

**Informal Incentives, Labor Supply, and the Effect
of Immigration on Wages**

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Working Paper No. 2112
May 2021

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May 14, 2021

Abstract

This paper theoretically investigates how an increase in the supply of homogenous workers can raise wages, generating new insights on potential drivers for the observed non-negative wage effects of immigration. We develop a model of a labor market with frictions in which firms can motivate workers only through informal incentives. A higher labor supply increases firms' chances of filling a vacancy, which reduces their credibility to compensate workers for their effort. As a response, firms *endogenously* generate costs of turnover by paying workers a rent, and this rent is higher if an increase in labor supply reduces a firm's credibility. By this effect, a higher labor supply — for example caused by immigration — can *increase* workers' compensation. Moreover, an asymmetric equilibrium exists in which native workers are paid higher wages than immigrants and work harder. In such an equilibrium, an inflow of immigrants increases productivity, profits, and employment.

Keywords: Informal Incentives, Labor Supply, Immigration.

JEL-Classification: D21, D86, F22, J21, J61, L22.

*We thank Alex Ahammer, Heski Bar-Isaac, Ben Elsner, Ulrich Glogowsky, Shingo Ishiguro, Michael Irlacher, Nadzeya Laurentsyeva, Noriaki Matsushima, Dominik Sachs, Andreas Steinmayr, Flora Stiftinger, as well as seminar participants at JKU Linz, OSIPP, and Japan Fair Trade Commission for helpful comments. We acknowledge financial support from Japan Fair Trade Commission and JSPS KAKENHI (JP16K21740, JP18H03640, JP19K01568, JP20K13451). The views expressed in this paper are those of the authors only and do not represent the views of any of the above institutions. All remaining errors are our own.

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1 Introduction

Recent evidence suggests that, contrary to the standard model of a competitive labor market, a higher labor supply induced by immigration can *increase* the wages of native workers (see Dustmann et al., 2008, and Peri, 2016, for surveys). Suggested mechanisms mostly rely on skill heterogeneity among workers. In these models, immigration not only increases total labor supply, but also changes the relative skill composition of the workforce and therefore raises the wages of some (mostly high-skilled) workers whose relative supply has gone down (Peri and Sparber, 2009; Dustmann et al., 2012).

While these studies assume that workers are compensated according to their marginal productivity, evidence points towards considerable wage-setting power of firms (Manning, 2003; Dube et al., 2020; Manning, 2021). Furthermore, immigrants are paid less than natives even when conducting the same kinds of tasks, and this wage gap — although declining over time — persists in the long run (Kerr and Kerr, 2011; Battisti et al., 2018).¹

Building upon these insights, this paper theoretically investigates the following questions. First, why do some studies find non-negative wage effects even for low-skilled native workers, whose task composition does not change and who are particularly affected by more intense competition?² Second, why are immigrants persistently paid lower wages than natives with similar skills? Third, how does immigration affect the optimal provision of incentives for workers to exert effort?

We show that an inflow of immigrants can increase wages in a setting where firms have wage-setting power and homogeneous workers need to be incentivized by self-enforcing agreements. By the presence of labor-market frictions, firms may not be able to fill a vacancy immediately, but the chances are better with a higher labor supply. In self-enforcing agreements, a firm's credibility to compensate a worker as promised is lower if a vacancy is easier to fill. Then, it can be optimal for firms

¹Studies incorporating this evidence have focused on search-and-matching models of the labor market, where the reservation wages of immigrants are smaller by assumption. In these models, positive effects of immigration on native workers' wages rely on both, task complementarity between native and immigrant workers and the creation of new jobs (Battisti et al., 2018).

²For example, Fogel and Peri (2016) report that immigration to Denmark increased the wages of natives who switched to more complex jobs, but also of those who continued to work on the same kinds of (mostly low-skilled) tasks where the increase of labor supply by immigration was particularly pronounced. Tabellini (2019) finds that immigration to the US in the early 20th century did not generate losses even among those working in highly exposed sectors. Clemens and Hunt (2019) reject the hypothesis that immigration has substantial negative effects on low-skilled native workers.

to *endogenously* increase costs of turnover by paying workers a rent, who then earn more than their outside option even though firms can fully determine the terms of employment. By reducing a firm’s credibility, a higher labor supply can *increase* workers’ compensation, which is in contrast to efficiency-wage models in the spirit of Shapiro and Stiglitz (1984).³ Since immigration leads to a higher labor supply, it thus can induce upward pressure on workers’ compensation. We further show that, even though native and immigrant workers are ex-ante identical, an *asymmetric* equilibrium exists in which natives are offered higher wages than immigrants and have a higher endogenous outside option. In such an equilibrium, immigration increases the productivity of the average employment relationship.

