) \Pi^F + \delta^{\kappa+1} \frac{\delta^T}{1-\delta} \Pi^C.$$

As in all the above cases, the punishment must be $w^P = 0$ because there is no unilateral action in a punishment phase that has any effect on future beliefs. Because the future beliefs and actions of the rival manufacturer are fixed in any punishment period, each manufacturer must play the one-shot best response of $w^P = 0$.

The deviation of one manufacturer triggers a punishment of length $\kappa \in [0, \infty)$. After the punishment phase, collusion is resumed, but the adaptive retailer beliefs require a formation phase of length T. A deviation in the formation phase is not profitable if

$$\frac{1-\delta^T}{1-\delta} \Pi^F + \frac{\delta^T}{1-\delta} \Pi^C \geq \Pi^{F,D} + \delta \left(\frac{1-\delta^\kappa}{1-\delta} \right) \Pi^P + \delta^{\kappa+1} \left(\frac{1-\delta^T}{1-\delta} \right) \Pi^F + \delta^{\kappa+1} \frac{\delta^T}{1-\delta} \Pi^C, \quad (20)$$

which simplifies to

$$\delta^T \left(\Pi^C - \Pi^F \right) \geq \left(\frac{(1-\delta)}{1-\delta^{\kappa+1}} \right) \Pi^{F,D} - \Pi^F + \delta \left(\frac{1-\delta^\kappa}{1-\delta^{\kappa+1}} \right) \Pi^P. \quad (21)$$

Next, we show that condition (15) is always stricter than condition (20). We also demonstrate that this is the relevant condition for formation. To see this, we consider the polar cases that correspond to a strategy "always collude" for $\kappa = 0$ and the grim-trigger strategy as $\kappa \to \infty$. We already know from the analysis before the proposition that the postulated relation of the conditions holds for $\kappa \to \infty$.

For $\kappa = 0$, the incentive constraint for formation can be written as

$$\delta^T \left(\Pi^C - \Pi^F \right) \geq \Pi^P - \Pi^F.$$

A deviation from the collusive price in the formation phase results in a profit of $\Pi^{F,D}$. However, for $\kappa = 0$, no punishment is triggered. The stability condition for formation becomes

$$\frac{1-\delta^T}{1-\delta} \Pi^F + \frac{\delta^T}{1-\delta} \Pi^C \geq \Pi^{F,D} + \delta \left(\frac{1-\delta^T}{1-\delta} \frac{\Pi^F}{1-\delta} + \frac{\delta^T}{1-\delta} \Pi^C \right),$$

which simplifies to

$$\delta^T \left(\Pi^C - \Pi^F \right) \geq \Pi^{F,D} - \Pi^F. \quad (22)$$

Observe that a deviation in the formation phase is not profitable under a stricter condition than the incentive condition for formation if $\Pi^{F,D} \geq \Pi^P$. Additionally, a necessary condition is $\Pi^C \geq \Pi^{F,D}$ because the discount factor δ is in the range $(0,1)$. Because $\delta < 1$, the left-hand side of condition (22) decreases in T, such that formability becomes harder to satisfy the larger T.

Thus, for both cases of $\kappa = 0$ and $\kappa \to \infty$, we demonstrated that sticking to the formation phase requires higher values of δ (the condition is more strict) if $\Pi^{F,D} \geq \Pi^P$. Let us now analyze intermediary values of κ.

Inequality (21) differs from the formation incentives. Recall that the formation condition (15) is

$$\delta^T \left(\Pi^C - \Pi^F\right) \geq \Pi^P - \Pi^F$$

and thus independent of κ. For a given value of δ, the weight of the term $\Pi^{F,D}$ is strictly monotonically decreasing in κ, whereas the weight of Π^P is strictly monotonically increasing in κ. Hence, the condition $\Pi^{F,D} \geq \Pi^P$ is sufficient for any $\kappa \in [0,\infty)$ to guarantee that condition (21) is tighter than condition (15). This implies that the stability condition for formation is the relevant condition for general trigger strategies.

Next, we show that

$$\Pi^{F,D} \geq \Pi^P$$

holds because the profit of i increases in the wholesale price of the competitor for given beliefs due to the fact that this increases the price of the competing retailer and increases demand for i. The profit under competition is

$$\Pi^P\left(w_i = 0, p^e_{-i} = p^P\right) = \underbrace{p_i\left(0, p^P\right)}_{p_i^P} \cdot D_i\left(p^P, p^P\right).$$

A firm that deviates form formation plays $w_i = 0$ because this maximizes the unilateral spot profit. The deviation profit $\Pi^{F,D}$ is similar to the punishment profit and only consists of the revenue $p_i(\cdot) \cdot D_i(\cdot)$. For $\Pi^{F,D}$, however, both the manufacturer and the retailer expect a higher wholesale price of w^C of the other manufacturer, yielding a higher price at the competing retailer. Because profits increase in the rival's price, $\Pi^{F,D} > \Pi^P$ holds.

