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Opportunism Problems of Colluding Manufacturers*

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Abstract

In a market with two exclusive manufacturer-retailer pairs, we show that colluding manufacturers may not be able to attain supra-competitive profits when contracts with retailers are secret. The stability of manufacturer collusion depends on the retailers' beliefs. We consider various dynamic beliefs and find that industry-profit-maximizing collusion is feasible for some. Collusion is even renegotiation-proof under trigger beliefs if a novel condition of opportunism-proofness holds, which can be more demanding than the standard stability condition. Trigger beliefs are not flexible enough to allow for formation of collusion. We demonstrate that adaptive beliefs may be necessary for the formation of manufacturer collusion in a non-collusive industry.

JEL classification: C73, D43, L13, L41, L81.

Keywords: opportunism, credible punishment, cartel formation, manufacturer collusion, vertical relations, renegotiation-proof, secret contracting.

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1 Introduction

In vertically related markets, a monopolist may face an opportunism problem. A monopolist may fail to implement monopoly prices because retailers facing secret contract offers may fear that the monopolist has offered better terms to the rival retailers (Hart et al., 1990; McAfee and Schwartz, 1994; Rey and Vergé, 2004). We address the question whether – absent any coordination with retailers – such an opportunism problem also exists when two (or more) manufacturers try to collude. There is evidence from a number of cartel cases where manufacturers failed to achieve a retail price increase when they did not communicate with their retailers that were not part of the cartel. For example, in the German coffee cartel, the coffee roasters only sustained higher wholesale prices after coordinating a retail price increase with retailers.[1] The same issue to convince retailers to accept higher wholesale prices appears to be the underlying problem in a number of so-called hub-and-spoke cartels.[2] The evidence points to the problem that colluding manufacturers may struggle to implement higher prices without sharing information on vertical contracts, or vertical price coordination.

To study whether an opportunism problem also exists for manufacturers aiming to collude, we set up an infinitely repeated pricing game that features private contracting within each of two exclusive manufacturer-retailer pairs. From the literature we know that the market outcome with a monopoly manufacturer depends on the beliefs that retailers have about the rival retailer's contract. Similarly, the answer also depends on retailers' beliefs in the dynamic context. The equilibrium concept of perfect Bayesian equilibria requires that retailers' beliefs are correct on the equilibrium path. However, the market outcome depends on the retailers' beliefs in case of deviations from the equilibrium strategy profile. In line with the literature, we consider beliefs that are passive within a period as well as symmetric beliefs.

In a repeated game in which the manufacturers may collude, we also distinguish how the retailers' beliefs may react dynamically to the observed actions in previous periods. We first consider beliefs that do not react to past behavior. For these beliefs, we find that manufacturers employing grim-trigger strategies are not able to sustain any price above the competitive price level. This result highlights that the opportunism problem of a monopoly manufacturer can carry over into a dynamic model and render collusion impossible.

In a next step, we investigate beliefs that react to the observed behavior of the manufacturers. First, we consider beliefs that anticipate the manufacturers' trigger strategies. Thus, the retailers have an understanding that manufacturers collude and how they do it. In this context, collusion at industry-profit-maximizing prices may be stable with respect to unilateral deviations. However, at the same time, collusion may not be *opportunism-proof* because the manufacturers may have a joint incentive to lower prices, similarly to the monopolist in Hart et al. (1990). We find that opportunism-proofness is a more restrictive condition to satisfy than the usual stability condition of collusion. Interestingly, albeit punishment with trigger strategies is usually not renegotiation-proof in a purely horizontal model without re-

[1]Further cases include Anheuser Busch, Haribo, Ritter, and Melitta; (last access 2020/02/03).
[2]See, for example, Harrington (2018) for a description of the cheese cartel in the UK.

tailers, retailer beliefs that anticipate punishment can render punishment credible. Hence, with beliefs that anticipate a collusive strategy, collusion can become a renegotiation-proof equilibrium of the game, provided that the manufacturers are patient enough, such that the stricter opportunism-proofness condition holds.

Case descriptions reveal that manufacturer cartels occasionally struggle to establish collusion because retailers do not accept the increased wholesale prices.[3] This raises the question under which condition retailers' beliefs allow for the formation of collusion. Formally, we introduce a definition of *formability* of collusion that requires that there exists a potential path from a non-collusive history to collusion. For example, beliefs that anticipate collusion with grim-trigger strategies would never allow for collusion after observing a single period of competition. Hence, pure grim-trigger beliefs do not support the formation of collusion. We introduce an adaptive belief whereby retailers expect collusion or competition depending on the past behavior of the manufacturers. We parameterize the adaption speed that specifies how fast a retailer believing in competitive prices, but receiving collusive wholesale price offers, switches to believing in collusive prices . The adaption speed may range from a single period to many periods. We find that collusion can be formable with these adaptive beliefs, but they perform poorly in the punishment phase. We find that a faster speed of adaption makes collusion easier to form but harder to sustain; particularly, collusion becomes less opportunism-proof.

Finally, we consider symmetric beliefs that immediately adapt to the actions of the manufacturers in each period. With symmetric beliefs, stable collusion is also opportunism-proof. However, the punishment phase of trigger strategies is not credible, such that there are no renegotiation-proof equilibria with symmetric beliefs and trigger strategies.

At least since Bernheim and Ray (1989) and Farrell and Maskin (1989) it is known that collusion with trigger strategies is usually not renegotiation-proof. The reason is that firms would have an incentive to jointly deviate in the punishment phases by immediately raising prices again, which undermines the credibility of punishment that is necessary to support the collusive equilibrium path. Although a few solutions for certain settings are known, such as asymmetric punishment, the problem of renegotiation seems to be largely ignored in the applied literature.[4] We find that if the colluding firms sell via retailers using private contracts, renegotiation-proof equilibria with the usual symmetric punishment can exist, and we offer different dynamic beliefs for which this holds. This has the flavor of 'strategic delegation' which solves the commitment problem of manufacturers to punish each other.

In summary, we find that the opportunism problem of a monopolist can also befall colluding manufacturers and impose additional conditions that are stricter than the standard stability conditions. In case of adverse beliefs (for example, history-independent passive beliefs), collusion can be entirely impossible due to opportunism. Colluding manufacturers might thus have an incentive to influence retailers' beliefs towards a high price level.[5] In prac-

[3]See footnote 1.

[4]An exception is McCutcheon (1997).

[5]A trade-off may arise between this incentive of setting the stage for supra-competitive supply conditions

tice, this may show when manufacturers communicate price expectations towards retailers or organize industry-wide resale price maintenance.[6]

The remainder of the paper is structured as follows. Section 2 relates our model to the relevant literature. We set up the model in Section 3 and subsequently analyze beliefs that are independent of the history of the game in Section 4. In Section 5, we study trigger beliefs as well as adaptive beliefs. We review symmetric beliefs in Section 6. Section 7 concludes.

2 Related Literature

Our paper contributes to four aspects that have been analyzed in the literature: (i) manufacturer collusion, (ii) a monopolist's opportunism problem , (iii) cartel formation, and (iv) downstream retailers' types of beliefs.

Manufacturer collusion. Jullien and Rey (2007) investigate the effects of resale price maintenance (RPM) on tacit collusion when only the retailers observe local shocks on demand or retail costs. The authors show that because of demand uncertainty, manufacturers never make use of RPM under competition; however, the use of RPM can facilitate collusion. This is due to the fact that without RPM, retailers set prices based on their information, which means that deviating behavior cannot be detected easily. Abolishing price flexibility though RPM has opposing effects on collusive stability: One the one hand, deviation can be readily detected. On the other hand, profits are reduced, which increases the short-run gains from deviating. The authors find that overall, the first effect can dominate. Hunold and Muthers, 2020 show that, absent any uncertainty and asymmetric information, RPM can still facilitate manufacturer collusion. RPM can make collusion successful when retail bargaining power renders supra-competitive increases of the wholesale prices impossible without RPM.

Nocke and White (2007) and Normann (2009) also analyze tacit collusion among manufacturers in vertical relationships but focus on whether vertical integration makes tacit collusion easier to sustain. Both articles consider perfect Bertrand competition among manufactures and compare an industry with no integration to one in which one pair of firms is vertically integrated. In Nocke and White (2007), manufacturers compete in two-part tariffs. The authors show that it is easier for manufacturers to sustain collusion in a scenario with vertical integration. Normann (2009) shows that this finding carries over to a situation in which manufacturers set linear prices, even though double marginalization leads to different collusive and deviation profits. Under public contracts, Schinkel et al. (2008) show that, when

while contracting on the one hand and hiding collusion from competition authorities on the other handy.

[6]In the German coffee cartel, manufacturers eventually used a retail price maintenance to fix retail prices (Holler and Rickert, 2019). Fink et al. (2017) document cases in which manufacturer cartels implemented additional clauses, for example, payment conditions. In the US sugar cartel, complex vertical contractual arrangements were also in place in addition to the set of rules that the members agreed on. The cartel members needed to use (often exclusive) brokers to deal with sales agents. These brokers were sought to adhere to the agreements made by the cartel and were blacklisted if they deviated (Genesove and Mullin, 2001).

manufacturers have full bargaining power, upstream collusion requires low wholesale prices and possibly negative franchise fees. Piccolo and Miklós-Thal (2012) finds similar results for the case in which retailers have full bargaining power. Piccolo and Reisinger (2011) analyze the impact of exclusive territories on manufacturers' possibility to collude. Under observable contracts, establishing exclusive territories has two opposing effects on collusive stability. Exclusive territories soften punishment, but they also reduce deviation profits. The second effect is due to the fact that when a manufacturer deviates, retailers of competing products adjust their prices, whereas retailers of the same product do not do so. As the effect on deviation tends to dominate, exclusive territories tend to facilitate tacit collusion.

Our contribution to this literature is twofold: First, we study the opportunism problems that colluding manufacturers face with secret contracting. Second, we demonstrate the relevance of retailers' beliefs about wholesale prices for the success of collusion.