Section 2 discusses the related literature in detail, including the studies mentioned above. Sections 3 and 4 set up and analyze an infinite-horizon model of an industry with many workers and firms. This model builds upon the setup introduced by MacLeod and Malcomson (1998) and extends it by introducing a matching friction on the labor market and allowing for continuous (instead of binary) effort. More precisely, in every period, each firm can employ exactly one worker. If a firm has a vacancy, it is randomly matched with an unemployed worker with probability α^F . We assume that α^F increases in the extent of unemployed workers and decreases in the extent of open vacancies. With probability $1 - \alpha^F$, the vacancy remains open until the next period. If a firm is matched with an unemployed worker, the firm makes a take-it-or-leave-it offer which contains an upfront wage and a discretionary bonus potentially paid after a worker’s effort choice. Effort increases the firm’s revenues but is costly for workers.

We assume that formal (i.e., court-enforceable) incentive contracts are not feasible, but a worker’s effort is observable to his employer. Given this, a firm must use a relational contract to motivate a worker, in which not only the worker has to be incentivized to exert effort, but also the firm to compensate the worker as promised (i.e., a contract must be self-enforcing). In this case, a firm which reneges on a promised payment is punished by the employed worker who subsequently does not exert effort anymore.⁴ Still, a firm can replace a worker after renegeing and start a new employment relationship. Therefore, a firm can make a credible promise

³Note that the above endogenous turnover costs have also been investigated by MacLeod and Malcomson (1998). However, they would predict that a higher labor supply reduces (or does not affect) wages, whereas we show that, with labor-market frictions, the opposite can occur (see Section 2).

⁴We exclude multilateral punishments as in Levin (2002) by considering a setting in which deviations cannot be observed by non-involved parties.

only if such turnover is sufficiently costly for the firm. Because a vacancy causes a production loss, a lower probability of filling a vacancy α^F increases the firm's cost of turnover. When α^F is high, however, the temptation to start a new relationship is large and reduces the willingness to compensate for effort as promised. In this case, there is an equilibrium in which, to maximize an individual firm's profits, firms pay a rent to newly employed workers to *endogenously* increase the cost of turnover.⁵ Thereby, a firm's reservation payoff is reduced and its commitment is increased. This allows for higher equilibrium effort, and employed workers' payoffs are strictly positive. Different from approaches with one principal and one agent (such as Levin, 2003), where each player's reservation payoff is exogenously given, equilibrium transfers can affect the relationship surplus in our setting. However, a firm needs to pay a rent even to a worker whose predecessor has left for an exogenous reason. The total turnover costs (consisting of the cost of not filling a vacancy and rents to new workers) optimally balance a firm's commitment with equilibrium costs when having to replace a worker, and equilibrium effort in this case is below the first best.⁶

A higher labor supply increases α^F and thus makes it easier for firms to fill a vacancy.⁷ Because of the self-enforcing nature of contracts, each firm increases compensation to keep the total turnover cost (and consequently equilibrium effort) constant. However, an inflow of workers also reduces the chances of unemployed workers to find a job, which in turn lowers a worker's outside option. This "efficiency-wage" mechanism, which can be seen in MacLeod and Malcomson (1998), puts downward pressure on workers' compensation, so the total effect of an increased labor supply can go either direction. We show that the total effect is positive if the mass of firms is small. Furthermore, if we endogenize the mass of firms (determined by a zero-profit condition subject to entry costs), the resulting entry or exit of firms keeps workers' outside options constant. Then, the effect of a higher labor supply on the compensation and utility of employed workers becomes *unambiguously positive*. Therefore, our results are qualitatively different from the efficiency-wage models in the spirit of Shapiro and Stiglitz (1984) or MacLeod and Malcomson (1998), where compensation either decreases in or is unaffected by a higher labor supply.

For lower values of α^F , workers are not paid a rent because the commitment

⁵We show that it is without loss to distribute this rent evenly over time, so workers receive a rent in each period of employment.

⁶Note that $\alpha^F = 1$ in MacLeod and Malcomson (1998) with $N > F$, so all turnover costs in their paper are endogenous and take the form of upfront rents.

⁷Battisti et al. (2018) find that immigration indeed reduces labor-market frictions.

provided by the low chances of filling a vacancy is sufficient for firms. Effort is below the first best for intermediate values of α^F , and at the first best for low values of α^F . In the former case and with a fixed number of firms, an inflow of workers increases each firm’s profits but reduces equilibrium levels of effort and compensation. Allowing for firm entry, though, effort and compensation are pushed up to their original levels. In the latter case, effort and compensation are unaffected by α^F , but higher profits due to a lower turnover cost still yield firm entry. Thus, for small and intermediate levels of α^F , the entry of firms induced by a higher labor supply increases employment opportunities, whereas a worker’s compensation is not (or negatively) affected.

We argue that these results — an increase in the labor supply might not only have negative effects on wages and employment even if workers are homogeneous and firms have wage-setting power — help understand the consequences of immigration and complement other theoretical explanations. As we discuss in Section 2, an abundance of evidence beginning with Card (1990) has found that immigration does not necessarily worsen the labor market conditions for native workers; rather, it might improve them.⁸ Although recent studies mostly focus on heterogeneity in worker skills, there is evidence that immigration can benefit native workers even when they work on the same kinds of, mostly low-skilled, tasks. By investigating the optimal provision of informal incentives for homogeneous workers, our model provides conditions for these results to occur. Moreover, our mechanism builds upon firms having considerable wage-setting power, which is in line with recent evidence that workers are not paid their marginal productivity (Manning, 2003; Dube et al., 2020; Manning, 2021).