Finally, observe that punishment is not credible whenever collusion is formable. To see this consider that the formability condition implies that is better to jointly deviate to a new formation of collusion in any period in which punishment profits are expected . This also implies that only in the case $\kappa = 0$, formability and credible punishment are both met because punishment is not part of the strategy. □

Proof of Proposition 5.

Proof. We will show all results for general trigger strategies as defined in Subsection 5.3, which includes grim-trigger strategies as a subcase for $\kappa \to \infty$. Let us first consider the stability condition. As argued in the text before the proposition, manufacturers collude on the industry-maximizing level and earn profits of $\Pi^C = \Pi^M/2$. If the manufacturers make a one-sided deviation from the collusive agreement, they must consider that changing their wholesale price also influences the belief of their own retailer. A deviating manufacturer maximizes the following problem:

$$w^D = \arg\max_{w_i} w_{it} \cdot D_i \left(p_i(w_{it}, p(w_{it})), p_{-i}(w^C, p(w^C)) \right) \\ + \left(p_i(w_{it}, p(w_{it})) - w_{it} \right) \cdot D_i \left(p_i(w_{it}, p(w_{it})), p(w_{it}) \right)).$$

This results in a profit Π^D for the deviating manufacturer. Note that $w^D > 0$. Punishment is assumed to be carried out on the competitive wholesale price level. The competitive benchmark corresponds to the case analyzed in Pagnozzi and Piccolo (2011). Under symmetric beliefs, manufactures solve

$$\max_{w_i} w_{it} \cdot D_i \left(p_i \left(w_{it}, p(w_{it}) \right), p_{-i} \left(w_{-it}, p(w_{-it}) \right) \right) + \left(p_i(w_{it}, p(w_{it})) - w_{it} \right) \cdot D_i \left(p_i \left(w_{it}, p(w_{it}) \right), p(w_{it}) \right).$$

This results in Π^P. By using the envelope theorem, the first-order condition simplifies to

$$\underbrace{w_i \underbrace{\frac{\partial D_i(\cdot)}{\partial p_i}}_{<0}}_{<0} + \underbrace{\left(p(w_i, p(w_i)) - w_i \right) \cdot \underbrace{\frac{\partial D_i(\cdot)}{\partial p_{-i}}}_{>0}}_{\text{belief effect:}>0} = 0. \tag{23}$$

Applying symmetry to (23) defines the equilibrium wholesale prices w^P. Note that the right-hand side of37 is positive at $w = 0$, which implies that $w^P > 0$. Under competition, with symmetric beliefs, prices are above the price level under competition with passive beliefs. The stability condition is given by

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \delta \left(\frac{1-\delta^\kappa}{1-\delta} \Pi^P + \frac{\delta^\kappa}{1-\delta} \Pi^C \right). \tag{24}$$

To see why collusion is formable with symmetric beliefs, recall the definition of formability. Symmetric beliefs allow for forming collusion if there exists a strategy profile, such that for this belief, best responses are played in period $t \geq s$, and there exists a weak PBE that results in payoff V^C. Given any history before period s – the period in which collusion is about to be formed –, manufacturers play mutually best responses in the following periods when setting wholesale prices that result in the collusive price defined by equation (3). The joint profit maximization of both manufacturers and setting w^C is a weak PBE if the stability condition for collusion – equation (24) – is fulfilled. In this equilibrium, both manufacturers earn a payoff of $V^C = \Pi^C/(1-\delta)$. Hence, collusion is formable according to our definition.

As in Subsection 5.4, it should be considered whether there exist incentives to deviate from formation. Due to symmetric beliefs, collusion is in place directly after it is formed, such that the condition for deviating from formation is identical to deviating from collusion in equation (24).

To check whether collusion is opportunism-proof, we consider the joint maximization problem of the manufacturers. As shown in the text, this leads to the Pareto-efficient wholesale price level. That is, manufacturers always prefer to set w^C when jointly maximizing. Thus, there is no scope for opportunistic behavior because the beliefs directly react to any change in wholesale prices. Following the same argument, punishment is not credible because manufacturers would prefer to renegotiate and revert to setting w^C jointly. $\square$