Opportunism problem. We relate to the classic opportunism problem of a monopolist in a vertical structure with secret contracting (Hart et al., 1990; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994). In such a scenario, the upstream firm that deals with multiple competing downstream firms through bilateral contracts may encounter the following problem: The upstream firm is interested in maintaining high prices and profits, but it cannot commit to refraining from opportunistic moves. Indeed, the upstream firm has an incentive to increase bilateral surplus with one downstream firm, which is anticipated by the other downstream firm(s). The existence of this opportunism problem has been evoked as an explanation for vertical mergers and various vertical restraints as measures aimed at restoring the upstream firm's market power (O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Vergé, 2004). The restraints include exclusive dealing, non-discrimination clauses, and industrywide RPM. Do and Miklos-Thal (2021) explore short-comings of the seminal papers by considering a version of sequential (re-)contracting between upstream and downstream firms. The opportunism problem has been neglected in the collusion literature so far. This is important because manufacturers, when colluding efficiently, should behave like a monopolist.

Beliefs. The literature on secret contracting between manufacturers and retailers, which dates back to Hart et al. (1990); McAfee and Schwartz (1994); Segal (1999), emphasizes the relevance of the retailers' beliefs. Passive, symmetric, and wary beliefs are the typical variants. Whereas passive beliefs reflect that the agents treat unexpected offers as mistakes, symmetric beliefs could correspond to a rule of thumb, where agents conjecture that identical principals make identical offers (Pagnozzi and Piccolo, 2011). Wary beliefs are often used when passive beliefs implausible or induce non-existence of equilibria (Rey and Vergé, 2004; Rey and Tirole, 2007; Miklós-Thal and Shaffer, 2016). Empirical evidence on passive and symmetric beliefs was tested by Zhang (2021) and by Martin et al. (2001) in an experiment. We contribute to this literature by providing an in-depth analysis of how retailers' beliefs affect manufacturer collusion in a setting of secret contracting. In our analysis, we use different beliefs, including history-dependent passive beliefs that reflect trigger-strategies as well as beliefs that adapt to changing manufacturer strategies.

Cartel formation. The literature on cartel membership formation has focused on a variety of different aspects. In this literature, the question of how to initiate cartels typically focuses, among other things, on contracts (Selten, 1973; d'Aspremont et al., 1983), on stochastic opportunities to form a cartel (Harrington and Chang, 2009), on the heterogeneity with regard to capacities and umbrella pricing (Bos and Harrington, 2010), on signals and beliefs of producers (Harrington Jr and Zhao, 2012; Harrington, 2017), and on quality differentiation (Bos et al., 2020). Selten (1973)) analyzes the case of quantity competition and assumes that cartels can be enforced via contracts and that a cartel acts as a Stackelberg leader. He shows that a cartel is stable in the sense that outsiders do not want to be part of the cartel, and insiders cannot profitably leave the cartel as long as the number of cartel members is relatively small. d'Aspremont et al. (1983) obtain a similar result for the case with price leadership.

Harrington and Chang (2009) consider a set of heterogeneous industries in which stochastic opportunities to form a cartel arise to explain the birth and death of cartels and to inform antitrust authorities about the extent of cartels that have not been discovered. Bos and Harrington (2010) endogenize the composition of a cartel in an industry in which heterogeneous firms differ in their capacities. They show that non-all-inclusive cartels set umbrella prices, and that mergers involving moderate-sized firms may result in the most severe coordinated effects. Harrington Jr and Zhao (2012) analyze whether different types of players (patient and impatient) manage to cooperate via grim-trigger strategies when players signal and coordinate through their actions. The authors show that there is always a positive chance of cooperation, but cooperation may fail altogether. Moreover, the longer cooperation does not occur, the less likely it is to occur in the next period. Harrington (2017) focuses on a mutual beliefs to coordinate prices. In the context of price leadership, firms are assumed to commonly believe that price increases will be at least matched. However, the firms lack any shared understanding about who will lead, when they will, and at what prices. Sufficient conditions are derived which ensure that supra-competitive prices emerge. However, price is bounded below the maximal equilibrium price.

In contrast to the literature on cartel membership formation, we address the question whether firms can transit to a collusive equilibrium once they have reached – implicitly or explicitly – a common understanding to collude in a currently non-collusive industry. We thereby focus on the process of collusion that firms need to go through until they may reach a stable collusive equilibrium. At the core of this analysis is the adaption of the retailers' beliefs in the transition process.

3 Model

We study manufacturer collusion in an infinitely repeated stage game with time $t = 0, \ldots, \infty$.

There are two symmetric manufacturers, M_A and M_B that compete by selling imperfect substitutes to their exclusive retailers R_A and R_B. Each manufacturer makes an exclusive and secret two-part tariff offer with a unit wholesale price w_i and a franchise fee F_i to its retailer. The retailers compete in retail prices. There is horizontal differentiation between the two retailer-product combinations.

Timing and information. In each period, the following stage game unfolds:

1. Each manufacturer makes a private contract offer to its retailer.
2. Retailers decide whether to accept offers. Post contract acceptance, the fixed fees are sunk.
3. The retailers simultaneously and non-cooperatively set their retail prices p_i.

The manufacturers' contract offers are secret. This means that when retailers receive a contract offer, they cannot observe the contract offered to their rival. When competing in the downstream market, the retailers are unable to observe each others' input prices.

At the end of each period, all actions are revealed to all players. All players thus know the complete history of the game at this point.

Manufacturers are long-lived and discount next period profits with the common discount factor $\delta > 0$. The retailers are short-lived and have a discount factor of zero, such that they do not take future profits into account.

Assumptions on costs, demand, and profits. All (fundamental) costs are zero. We normalize the outside option (opportunity costs) of each retailer to zero. We consider general demand functions that fulfill standard properties in the relevant range which we summarize in

Assumption 1. *Demand* $D_i(p_i, p_{-i})$

- *decreases in the own price* $(\partial D_i(p_i, p_{-i})/\partial p_i < 0)$,
- *increases in the other product's price* $(\partial D_i(p_i, p_{-i})/\partial p_{-i} > 0)$, *and*
- *decreases when all prices increase* $(\partial D_i(p_i, p_{-i})/\partial p_i + \partial D_i(p_i, p_{-i})/\partial p_{-i} < 0)$.

We focus on well-behaved profits that are twice differentiable and concave. Note that this implies that the retailers' reactions behave normally, such that $\frac{\partial p_i\left(w_{it}, p^e_{-i}\right)}{\partial w_{it}} > 0$ and, consequently, $\frac{\partial D_i\left(p_i\left(w_{it}, p^e_{-i}\right), p^e_{-i}\right)}{\partial p_i} < 0$ hold.

Equilibrium concept. We consider (pure-strategy) perfect Bayesian equilibria and focus on symmetric equilibria. Retailers need to form beliefs about the contract offer made to the rival when accepting their own contract in the second stage. Without loss of generality, we focus on retailers having a belief about the retail price of the rival.[7] That is, the potential cost of the rival is not known to a retailer, because this cost depends on the contract offered by the other manufacturer. The super game is a game of complete information but unobservable actions. All contracts of the form (w, F) become observable after the stage game to exclude the possibility that imperfect information carries over from period to period.

Outlook. In the following sections, we consider Perfect Bayesian Equilibria (PBE) of the game and focus on symmetric equilibria with constant equilibrium paths. On the collusive equilibrium path, beliefs are correct and thus pinned down by the equilibrium strategy profile. However, off the equilibrium path, the equilibrium concept of PBE has no bite. We restrict off-equilibrium beliefs in different forms (for example, passive) and develop novel criteria like opportunism-proofness to assess the plausibility of a given PBE.

4 History-independent Passive Beliefs

In this section, we focus on short-lived retailers whose beliefs are passive and independent of the history of actions in the game. With these history-independent, passive beliefs, we will show that an opportunism problem arises if manufacturers try to collude.

Definition 1. (History-Independent, Passive Beliefs) The price expectation p^e_{-it} of retailer i in period t about the price of retailer $-i$ is independent of the history of the actions in the game and independent of the offer (w_{it}, F_{it}) made by its supplier in period t.

First, we solve for the competitive equilibrium of the game that results when both manufacturers maximize stage-game profits and the retailers hold passive beliefs. In the last stage, on the equilibrium path, each retailer has accepted the contract, but the rival's wholesale price remains secret. Each retailer $i \in \{1, 2\}$ faces the own wholesale price w_{it} and holds a belief p^e_{-it} about the retail price of the rival. The retailers set the retail prices p_{it} simultaneously. Thus, the profit of retailer i is

$$\pi_{it}\left(p_{it}, p^e_{-it}\right) = \left(p_{it} - w_{it}\right) D_i\left(p_{it}, p^e_{-it}\right) - F_{it}.$$

Retailer i maximizes its profit with respect to p_{it}. The first-order condition is

$$D_i\left(p_{it}, p^e_{-it}\right) + \left(p_{it} - w_{it}\right) \frac{\partial D_i\left(p_{it}, p^e_{-it}\right)}{\partial p_{it}} = 0$$

[7] The retail price of the rival is the pay-off relevant information that retailers are lacking. Alternatively they could have beliefs about the wholesale price that the retailer faces and about the belief that the other retailer has about their wholesale price.

and defines retailer i's reaction function $p_i\left(w_{it}, p^e_{-it}\right)$. Anticipating the pricing outcome, in the second stage, each retailer decides whether to accept the wholesale tariff offered by its manufacturer.

In the first stage, the manufacturers offer their wholesale tariffs. Under manufacturer competition, each manufacturer i maximizes its profit, anticipating that its retailer sets a price of $p_i\left(w_{it}, p^e_{-i}\right)$:

$$\max_{w_{it}, F_{it}} \Pi_{it} = w_{it} \cdot D_i\left(p_i\left(w_{it}, p^e_{-it}\right), p_{-i}(w_{-it}, p^e_{it})\right) + F_{it},$$

subject to the retailer's participation constraint

$$\left(p_i\left(w_{it}, p^e_{-it}\right) - w_{it}\right) \cdot D_i\left(p_i\left(w_{it}, p^e_{-it}\right), p^e_{-it}\right) - F_{it} \geq 0.$$

The profit is determined by two parts. The variable part consists of the units sold times the unit wholesale price, and the fixed part consists of an up-front payment from the retailer to the manufacturer. The fixed payment is restricted to the revenue that the retailer earns. This participation constraint binds in equilibrium.

Beliefs have to be correct on the equilibrium path in any perfect Bayesian equilibrium and we focus on time history independent beliefs in this section. Whenever the manufacturers' strategies yield a time-constant equilibrium path, beliefs consistent with that have to be time constant. The only alternative would be that beliefs change over time independent of past actions, but this would not support a constant equilibrium path. Thus, in a competitive equilibrium, retailer i has a belief $p^e_{-it} = p^e_{-i}$ that is time constant.

Let us first characterize the competitive equilibrium. This corresponds to the equilibrium of the one-shot game. In equilibrium, the belief of the retailer about the retail price of the rival coincides with the expectation of the manufacturer about the retail prices. This implies $p_{-i}(w_{-it}, p^e_{it}) = p^e_{-i}$. Inserting the price and simplifying yields

$$\Pi_{it}\left(w_{it}, p^e_{-i}\right) = p_i\left(w_{it}, p^e_{-i}\right) \cdot D_i\left(p_{it}\left(w_{it}, p^e_{-i}\right), p^e_{-i}\right).$$

Maximizing with respect to w_i gives the first-order condition

$$\begin{aligned}\frac{\partial p_i\left(w_{it}, p^e_{-i}\right)}{\partial w_{it}} D_i\left(p_i\left(w_{it}, p^e_{-i}\right), p^e_{-i}\right) + \frac{\partial D_i\left(p_i\left(w_{it}, p^e_{-i}\right), p^e_{-i}\right)}{\partial p_i}\frac{\partial p_i\left(w_{it}, p^e_{-i}\right)}{\partial w_{it}} &= 0\\ \iff \frac{\partial p_i\left(w_{it}, p^e_{-i}\right)}{\partial w_{it}}\frac{\partial D_i\left(p_i\left(w_{it}, p^e_{-i}\right), p^e_{-i}\right)}{\partial p_i} w_{it} &= 0.\end{aligned}$$

Because the first term on the left-hand side in the previous line is assumed to be strictly positive and the second term is assumed to be strictly negative (Assumption 1), the only solution to the first-order condition is $w_i = 0$. This yields

Lemma 1. *With history-independent and passive beliefs, the wholesale prices in the equilibrium with manufacturer competition are equal to zero.*

To show that firms are unable to collude on a price different than the competitive wholesale price of zero, we take the whole dynamic game into account. Manufacturers collude using grim-trigger strategies:

- In the first period, manufacturers set the collusive wholesale price w^C;
- in the t^{th} period,
  - if both manufacturers have set the collusive price in each of the $t-1$ previous periods, they set the collusive wholesale price w^C,
  - otherwise manufacturers set the price of the weak Bayesian Nash equilibrium with passive beliefs ($w^P = 0$) of the stage game.

With grim-trigger strategies, any deviation results in a constant punitive action that the manufacturers play forever. This action corresponds to the action of the competitive equilibrium. Like in the competitive equilibrium, the grim-trigger strategies imply a time-constant equilibrium path for the wholesale prices. Thus, again, in a perfect Bayesian Equilibrium of the game, any history independent beliefs has to be time constant. Note that the only alternative would be belief that changes with t but independent of the actions of the players. As we focus on symmetric equilibria, the beliefs are also identical for both retailers in any equilibrium.

Proposition 1. *There exists a unique symmetric grim-trigger equilibrium with a constant equilibrium path: the competitive equilibrium. Collusion cannot effectively increase prices, that is, manufacturers face an opportunism problem.*

Proof. See Appendix A. □

Due to the nature of the history-independent passive beliefs, manufacturers face an opportunism problem.[8] Surprisingly, the opportunism problem arises even for colluding manufacturers that use grim-trigger strategies that usually can support a larger set of equilibria. However, in this case, the punishment is limited by the assumption that retailers do not update their beliefs after a deviation. This allows deviating manufacturers to maintain a high profit level and destroys the commitment power of the grim-trigger strategies. In contrast, if retailers' beliefs react to deviations, punishment might become more effective. We consider this case in the next sections where we introduce history-dependent beliefs.

5 History-Dependent Passive Beliefs

Second, consider a dynamic version of passive beliefs that depend on the history of the game. We focus on the history of wholesale prices and disregard the history of retail prices, such

[8]Note that this result is more general, it also extends to general trigger strategies, which we define in Subsection 5.3.

that the retailers' beliefs cannot directly depend on the competing retailers' past actions. Otherwise, such beliefs could support retail collusion, which we want to abstract from.[9] Thus, the relevant history of the game in period t is $\mathcal{H}_t \equiv [(w_{A0}, w_{B0}); \ldots; (w_{At-1}, w_{Bt-1})]$, that is, the set of the pairs of wholesale prices that the manufacturers have set in all periods up to period $t-1$.

In contrast to constant passive beliefs, the question is whether it is possible for the manufacturers to support the optimal collusive price in equilibrium. We define history-dependent passive beliefs as follows:

Definition 2. (History-Dependent Passive Beliefs) The beliefs are passive within each period t, that is, the offer about w_{it} does not affect the belief p^e_{-it} about the competitor's retail price. The beliefs are dynamic: For each retailer i, the belief is a function $p^e_{-it}(\mathcal{H}_t)$ that depends on the history of past wholesale prices.

There exist multiple variants of history-dependent passive beliefs. We derive beliefs that are in line with the strategies of the manufacturers (for example, grim-trigger strategies).

Definition 3. (Opportunism-Proofness) An equilibrium collusive strategy for given beliefs (a Perfect Bayesian Equilibrium) is said to be opportunism-proof if the manufacturers, on the equilibrium path, do not benefit if they jointly reduce their wholesale prices.

Opportunism-proofness is not a necessary condition for a PBE to exist. It is an additional condition that, in our opinion, increases the plausibility that an equilibrium can be sustained in a collusive environment. Without opportunism-proofness, the implicit "collusive agreement" between the manufacturers is not renegotiation-proof.

Definition 4. (Credible Punishment) A collusive strategy for given beliefs has the property of credible punishment if there is no punishment period in which the manufacturers would benefit if they jointly change their wholesale prices.

Whereas opportunism-proofness considers renegotiation proofness of a the collusive strategy on the collusive path, the criterion of credible punishment does so for the punishment phase. Together, opportunism-proofness and credible punishment ensure weak renegotiation-proofness as defined in Farrell and Maskin (1989).

For reference, consider a standard model of horizontal collusion absent a vertical dimension, such as our model when each manufacturer-retailer pair is vertically integrated. In this industry structure, punishment is typically not credible because the firms would prefer to renegotiate to return to the collusive pricing scheme. This property can prevent the existence of renegotiation-proof equilibria in the case of standard horizontal collusion. Instead, as we will prove later, manufacturer collusion with trigger strategies in a setting with retailers and secret contracting can have the property of credible punishment, depending on the retailers' beliefs.

[9]Suppose that each retailer believes that the price of the competitor is the monopoly price unless they have observed a different price in the past, in which case they would believe that the price is the competitive price. Such beliefs could support a collusive action profile even with otherwise history-independent strategies.

We also define when collusion is formable in the sense that collusion can be arrived at.

Definition 5. (Formable) Consider a PBE with a collusive strategy profile and given beliefs resulting in discounted collusive payoffs on the equilibrium path denoted by V^C. Collusion is formable if there exists a strategy profile, such that for given beliefs:

1. Starting in some period s with an arbitrary history $\mathcal{H}_s$, the action profile consists of mutually best responses in $t \geq s$,

2. the continuation game starting in period $s + x$, with $x < \infty$ constitutes a PBE with payoff V^C.

This definition is stronger than the condition for sustainability of collusion, because the latter is necessary but not sufficient for formation. The definition implies a property that beliefs 'forgive' any non-collusive behavior after some periods; this includes that beliefs do not react to opportunistic behavior. The number of transition periods x can be very long, such that our definition is not restrictive with regard to the speed of formation.

With formability we address the question whether firms can transit to a collusive equilibrium once they have reached a common understanding to collude in a currently non-collusive industry. This reflects the process of collusion that firms need to go through until they may reach a stable collusive equilibrium. We take a "meeting of the minds", that is how firms reach a common understanding to collude among themselves, as the starting point. Formability requires that there is feasible path towards collusion which is incentive compatible (self-enforcing). The underlying complication for this is that the environment may first need to adapt over time to make collusion profitable. Within our model, this is reflected in the retailers' beliefs which may need time to adapt to collusion among manufacturers out of a situation of manufacturer competition.

In a sense the property of formability is similar to the property of stability of static Nash equilibria as both properties consider weather there exist a path of mutual best responses that lead to the proposed equilibrium.

For example, consider a standard model of horizontal collusion with grim-trigger strategies and without private information. Suppose firms collude at the industry-profit-maximizing level and the usual stability condition is met. Such a collusion is formable , because switching to grim-trigger strategies is an equilibrium of the continuation game independent of the history of actions in the game. Collusion is then also opportunism-proof, because joint profits are maximized. However, punishment is not credible, because jointly reverting to collusion yields larger profits than continuing punishment.

5.1 Collusive Strategy

In the following, we will consider collusion at industry-profit-maximizing prices. Thus, let us derive the conditions for the industry profit maximum. First, we define the optimal collusive price provided that the retailers understand the grim-trigger strategy. The optimal collusive wholesale price is determined by the price that maximizes the joint profit of the manufacturers given that the retailers' belief is identical to that price. As we focus on manufacturers colluding on the industry profit maximum, collusive profits are Pareto-optimal if there is no joint deviation that is more profitable. If weak renegotiation-proofness is fulfilled, then the equilibrium is also strong renegotiation-proof, as the profit on the equilibrium path would be Pareto-efficient for the colluding manufacturers.

Optimal Collusive Price: Integrated Industry Solution

We consider collusion at the prices that maximize the integrated industry profit:

$$p^M = \max_{p_i} p_i \cdot D_i\left(p_i, p_{-i}\right) + p_{-i} \cdot D_{-i}\left(p_{-i}, p_i\right), \tag{1}$$

where w^M is defined by $p^M = p_i\left(w^M, w^M\right)$. Denote the maximal industry profit by

$$\Pi^M := p^M \cdot D_i\left(p^M, p^M\right) + p^M \cdot D_{-i}\left(p^M, p^M\right). \tag{2}$$

The optimal collusive wholesale price w^M and the respective belief about the retail price, p^M, are then determined by equation (1) as well. Thus, whenever the manufacturers collude at the industry-profit-maximizing wholesale prices of w^M, each manufacturer earns a profit of

$$\Pi^C := \Pi^M/2.$$

5.2 Grim-Trigger Beliefs

We construct history-dependent passive beliefs (Definition 2) that have a grim-trigger property. For this, consider that manufacturers play grim-trigger strategies as described above.

We focus on industry-profit-maximizing collusion, such that $w^C = w^M$. Any deviation by one manufacturer causes the other manufacturer to set the punitive price w^P forever. Because we are solving for perfect Bayesian Nash equilibria, the punishment with w^P must be individually rational. This implies that $w^P = 0$ must hold in equilibrium. This follows from the logic presented in the proof of Proposition 1: Otherwise, each manufacturer would have an incentive deviate from a $w^P \neq 0$ in a punishment period, in which future beliefs and action are fixed and unaffected by current actions. In punishment periods, manufacturers thus behave as if they maximize short-term profits.

Histories and beliefs. If manufacturers play grim-trigger strategies, actions are only conditional on two kinds of histories: the collusive history $\mathcal{H}^{\mathcal{C}}$, in which both manufacturers have only played w^C, and the deviation histories $\mathcal{H}^{\mathcal{D}}$ (any history other than $\mathcal{H}^{\mathcal{C}}$). Grim-trigger beliefs match these strategies by assigning two different beliefs to these histories. Grim-trigger beliefs $p_i^e(\mathcal{H})$ are thus history-dependent (Definition 2) and differentiate between the two histories $\mathcal{H}^{\mathcal{C}}$ and $\mathcal{H}^{\mathcal{D}}$.

Definition 6. (Grim-trigger beliefs)

In the first period, retailers believe in a rival's retail price of p^C.

As long as both manufacturers play w^C, that is, the collusive history $\mathcal{H}^C$ prevails, each retailer believes that the other retailer sets p^C in the current period.

Once one manufacturer has deviated, the history is $\mathcal{H}^\mathcal{D}$, and both retailers believe that the other retailer sets the competitive price p^P. This corresponds to a situation in which both retailers have common knowledge that the wholesale prices are $w^P = 0$ in the current period.

Given the passive nature of beliefs in a period (Definition 2), the beliefs are not correct in deviation periods. Neither a retailer that is offered the equilibrium contract updates its belief nor does the retailer that receives a deviating offer.

Because a deviation does not occur on the equilibrium path, the beliefs are nevertheless correct on the equilibrium path. Off the equilibrium path, the retailers' beliefs anticipate that manufacturers play grim-trigger strategies in that they also punish a deviation of one manufacturer in period t in all future periods.

Equilibrium. To determine an equilibrium of the dynamic game, we need to consider deviations from the collusive strategy. In equilibrium, each manufacturer realizes profits of $\Pi^C = {}^{\Pi^M}/2$. In a deviation period, both retailers believe that the wholesale price is w^C and anticipate that the other retailer sets p^C. This results in a belief $p^e_{it} := p_i\left(w_{it}, p^C\right)$. Suppose that manufacturer i maximizes its deviating profit in period t in view of history $\mathcal{H}^C$. When there is a deviation, that is $w_{it} \neq w^C$ holds, grim-trigger beliefs imply that the level of w_{it} has no impact on future beliefs. The deviation profit is given by

$$\begin{aligned} & w_{it} \cdot D_i\left(p_i\left(w_{it}, p^C\right), p^C\right) + \left[p_i\left(w_{it}, p^C\right) - w_{it}\right] \cdot D_i\left(p_i\left(w_{it}, p^C\right), p^C\right) \\ & = p_i\left(w_{it}, p^C\right) \cdot D_i\left(p_i\left(w_{it}, p^C\right), p^C\right). \end{aligned} \tag{3}$$

Maximizing with respect to w_{it} yields the first-order condition

$$\begin{aligned} \frac{\partial p_i\left(w_{it}, p^C\right)}{\partial w_{it}} D_i\left(p_i\left(w_{it}, p^C\right), p^C\right) + \frac{\partial D_i\left(p_i\left(w_{it}, p^C\right), p^C\right)}{\partial p_i} \frac{\partial p_i\left(w_{it}, p^C\right)}{\partial w_{it}} = 0 \\ \iff \frac{\partial p_{it}\left(w_{it}, p^e_{-i}\right)}{\partial w_{it}} \frac{\partial D_{it}\left(p_{it}\left(w_{it}, p^e_{-i}\right), p^e_{-i}\right)}{\partial p_{it}} w_{it} = 0. \end{aligned} \tag{4}$$

The last step follows from the first-order condition of the retailers and applying the envelope theorem (see Pagnozzi and Piccolo, 2011, p. 180). Note that the first two factors in equation (4) are non-zero. Therefore, the manufacturer optimally deviates to $w^D = 0$. This results in

$$\Pi^D := p_i(0, p^C) \cdot D_i\left(p_i(0, p^C), p^C\right).$$

After any deviation by a manufacturers, the beliefs revert to $p^e_{-i} = p^P$, that is, the belief in the punishment period, forever. This results in profits of

$$\Pi^P := p^P \cdot D_i(p^P, p^P). \tag{5}$$

Recall that $p^P = p_{it}(0, p^P)$ is the competitive price.

Collusion is sustainable when no manufacturer wants to deviate from the grim-trigger strategy. Using the one-shot deviation principle, the relevant incentive constraint for stability is

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \frac{\delta \Pi^P}{1-\delta}. \tag{6}$$

The left-hand side contains the present value on the equilibrium path and the the right-hand side the present value of a deviation. We can rewrite the incentive constraint for manufacturer i as follows:

$$\frac{p^C \cdot D_i(p^C, p^C)}{(1-\delta)} \geq p_i(0, p^C) \cdot D_i\left(p_i(0, p^C), p^C\right) + \frac{\delta}{1-\delta}\left(p^P \cdot D_i(p^P, p^P)\right). \tag{7}$$

This is equivalent to the incentive constraint for standard horizontal collusion (when manufacturers and retailers are pairwise integrated), where the following order holds: $\Pi^D > \Pi^C = \Pi^M/2 > \Pi^P$.

Let us check whether the equilibrium is opportunism-proof (see Definition 3). Jointly, the manufacturers may have an incentive to reduce their wholesale prices for given beliefs. Suppose $\mathcal{H}_C$ is the history of the game, such that each retailer believes that its competitor sets p^C and anticipates to set $p_i\left(w_{it}, p^C\right)$. First, we show that jointly deviating manufacturers set a wholesale price of $w_{it} = w_{-it} < 0$ in order to maximize spot profits. To see this, let us inspect the profit per manufacturer in the case of a joint deviation:

$$\begin{aligned}\Pi^{JD} :=& \frac{1}{2}\max_{w_{it}, w_{-it}} w_{it} \cdot D_i\left(p_i(w_{it}, p^C), p_{-i}(w_{-it}, p^C)\right) + \left[p_i(w_{it}, p^C) - w_{it}\right] \cdot D_i\left(p_i(w_{it}, p^C), p^C\right) \\ &+ w_{-it} \cdot D_{-i}\left(p_{-i}(w_{-it}, p^C), p_i(w_{it}, p^C)\right) + \left[p_{-i}(w_{-it}, p^C) - w_{-it}\right] \cdot D_{-i}\left(p_{-i}(w_{-it}, p^C), p^C\right).\end{aligned}$$

Applying the envelope theorem and symmetry, the first-order conditions can be rewritten as

$$\left[\underbrace{D_i\left(p_i(w, p^C), p_{-i}(w, p^C)\right) - D_i\left(p_i(w, p^C), p^C\right)}_{<0}\right] + w\underbrace{\frac{\partial p_i}{\partial w_{it}}}_{>0}\left[\underbrace{\frac{\partial D_i}{\partial p_i} + \frac{\partial D_{-i}}{\partial p_i}}_{<0}\right] = 0, \forall i. \tag{8}$$

Equation (8) only holds for $w < 0$. Hence, manufacturers optimally deviate to $w < 0$ jointly. A manufacturer makes a higher profit in the case of a joint deviation than when deviating unilaterally: $\Pi^{JD} > \Pi^D$. To explain the last inequality, note that the manufacturers could replicate the profit of Π^D for each of them by setting $w = 0$. The manufacturers, however, optimally set a price w below zero as this yields strictly larger profits. Following a deviation, the manufacturers make profits Π^P in future periods due to the grim-trigger beliefs. This results in the opportunism-proofness condition

$$\frac{\Pi^C}{1-\delta} \geq \Pi^{JD} + \frac{\delta \Pi^P}{1-\delta}. \tag{9}$$

Comparing equation (9) with equation (6), the only difference is the deviation profit on the right-hand side. As $\Pi^{JD} > \Pi^D$, the condition for opportunism proofness is harder to satisfy than the the stability condition above.

Proposition 2. *With grim-trigger beliefs, there exists an equilibrium in which manufacturers are able to sustain collusion on the industry-profit-maximizing collusive wholesale price using grim trigger-strategies if the discount factor is high enough to satisfy equation* (7)*. This condition is equivalent to the incentive constraint when manufacturers and retailers are pairwise integrated, that is, horizontal collusion. Under the stronger condition* (9)*, collusion is also opportunism-proof. The punishment is credible, but collusion is not formable.*

Proof. See Appendix A. □

As derived above, the opportunism problem gives rise to incentive condition (9) which is harder to satisfy, that is, only for more patient firms, than the condition (6) for stable collusion. Hence, opportunism can make cartels less sustainable. Similar to the literature on the opportunism problem in single-shot games, this result depends on the beliefs. Due to their passive beliefs, the retailers do not react immediately to opportunism, which allows the manufacturers to 'trick' the retailers, because the price that retailers expect will turn out to be incorrect if manufacturers jointly deviate. This joint deviation differs from a unilateral deviation of one manufacturer-retailer pair from a candidate equilibrium in which instead the belief of the deviating retailer is correct. In the latter case, it is optimal for the deviating manufacturer to set the wholesale price equal to the true costs. In the former case of a joint deviation, each retailer wrongly believes that the other retailer will buy at a high wholesale price and thus sell at a high price, such that demand is high. It is thus profitable for the manufacturer to demand a high fixed fee in return for a marginal wholesale price below costs, because the retailer believes that it can sell a large quantity. However, a marginal wholesale price below costs only becomes profitable for a manufacturer when the retailer has a wrong belief. It could thus signal the retailer that its belief is wrong, because, if it were correct, the manufacturer's offer would be dominated by an offer with a wholesale price of $w = 0$. The stricter condition for opportunism-proofness is thus an implication of the passive nature of the beliefs. We later consider symmetric beliefs, where retailers' beliefs instantly react to any change in wholesale prices.

If marginal wholesale prices below marginal costs are impossible (for instance, due to competition law), the incentive constraint for opportunism and for stability are identical, implying that any stable cartel is also opportunism-proof.

In any case, the opportunism problem is mitigated under grim-trigger beliefs compared to the one-shot game, or history-independent beliefs, because retailers with grim-trigger beliefs react to the deviation from the expected cartel wholesale prices by adapting their beliefs from the next period on to the level of competitive prices. This effectively punishes manufacturer opportunism, such that joint deviations are not profitable when manufacturers are sufficiently patient.

Interestingly, we find that under grim-trigger beliefs, collusion is *not* formable. Because these beliefs are unforgiving, they are beneficial in supporting the collusive equilibrium, but do not allow for new cycles of collusion, even after a long time.

The characterized equilibrium is also renegotiation-proof in the sense of (Farrell and Maskin, 1989). As the opportunism-proofness condition (9) ensures that manufacturers have no incentives to deviate jointly from the collusive agreement and credible punishment is fulfilled as well, the equilibrium is *weakly* renegotiation-proof. Strong renegotiation-proofness requires that not only all the continuation game equilibria do not invite joint deviation but also that there is no other renegotiation-proof strategy profile that Pareto dominates the candidate equilibrium of the whole game. As the equilibrium features collusion on the industry-profit maximizing prices, which are Pareto-optimal for the manufacturers, the equilibrium is also *strongly* renegotiation-proof.

Corollary 1. *The equilibrium with grim-tigger beliefs described in Proposition 2 is strongly renegotiation-proof if the opportunism-proofness condition* (9) *is satisfied.*

Note that this corollary extends naturally to general trigger-strategies and more general beliefs as employed in the next section.

5.3 Trigger beliefs

In this section, we define trigger beliefs and derive the conditions for the sustainability and other properties of collusion. First, consider that the manufacturers play trigger-strategies similar to the grim-trigger strategy above, but a deviation is forgiven after κ periods.

We define trigger-strategies as in Green and Porter (1984):

Formally, let $y = (w^C, w^C)$ be a profile of collusive wholesale prices and let $z = (w^P, w^P)$ be a punishment wholesale price profile. Let κ denote a time length measured in periods. Define period t to be collusive if

(a) $t = 0$, or

(b) $t - 1$ was collusive and $w_{it-1} = w^C$ for all i, or

(c) $t - \kappa$ was collusive and $w_{it-\kappa} \neq w^C$ for some i.

Define t to be reversionary otherwise. Manufacturer i sets

$$w_{it} = \begin{cases} y_i & \text{if } t \text{ is collusive,} \\ z_i & \text{if } t \text{ is reversionary.} \end{cases}$$

Assume that the collusive price equals the monopoly price ($w^C = w^M$), as defined in Subsection 5.1. Any deviation by one manufacturer causes the other manufacturer to carry out a punitive action of $w^P = 0$. Again, $w^P = 0$ must hold because a deviation during the reversionary periods is not punished because the future actions and beliefs are fixed. Hence, the punishment action must be the same as the short-term optimal action, which is $w^P = 0$ as we demonstrated before. This maximizes the manufacturer profits because it aligns the incentives of retailer and manufacturer.

Definition 7. (Trigger beliefs)

Choose a collusive price level p^C and punishment price level p^P. Formally, the retailers' beliefs correspond to the manufacturer strategies for collusive and reversionary periods as defined above:

$$p^e_{-it} = \begin{cases} p^C & \text{if } t \text{ is collusive} \\ p^P & \text{if } t \text{ is reversionary} \end{cases}$$

When both manufacturers play trigger-strategies with actions w^C and w^P, the retailers' corresponding trigger-beliefs are correct on the collusive equilibrium path. They are also correct in the punishment phase as well.

To analyze the equilibrium of the dynamic game with trigger beliefs, it is necessary to consider the conditions that are needed to sustain collusion. After any deviation by a manufacturers, the beliefs revert to $p^e_{-i} = p^P$ for κ periods, which results in profits of $\Pi^P = p^P \cdot D_i(p^P, p^P)$. After κ periods, however, the retailers believe in collusion p^C again. Collusion is sustainable when no manufacturer wants to deviate from the trigger strategy given by the incentive condition:

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \delta \left(\frac{1-\delta^\kappa}{1-\delta} \Pi^P + \frac{\delta^\kappa}{1-\delta} \Pi^C \right). \tag{10}$$

This condition is harder to fulfill than the incentive condition (7) for grim-trigger strategies and beliefs. The punishment in condition (10) is less harsh and ends after κ periods, such that the expression on the right-hand side is larger than in the condition with grim-trigger strategies. Recall that manufacturers make the same profits as pairwise integrated manufacturer-retailer pairs (Proposition 2). Note that the individual profits, Π^C, Π^D, and Π^P are still identical to an integrated firm's profits. This implies that the stability condition (10) is the same for vertically separated and integrated manufacturer-retailer pairs whenever they play trigger strategies of length κ.

To check whether the equilibrium is opportunism-proof, we must consider a revised version of condition (9) that applies to trigger beliefs. Because trigger beliefs are forgiving after κ periods, a joint deviation of both manufacturers is not "punished by the beliefs" forever.[10] Hence, the conditions can be written as

$$\frac{\Pi^C}{1-\delta} \geq \Pi^{JD} + \delta \left(\frac{1-\delta^\kappa}{1-\delta} \Pi^P + \frac{\delta^\kappa}{1-\delta} \Pi^C \right). \tag{11}$$

[10] By contrast, the grim-trigger beliefs switch forever to the competitive price level in response to a deviation. This effectively punishes the manufacturers who from then on face pessimistic retailers.

Again, the opportunism-proofness condition (11) resembles the stability condition (10), except that $\Pi^{JD} > \Pi^{D}$, which makes the condition harder to meet. If condition (11) holds, collusion is robust against joint deviations by the manufacturers, that is, opportunistic behavior.

The punishment is credible, because in a punishment phase, both retailers believe $w^P = 0$ to which the in period joint best response is $w = 0$ as well. Beliefs are constant in the wholesale price played in the punishment phase. Thus, the argument in Subsection 5.2 applies, such that focusing on short-term best responses is valid.

To see that collusion is not formable with trigger beliefs, consider an argument by contradiction. Assume that in period $t = 0$, no collusion is played. It follows that it is impossible for retailers to hold collusive beliefs at any future point of the game. We summarize the above results in

Proposition 3. *With trigger beliefs, there exists an equilibrium in which manufacturers are able to sustain collusion on the industry-profit-maximizing collusive wholesale price (with trigger strategies) if the discount factor is high enough to satisfy condition* (10)*. This condition is equivalent to the incentive constraint when manufacturers and retailers are pairwise integrated. Furthermore, the condition requires more patience than under grim-trigger beliefs and strategies. Only under the stricter condition* (11)*, collusion is also opportunism-proof. Punishment is credible and collusion is not formable.*

Qualitatively, our insights for grim-trigger strategies carry over to the more general trigger strategies. Trigger strategies with limited punishment imply stricter conditions for stability and opportunism-proofness compared to grim-trigger strategies. However, they may be attractive for potentially highly relevant reasons that we do not model, including cost and demand shocks as well as other uncertainty that could result in unwarranted punishment, which in case of grim-trigger strategies would be very costly. The derived equilibrium with trigger beliefs is Pareto-efficient and has credible punishment. It, in addition, the manufacturers are patient enough for the if the condition of opportunism-proofness to hold, the equilibrium is strongly renegotiation-proof .

Corollary 2. *The equilibrium with trigger beliefs described in Proposition 3 is strongly renegotiation-proof if the opportunism-proofness condition* (11) *is satisfied.*

5.4 Adaptive beliefs

After analyzing beliefs that follow (grim-)trigger strategies, next we provide a simple example of history-dependent passive beliefs that allow for collusion that is stable, formable, and opportunism-proof. Formability intuitively requires that beliefs can switch between competitive and collusive phases. The following beliefs allow this by adapting to the behavior of manufacturers. These adaptive beliefs also can be used to analyze equilibria with (grim-)trigger strategies. We define the beliefs in a way that they only depend on the actions of the manufacturers in the last T periods and not on the full history of the game.

Definition 8. (Adaptive Beliefs)

Beliefs are passive within each period t, that is, w_{it} does not affect the belief p^e_{-it}. Beliefs are dynamic: p^e_{-it} can depend on the history of past wholesale prices. There are three relevant histories:

1. In period t, the manufacturers offer contracts which are identical to the ex-ante beliefs in period t. Then, both retailers retain the same belief in period $t+1$.

2. In period t, both manufacturers play the same $w \in W^*$, which differs from the ex-ante beliefs of the retailers in period t. The same holds for all previous periods up to $t-(T-1)$, with $T \in \{1, 2, ...\}$ being a parameter measuring the adaptation length in periods. In $t+1$, both retailers hold the new (passive) belief p^*. The set W^* contains wholesale prices that the retailers accept as possible equilibria (e.g., the collusive wholesale price w^C).

3. In period t, at least one of the manufacturers does not play a price consistent with a retailer's ex ante belief and it is not the case that both manufacturers play a price $w \in W^*$. In $t+1$, both retailers hold the new belief p^P, where p^P is the wholesale price of the perfect Bayesian Nash equilibrium of the stage game (Nash reversal).

An important feature of these beliefs is that they allow for the formation of collusion. Keeping in mind our definition of formability, suppose that, starting from any history in period s, the manufacturers start to play the collusive wholesale price w^C. If the manufacturers want to form collusion, the worst history in terms of our beliefs is that there was no collusion in $s-1$. This implies that, for the next T periods, the retailers' beliefs are fixed at p^P, such that the manufacturers can extract lower transfers. Denote the resulting manufacturer profit by Π^F. In each of these formation periods the retailers' beliefs remain at p^P but the manufacturers have to play w^C to establish collusion and to change the retailers' beliefs in the future. In such a period, each manufacturer could deviate to a lower wholesale price and realize a profit of $\Pi^{F,D}$.

To analyze the equilibria with adaptive beliefs, consider the constraints implied by opportunism proofness, credible punishment, and formability of collusion. In general, collusion can be sustainable if manufacturers are sufficiently patient.

In the following, we use the example of grim-trigger strategies to describe the necessary incentive constraints that arise from considering adaptive beliefs. We also solve for trigger strategies and present the results in the proposition, relegating the exposition to the proof.

Stability. The stability condition for collusion, once established, is

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \frac{\delta \Pi^P}{1-\delta}. \tag{12}$$

In this case retailers with adaptive beliefs already have the belief p^C and revert to the belief p^P after a deviation. Hence, with grim-trigger strategies the condition is the same as the incentive condition (6). Hence, collusion is stable if manufacturers are sufficiently patient (δ is large enough). Again, the condition is identical to the stability condition for a horizontal cartel.

Formability. Suppose the retailers' belief is p^P, that is the belief prevailing in periods of punishment and manufacturer competition. Hence, in the following inequalities , Π^P denotes the profit that manufacturers earn when they compete and play the perfect Bayesian equilibrium of the stage game. Formability is given if there is a transition path from competition to collusion where –on the path– the manufacturers' actions are mutually best responses.

Suppose the manufacturers decide to start colluding while the retailers believe p^P. Both manufacturers start to play w^C with the usual grim-trigger strategies that punish any deviation. During formation, the retailers believe in competition but the manufacturers set collusive prices. The retailers' beliefs are incorrect in transition periods (they adjust only after T periods). In the transition periods, the manufacturer profits are lower than under competition, $\Pi^F < \Pi^P$. The reason is that the beliefs are identical in both cases and whereas the manufacturers play their unique best response to the belief in periods of competition, which results in Π^P, in transition periods they play a worse action with respect to stage game profits as response to the same belief, resulting in a profit of Π^F.

With grim-trigger strategies, the the following condition implies that manufacturers have an incentive to jointly start to collude in a competitive period, such that they eventually arrive at the collusive equilibrium path:

$$\frac{\Pi^P}{1-\delta} \leq \frac{1-\delta^T}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C. \tag{13}$$

The left-hand-side contains the present value of perpetual competition. The first term on the right-hand-side is the discounted profit of the T periods in which retailers belief is p^P while manufacturers actually set w^C; the second term is the discounted profit of perpetual collusion starting after T formation periods.

Note that during the formation the manufacturer profits are lower than under competition: $\Pi^F < \Pi^P$.Thus each manufacturer may have an incentive to deviate during formation. Deviating during the formation phase yields a period prof if $\Pi^{F,D}$ but triggers a punitive action forever. Consider the incentives to stick to w^C in the formation phase. Under the following condition no manufacturer wants to deviate unilaterally in the formation phase, which implies that actions in the transition are mutual best-responses:

$$\frac{1-\delta^T}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C \geq \Pi^{F,D} + \frac{\delta\Pi^P}{1-\delta}. \tag{14}$$

Comparing the inequalities (13) and (14) shows that the latter is stricter if and only if $\Pi^{F,D} \geq \Pi^P$. This is always the case as we demonstrate in the proof of Proposition 4. Hence, the deviation condition (14) not only necessary but also sufficient for formability. Note that formability decreases in T, that is, holds for a smaller set of discount factors, as the left-hand-side decreases in T. This holds as an increase in T, which only affects the left-hand side of condition (14), shifts the weight Π^C to the smaller term Π^F.

Opportunism proofness. Collusion is opportunism-proof if the manufacturers have no incentive to deviate jointly from the collusive price. Suppose the manufacturers jointly behave opportunistically in the present period. They can earn an opportunism profits of Π^{JD} by lowering w_i and increasing F_i for each retailer. As a result, in the next period the retailers believe in competition. Confronted with competitive beliefs, the reformation phase starts so that the manufacturers need to play w^C for T periods to re-convince retailers of collusive prices. This yields the condition of opportunism-proofness

$$\frac{\Pi^C}{1-\delta} \geq \Pi^{JD} + \delta\left(\frac{\left(1-\delta^T\right)}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C\right). \tag{15}$$

For grim-trigger strategies, the stability condition is the same as for vertically integrated collusion. Collusion is formable and opportunism-proof if manufacturers are sufficiently patient, that is if the condition (14) for formability and the condition (15) for opportunism-proofness hold. Increasing the adaptation length T of the beliefs makes collusion less formable but more opportunism-proof, i.e., relaxes condition (15) but tightens condition (14). Opportunsim-proofness is harder to satisfy than stability whenever collusion is formable. To see this, compare the right-hand side of (15) with the stability condition (12) and note that $\Pi^{JD} > \Pi^D$.

Punishment is not credible whenever collusion is formable. To see this, note that whenever the formability condition holds and the manufacturers are supposed to punish, they are better off forming collusion again. This implies that formation dominates competitive pricing in the punishment phase.

Note that grim-trigger is a special case of trigger strategies with $\kappa \to \infty$. For trigger strategies we can generalize the result to

Proposition 4. *With adaptive beliefs and trigger strategies, the stability condition is the same as for vertically integrated collusion with the equivalent trigger strategies. Stability increases in the number of punishment periods κ. Collusion is formable and opportunism-proof if δ is sufficiently large. Increasing the adaptation periods T of the retailer beliefs makes collusion less formable but more opportunism-proof. Punishment is not credible for any $\kappa > 0$ whenever collusion is formable, but collusion may be sustained even without punishment, that is, for $\kappa = 0$.*

Proof. See Appendix A. □

The take-away from our analysis of adaptive beliefs is that beliefs exist which make collusion formable, sustainable and opportunism-proof. Formability intuitively requires that beliefs can adapt to collusion after a 'history' of competition or punishment. The adaptability can have a cost, the adaptable beliefs do make punishment non-credible as the credibility of punishment with trigger beliefs was supported by the belief alone. An interesting observation is that for adaptive beliefs there is a trade-off between opportunism-proofness and formability the longer beliefs take to adapt the harder it is to start collusion while opportunistic behavior that counts on restarting collusion becomes less of a problem.

As a polar case we find that the strategy to always collude, the degenerate trigger strategy with a punishment length of zero, can support a collusive equilibrium. If manufacturers play "always collusion", the incentive constraint for stability is

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \delta \left(\frac{1-\delta^T}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C \right). \tag{16}$$

Note that the stability is supported by the beliefs as deviation requires a new formation of collusion of length T. While this strategy is the least stable strategy, it is the only strategy that features formability and, in addition, features credible punishment.

Corollary 3. *For $\kappa > 0$. there exists a strongly renegotiation-proof equilibrium that is not formable if T and δ are sufficiently large. For $\kappa > 0$ and if formability is given, there is no renegotiation-proof equilibrium with trigger strategies. For $\kappa = 0$, a strongly renegotiation proof equilibrium exists if the opportunism-proofness condition* (15) *holds (implying also stability).*

Proof. The equilibrium path is always Pareto-efficient by assumption, such that any weakly renegotiation-proof equilibrium is also strongly renegotiation-proof.

To find the critical value of T such that collusion is not formable, but opportunism-proof for $\kappa > 0$, we must compare the relevant conditions. If collusion is not formable, punishment will be credible. The equilibrium is thus weakly renegotiation-proof for certain values of T. As shown in the proof of Proposition 4, the relevant condition for formation is the condition whereby a deviation from formation is unprofitable:

$$\delta^T \left(\Pi^C - \Pi^F \right) \geq \left(\frac{1-\delta}{1-\delta^{\kappa+1}} \right) \Pi^{F,D} - \Pi^F + \delta \left(\frac{1-\delta^{\kappa}}{1-\delta^{\kappa+1}} \right) \Pi^P > 0.$$

As $\delta < 1$, this condition is violated for sufficiently large T such that δ^T and thus the left-hand side of the formation condition becomes arbitrarily small.

The conditionfor opportunism-proofness 15can be written as

$$\delta^T \left(\Pi^C - \Pi^F \right) \leq \frac{1}{\delta} \left[\Pi^C - (1-\delta)\,\Pi^{JD} - \delta\Pi^F \right].$$

As for $\delta < 1$ the left-hand side of the opportunism-proofness condition also becomes arbitrarily small as T increases, there exists a T such that the opportunism condition holds, whenever the right-hand side is non-negative (which holds for sufficiently large δ), while the formation condition is violated.

□

6 Symmetric Beliefs

After considering passive beliefs, we turn to the analysis of the case that retailers have symmetric beliefs which we define as follows:

Definition 9. (Symmetric Beliefs) The price expectation p^e_{-it} of retailer i in period t about the price of retailer $-i$ is $p_i\left(w_{it}, p(w_{it})\right)$.

In other words, when a retailer receives an unexpected offer deviating from the candidate equilibrium, the retailer revises its belief and believes that its rival has received the same offer by its manufacturer. Symmetric beliefs are history-independent, because they only rely on the information contained in the current wholesale price offer. We assume that manufacturers play (grim-)trigger as in the previous sections. The case of symmetric beliefs as also analyzed in Liu and Thomes (2020)who consider the linear demand case and thus offer closed form solutions for the critical discount factor.

We focus again on industry-profit-maximizing collusion, which naturally arises when manufacturers jointly maximize their profits given symmetric beliefs. Denote the price expectation of retailer i with symmetric beliefs by $p^e_{it} := p_i\left(w_{it}, p(w_{it})\right)$. This allows the manufacturer to essentially choose the symmetric price level such that the joint maximization problem of the manufacturerscan be rewritten as

$$\Pi^C := \frac{1}{2} \max_p p \cdot D_i\left(p, p\right) + p \cdot D_{-i}\left(p, p\right) = \Pi^M.$$

Hence, the joint-profit-maximizing wholesale price of the manufacturers is equal to the industry-maximizing price. Moreover, this implies that any joint deviation of the manufacturers will always be the industry-profit-maximizing price if retailers hold symmetric beliefs. In contrast to (grim-)trigger beliefs, where manufacturers could jointly optimally deviate to a wholesale price below zero, such a deviation is not optimal, because only the collusive price maximizes the manufacturers' profits. As the manufacturers collude at the Pareto-efficient level, collusion is opportunism-proof. Additionally, punishment is not credible either because the manufacturers would prefer to revert back to collusion in every punishment period.

Collusion is formable because symmetric beliefs instantly adapt to the new wholesale price in every period. Manufacturers only need to agree on the collusive price and set it in any period. In a period where the manufacturers set wholesale prices of w^C, the retailers' expectations are immediately equal to $p_i\left(w^C, p(w^C)\right) = p_i(w^C, p^C) = p^C$, which corresponds to the expectation of collusion. Forming collusion immediately leads to stable to collusion as long as the stability condition is fulfilled.

From Pagnozzi and Piccolo (2011), we know that symmetric beliefs affect competition between vertically separated manufacturers. Competition is less fierce due to a so-called belief effect, which increases the competitive wholesale price above marginal costs. If punishment, however, relies on the competitive wholesale prices and profits, the stability of collusion is affected. Manufacturers must be more patient to satisfy the condition of stable collusion.

Symmetric beliefs violate the 'no signaling what you don't know' restriction imposed by perfect Bayesian equilibria as defined in Fudenberg and Tirole (1991).[11] We thus look for a weak perfect Bayesian equilibrium in the context of symmetric beliefs.

Proposition 5. *With symmetric beliefs, collusion with (grim-)trigger strategies is stable if condition 24 holds. Collusion is also formable if the stability condition holds. Collusion is always opportunism-proof but punishment is never credible.*

Proof. See Appendix A. □

Symmetric beliefs solve the opportunism problem by making it impossible to lower the wholesale prices without negatively affecting the price belief. As the symmetric belief follows the manufacturers' actions, it does not restrict and commit the manufacturers, as, for example, the trigger-beliefs do. This implies that formation, in the sense of needing to convince retailers, is not an issue with symmetric beliefs. However, as a downside for collusion, symmetric beliefs do not support the credibility of punishment as they make the punishment phase prone to renegotiation incentives. This yields

Corollary 4. *The equilibrium with symmetric beliefs described in Proposition 5 is not renegotiation-proof.*

7 Conclusion

We first analyze beliefs that do not react to present and past behavior of the manufacturers. We find that the classic opportunism problem found in one-shot games with secret vertical contracts carries over to an infinitely repeated game. For example, such beliefs would correspond to a situation with short-lived retailers that do not observe past wholesale prices. These simple beliefs may also arise in industries that have long-standing competitive conduct. Thus, an implication could be that belief differences over industries may explain why some industries give birth to collusion over and over again, while other industries stay competitive. Because these beliefs give rise to a perfect Bayesian equilibrium of the infinitely repeated game, and consequently are correct on the equilibrium path, they are self-enforcing and may never be challenged.

We then analyze beliefs that react to observed past actions. These beliefs can support trigger-based collusive equilibria and even make the punishment credible in situations in which this would not be the case in a vertically integrated industry. We show that opportunism can still be the most important challenge for the colluding firms, more so than the usual unilateral deviation incentives.

We find that beliefs that mirror the collusive strategy might not be flexible enough to allow for the creation of collusion. The intuition here is that such beliefs cannot handle

[11] Pagnozzi and Piccolo (2011) discuss this observation and also show that, with common cost shocks, symmetric beliefs can be consistent with a PBE.

the transition from a non-collusive state to working collusion because they would be too "pessimistic" about the future whenever they observe a non-collusive price in the history of the game.

The retailer beliefs studied in this paper have opposing effects on the ability of manufacturers to collude. Trigger beliefs and particularly grim-trigger beliefs feature credible punishment and opportunism-proofness because they make it difficult for manufacturers to again obtain high wholesale price after a breakdown. However, these beliefs do not allow for the formation of collusion.

We introduce a new belief that is adaptive over time to the manufacturers' pricing. From the manufacturers' point of view, the advantage of adaptive retailer beliefs is that they facilitate the formation of collusion. Because this belief does not mirror the collusive strategy, the belief does not support credible punishment; it may still support a credible collusive strategy because it 'punishes' deviation by believing in competitive conduct for a certain number of periods. Albeit adaptive beliefs can also satisfy the conditions for stability and opportunism-proofness.

Our setting would, furthermore, allow for a combination of adaptive beliefs and trigger beliefs that could improve manufacturer collusion. These retailer beliefs, which one could call *adaptive trigger beliefs*, would inherit the credibility of punishment from the trigger beliefs and combine it with formability of the adaptive beliefs. The idea is that the beliefs are adaptive in competitive periods that are not punishment periods, such that the formation of collusion is feasible. Once the formation process is complete, the adaptive beliefs switch to the trigger beliefs. Consequently, in punishment periods that are triggered by a deviation from the collusive prices, the belief is not adaptive. This lasts until the punishment phase is over; at this point, the beliefs switch from trigger beliefs to adaptive beliefs again. Together, these beliefs would be able to meet all the criteria, making collusion both renegotiation-proof and formable.

Because manufacturer cartels are ubiquitous, our results may help competition authorities to screen markets. Whenever supply contracts are not public or easily renegotiable, we find that the ability to form and sustain collusion critically depends on retailers' beliefs about the supply conditions of other retailers. This may make it easier to sustain collusion in markets in which the retailers are used to manufacturer collusion. Our findings suggest that it should be in the interest of colluding manufacturers to manage and influence their retailers' beliefs about the conditions in the wholesale market. One conjecture is thus that the opportunism problem may be one of the causes behind the widespread use of resale price maintenance and hub-and-spoke arrangements when manufacturers collude. Consequently, it might be useful for competition authorities to target retailers and provide incentives for retailers to report manufacturer cartels because they seem to be essential for making these cartels work.

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Appendix A: Proofs

Proof of Proposition 1.

Proof. We will show that any collusive grim-trigger strategy involving $w^C \neq 0$ cannot be supported by history-independent, passive beliefs. Take one retailer that holds a history-independent, passive belief about the retail price of the other retailer. Manufacturers play grim-trigger strategies. Suppose there exists a candidate equilibrium with grim-trigger strategies in which manufacturers agreed on some $w^C \neq w^P$. In this candidate equilibrium, beliefs must be correct on the equilibrium path (due to perfect Bayesian equilibrium). This belief must also be time constant, because the behavior on the equilibrium path is constant in any equilibrium with grim-trigger strategies. This belief about the rival's retail price is given by the equilibrium price p^C in any period. As usual, a grim-trigger equilibrium only exists if the stability condition holds, i.e.,

$$\begin{gathered}\tfrac{1}{1-\delta}\left[p^C \cdot D_i\left(p^C, p^C\right)\right] > p_i(0,p^C)D_i\left(p_i(0,p^C),p^C\right) + \\ \tfrac{1}{1-\delta}\left[p_i(w^P,p^C)D_i\left(p_i(w^P,p^C),p^C\right) + w^P\left[D_i\left(p_i(w^P,p^C),p_{-i}(w^P,p^C)\right) - D_i\left(p_i(w^P,p^C),p^C\right)\right]\right].\end{gathered} \tag{17}$$

We use the following short-hand expressions for the different profits to present (17) in a reduced form:

$$\frac{1}{1-\delta}\Pi\left(w^C,p^C\right) > \Pi(0,p^C) + \frac{1}{1-\delta}\Pi^P.$$

Note that w^P must be a best response for each retailer to w^P as otherwise each retailer would have an incentive to deviate in any punishment period where the current action has no effect on future beliefs or actions:

$$\begin{gathered}w^P \equiv \arg\max_{w_{it}}, p_i(w_{it},p^C)D_i\left(p_i(w_{it},p^C),p^C\right) \\ +w_{it}\left[D_i\left(p_i(w_{it},p^C),p_{-i}(w^P,p^C)\right) - D_i\left(p_i(w_{it},p^C),p^C\right)\right]\end{gathered} \tag{18}$$

Because we suppose that $w^C \neq w^P$, there are two cases to consider. First, $w^C > w^P$, which implies $p^C > p_{-i}(w^P,p^C)$ and, in turn, $D_i\left(p_i(w_{it},p^C),p_{-i}(w^P,p^C)\right) < D_i\left(p_i(w_{it},p^C),p^C\right)$. It follows that w^P must be smaller than zero as at the locus $w^P = 0$ the profits, given on the right-hand side of (18), decrease in w_{it}. This also implies that the punishment profits are larger than the profits on the candidate equilibrium path which violates the stability condition for the grim-trigger strategy and thus yields a contradiction. The profits rank $\Pi^P > \Pi(0,p^C) > \Pi\left(w^C,p^C\right)$, where the first inequality directly follows from the fact that $w^P < 0$ is preferred by the retailer over $w_{it} = 0$, and the second inequality follows from Lemma (1) which shows that the best response of a manufacturer to any constant equilibrium belief is $w_{it} = 0$.

The second case, $w^C < w^P$, results in a similar contradiction. $w^C < w^P$ implies $p^C < p_{-i}(w^P, p^C)$ and, evaluating the first order condition of (18) at the locus $w_{it} = 0$, implies $w^P > 0$. Still $\Pi^P > \Pi(0, p^C) > \Pi\left(w^C, p^C\right)$, which violates the stability condition 29. Hence, only $w^C = w^P$ is left as a candidate. From Lemma (1) and equation (18) follows that $w^C = w^P = 0$ must hold as w_{it} is the argument that maximizes the manufacturer profit on any equilibrium path. □

Proof of Proposition 2.

Proof. Manufacturers are able to *sustain* collusion as long as equation (7) is fulfilled. This condition is derived by inserting the respective profits into equation (6). To show that the incentive constraint is the same if manufacturers and retailers are pairwise integrated, it is sufficient to prove that the profits are the same: $\Pi_V^j = \Pi_I^j$ for $j = C, D, P$ (V: vertical, I: integrated). As we define Π_V^C to be half of the integrated industry-maximizing profit (2), the profit from collusion in the integrated case Π_I^C is the same.

The deviation profit Π_V^D is given by the first line of equation (3) and simplifies further in the second line as manufacturer and retailers' beliefs are aligned. The second line, however, is equal to the maximization problem of an integrated manufacturer, who maximizes the profit with respect to the retail price. Lastly, the punishment profits are aligned as well, following the same argument as before. The punishment profits are given by equation (5). Manufacturers set $w^P = 0$, which results in the same retail prices and profits as in a vertically integrated industry.

To see that punishment is *credible*, note that with grim-trigger beliefs any deviation leads to the belief p^P forever to which the best response is, in each period, for each manufacturer, to set $w = 0$. Consider the profit each manufacturer earns when jointly deviating in a punishment period:

$$\begin{aligned}\Pi =& w_{it} \cdot D_i\left(p_i(w_{it}, p^P), p_{-i}(w_{-it}, p^P)\right) + \left[p_i(w_{it}, p^P) - w_{it}\right] \cdot D_i\left(p_i(w_{it}, p^P), p^P\right) \\ &+ w_{-it} \cdot D_{-i}\left(p_{-i}(w_{-it}, p^P), p_i(w_{it}, p^P)\right) + \left[p_{-i}(w_{-it}, p^P) - w_{-it}\right] \cdot D_{-i}\left(p_{-i}(w_{-it}, p^P), p^P\right).\end{aligned}$$

Applying the envelope theorem and symmetry, the first-order conditions can be rewritten as

$$\left[\underbrace{D_i\left(p_i(w, p^P), p_{-i}(w, p^P)\right) - D_i\left(p_i(w, p^P), p^P\right)}_{\leq 0}\right] + w \underbrace{\frac{\partial p_i}{\partial w_{it}}}_{>0} \left[\underbrace{\frac{\partial D_i}{\partial p_i} + \frac{\partial D_{-i}}{\partial p_i}}_{<0}\right] = 0 \ \forall i. \quad (19)$$

This holds for or $w = 0$.

To see that collusion is *not formable*, consider any history with $w_i \neq w^C$ in $t = 0$. With this history, grim-trigger beliefs imply that the belief is p^P forever. Thus violating the condition two of the definition of formability that a collusive PBE can be obtained in some future period for any history. □

Proof of Proposition 4.

Proof. Similar to the incentive constraints above the proposition for grim-trigger strategies, we can also describe the incentive constraints when manufacturers play more general trigger strategies. The difference between these strategies is the length of the punishment phase κ. Because neither the formation condition (13) nor the opportunism-proofness condition (15) rely on punishment between the colluding firms, the conditions remain the same.

In the remainder of the proof, we characterize the stability condition and the stability condition of the formation phase for trigger strategies. If firms play trigger strategies with punishment length κ, the incentive constraint for stability is

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \delta\left(\frac{1-\delta^\kappa}{1-\delta}\right)\Pi^P + \delta^{\kappa+1}\left(\frac{1-\delta^T}{1-\delta}\right)\Pi^F + \delta^{\kappa+1}\frac{\delta^T}{1-\delta}\Pi^C.$$

As in all the above cases, the punishment must be $w^P = 0$ as there is no unilateral action in a punishment phase that has any effect on future beliefs. As the future beliefs and actions of the rival manufacturer are fixed in any punishment period, each manufacturer must play the one-shot best response of $w^P = 0$.

The deviation of one manufacturer triggers a punishment of length $\kappa \in [0, \infty)$. After the punishment phase, collusion is resumed but the adaptive retailer beliefs require a formation phase of length T. A deviation in the formation phase is not profitable if

$$\frac{1-\delta^T}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C \geq \Pi^{F,D} + \delta\left(\frac{1-\delta^\kappa}{1-\delta}\right)\Pi^P + \delta^{\kappa+1}\left(\frac{1-\delta^T}{1-\delta}\right)\Pi^F + \delta^{\kappa+1}\frac{\delta^T}{1-\delta}\Pi^C, \quad (20)$$

which simplifies to

$$\delta^T\left(\Pi^C - \Pi^F\right) \geq \left(\frac{(1-\delta)}{1-\delta^{\kappa+1}}\right)\Pi^{F,D} - \Pi^F + \delta\left(\frac{1-\delta^\kappa}{1-\delta^{\kappa+1}}\right)\Pi^P. \quad (21)$$

Next, we show that condition (13) is always stricter than condition (20). We also demonstrate that this is the relevant condition for formation. To see this, we consider the polar cases, which correspond to a strategy "always collude" for $\kappa = 0$ and the grim trigger strategy as $\kappa \to \infty$. We already know from the analysis before the proposition that the postulated relation of the conditions holds for $\kappa \to \infty$.

For $\kappa = 0$, the incentive constraint for formation can be written as

$$\delta^T\left(\Pi^C - \Pi^F\right) \geq \Pi^P - \Pi^F.$$

A deviation from the collusive price in the formation phase results in a profit of $\Pi^{F,D}$. However, for $\kappa = 0$, no punishment is triggered. The stability condition for formation becomes

$$\frac{1-\delta^T}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C \geq \Pi^{F,D} + \delta\left(\frac{1-\delta^T}{1-\delta}\frac{\Pi^F}{1-\delta} + \frac{\delta^T}{1-\delta}\Pi^C\right),$$

which simplifies to

$$\delta^T \left(\Pi^C - \Pi^F\right) \geq \Pi^{F,D} - \Pi^F. \tag{22}$$

Observe that a deviation in the formation phase is unprofitable under a stricter condition than the incentive condition for formation if $\Pi^{F,D} \geq \Pi^P$. Additionally, a necessary condition is $\Pi^C \geq \Pi^{F,D}$ as the discount factor δ is in the range $(0,1)$. As $\delta < 1$, the left-hand side of condition (22) decreases in T, such that formability becomes harder to satisfy the larger T.

Thus, for both cases of $\kappa = 0$ and $\kappa \to \infty$, we demonstrated that sticking to the formation phase requires higher values of δ (the condition is more strict) if $\Pi^{F,D} \geq \Pi^P$. Let us now analyze intermediary values of κ.

Inequality (21) differs from the formation incentives by term (1) and term (2). Recall that the formation condition (13) is

$$\delta^T \left(\Pi^C - \Pi^F\right) \geq \Pi^P - \Pi^F$$

and thus independent of κ. For a given value of δ, the weight of the term $\Pi^{F,D}$ is strictly monotonously decreasing in κ, whereas the weight of Π^P is strictly monotonously increasing in κ. Hence, the condition $\Pi^{F,D} \geq \Pi^P$ is sufficient for any $\kappa \in [0, \infty)$ to guarantee that condition (21) is tighter than condition (13). This implies that the stability condition for formation is the relevant condition for general trigger strategies.

Next, we show that

$$\Pi^{F,D} \geq \Pi^P$$

holds, because the profit of i increases in the wholesale price of the competitor for given beliefs, as this increases the price of the competing retailer and increases demand for i. The profit under competition is

$$\Pi^P \left(w_i = 0, p^e_{-i} = p^P\right) = \underbrace{p_i\left(0, p^P\right)}_{p_i^P} \cdot D_i\left(p^P, p^P\right).$$

A firm that deviates form formation plays $w_i = 0$ as this maximizes the unilateral spot profit. The deviation profit $\Pi^{F,D}$ is similar to the punishment profit and only consists of the revenue $p_i(\cdot) \cdot D_i(\cdot)$. However for $\Pi^{F,D}$ both the manufacturer and the retailer expect a higher wholesale price of w^C of the other manufacturer, yielding a higher price at the competing retailer. As profits increase in the rivals price $\Pi^{F,D} > \Pi^P$ holds.

Finally, observe that punishment is not credible whenever collusion is formable. To see this consider that the formability condition implies that is better to jointly deviate in any period where punishment profits are expected to a new formation of collusion. This also implies that only in the case $\kappa = 0$ formability and credible punishment are both met, as punishment is not part of the strategy. □

Proof of Proposition 5.

Proof. We will show all results for general trigger strategies as defined in Subsection 5.3, which contain grim-trigger strategies as a subcase for $\kappa \to \infty$. Let us first consider the stability condition. As argued in the text before the proposition, manufacturers collude on the industry-maximizing level and earn profits of $\Pi^C = {}^{\Pi^M}/_2$. If the manufacturers make a one-sided deviation from the collusive agreement, they must consider that changing their wholesale price also influences the belief of their own retailer. A deviating manufacturer maximizes the following problem

$$w^D = \arg\max_{w_i} w_{it} \cdot D_i\left(p_i(w_{it}, p(w_{it})), p_{-i}(w^C, p(w^C))\right) \\ + \left(p_i(w_{it}, p(w_{it})) - w_{it}\right) \cdot D_i\left(p_i(w_{it}, p(w_{it})), p(w_{it})\right)).$$

This results in a profit Π^D for the deviating manufacturer. Note that $w^D > 0$. Punishment is assumed to be carried out on the competitive wholesale price level. The competitive benchmark corresponds to the case analyzed in Pagnozzi and Piccolo (2011). Under symmetric beliefs manufactures solve

$$\max_{w_i} w_{it} \cdot D_i\left(p_i\left(w_{it}, p(w_{it})\right), p_{-i}\left(w_{-it}, p(w_{-it})\right)\right) + \left(p_i(w_{it}, p(w_{it})) - w_{it}\right) \cdot D_i\left(p_i\left(w_{it}, p(w_{it})\right), p(w_{it})\right)$$

This results in Π^P. By using the envelope theorem, the first-order condition simplifies to

$$\underbrace{w_i \underbrace{\frac{\partial D_i(\cdot)}{\partial p_i}}_{<0}}_{<0} + \underbrace{(p(w_i, p(w_i)) - w_i) \cdot \underbrace{\frac{\partial D_i(\cdot)}{\partial p_{-i}}}_{>0}}_{\text{belief effect:}>0} = 0. \tag{23}$$

Applying symmetry to (**??**) defines the equilibrium wholesale prices w^P. Note that the right hand side of33 is positive at $w = 0$ which implies that $w^P > 0$. Under competition, with symmetric beliefs, prices are above the price level than under competition with passive beliefs. The stability condition given by

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \delta\left(\frac{1-\delta^\kappa}{1-\delta}\Pi^P + \frac{\delta^\kappa}{1-\delta}\Pi^C\right). \tag{24}$$

To see why collusion is formable with symmetric beliefs, recall the definition of formability. Symmetric beliefs allow for forming collusion if there exists a strategy profile, such that for this belief, best responses are played in period $t \geq s$, and there exists a weak PBE that results in payoff V^C. Given any history before period s - the period in which collusion is about to be formed -, manufacturers play mutual best responses in the following periods when setting wholesale prices that result in the collusive price defined by equation (1). The joint profit maximization of both manufacturers and setting w^C is a weak PBE if the stability condition for collusion - equation (24) - is fulfilled. In this equilibrium, both manufacturers earn a payoff of $V^C = {}^1/_{1-\delta}\Pi^C$. Hence, collusion is formable according to our definition.

As in Subsection 5.4, it should be considered whether there exist incentives to deviate from formation. Due to symmetric beliefs, collusion is in place directly after it is formed, such that the condition for deviating from formation is identical to deviating from collusion in equation (24).

To check whether collusion is opportunism-proof, we consider the joint maximization problem of the manufacturers. As shown in the text, this leads to the Pareto-efficient wholesale price level. That is, manufacturers always prefer to set w^C when jointly maximizing. Thus, there is no scope for opportunistic behavior, because the beliefs directly react to any change in wholesale prices. Following the same argument, punishment is not credible, because manufacturers would prefer to renegotiate and revert to setting w^C jointly. $\square$