Section 5 investigates the possibility that native workers are treated better than immigrant workers, an outcome that has been empirically identified by Battisti et al. (2018), Dustmann et al. (2012), or Dustmann and Preston (2012). We show that such unfavorable outcomes for immigrants can emerge even if all types of workers are ex-ante identical. This is because there is a profit-maximizing equilibrium in which natives are better off and have a higher *endogenous* outside option, and in which immigrants work harder than natives. Firms potentially have different arrangements with “insiders” than with “outsiders,” which indeed is a profit-maximizing equilibrium if α^F is high. Then, a higher labor supply and the resulting higher α^F have no direct effect on firms’ profits because the higher rents paid to workers just offset

⁸In Section 2, we also discuss evidence for negative wage effects of immigration.

higher matching probabilities. For our mechanism, it is only important that the *expected* rent paid to a new worker goes up, and it does not depend on how this rent is allocated among different identities of workers. Therefore, an inflow of outsiders can result in paying the necessary rent increase only to insiders. Such an equilibrium benefits insiders at the expense of outsiders without affecting the expected profits of an unmatched firm.⁹ We discuss that social norms would determine whether insiders and outsiders are treated more equally or only insiders benefit from immigration. Such an asymmetric equilibrium also increases outsiders' effort levels above those of insiders and thus average effort and productivity, an outcome consistent with evidence of positive productivity effects of immigration.

In Section 6, we discuss the robustness of our main results: general levels of bargaining power between a firm and a worker, other forms of endogenous turnover costs, and different specifications of the probability of filling a vacancy. We also provide additional predictions based on the (un)availability of formal contracts, the severity of labor market frictions, and the allocation of bargaining power — which could help assess the importance of our mechanisms in further empirical research. All proofs are in the Appendix.

2 Related Theoretical and Empirical Literature

In this section, we discuss related theoretical and empirical research. This paper studies the consequences of an increase in the labor supply on the optimal provision of informal incentives. Applying it to the immigration of homogeneous-skill workers, we derive predictions on the consequences of an inflow of immigrant workers on the wages of native workers, as well as on productivity, profits, and employment. Importantly, we demonstrate that differences between natives and immigrants can sustain in the long run, even when both are ex-ante identical.

The standard model of the competitive labor market involves homogeneous-skill workers and complete contracts; as labor supply goes up, the equilibrium wage goes down (or stays constant after capital has been adjusted).¹⁰ Efficiency-wage models of the labor market acknowledge the need to incentivize workers and as-

⁹We show that, as long as the share of insiders is sufficiently large, there is no profit-maximizing equilibrium (among the set of equilibria we consider) in which outsiders are treated better than insiders.

¹⁰As an example, Borjas (2001) analyzes the consequences of immigration within the canonical model of the labor market.

sume that this is obtained by a combination of wages above the market-clearing level and a firing threat (Shapiro and Stiglitz, 1984; Yellen, 1984; MacLeod et al., 1994). In these models, a higher labor supply reduces workers’ utilities once they become unemployed and hence motivates them to work harder. In response, firms decrease wages. Therefore, this “efficiency-wage effect” predicts that a higher labor supply *reduces* equilibrium wages. Incorporating the labor-market friction α^F , we demonstrate that a higher labor supply can reduce a firm’s credibility when making promises, deriving the opposite of the efficiency-wage effect. We characterize conditions for our mechanism to dominate the efficiency-wage effect, which happens either if the number of firms is sufficiently small compared to that of workers, or if the number of firms is endogenously determined by firm entry or exit. Then, a higher labor supply *increases* workers’ compensation.

MacLeod and Malcomson (1998) take into account that incentives to workers are often informal and performance pay (such as bonuses) might be used to provide incentives. If firms are on the short side of the market, standard performance pay is not possible because firms would fire and replace workers when supposed to pay a bonus. Then, firms pay workers a rent to motivate them, which generates endogenous turnover costs because this rent has to be paid to new workers as well. Their mechanism involving endogenous turnover costs also appears in our model. However, since the labor market in MacLeod and Malcomson (1998) is frictionless (firms can fill a vacancy with probability one if there is unemployment), a higher labor supply reduces (or has no effect) on wages.

Yang (2008) extends the setting of MacLeod and Malcomson (1998) by assuming that turnover is costly. He demonstrates that higher (exogenous) turnover costs reduce total wage payments and unemployment. Fahn (2017) assumes that firms and workers bargain about the terms of the employment relationship. Workers’ incentives increase in their bargaining power, thus a minimum wage can increase effort and consequently the efficiency of employment relationships.

In the literature on immigration, the effects of a higher labor supply on wages have been extensively analyzed. A number of empirical studies lend support to the canonical model of the labor market, finding negative wage effects of immigration (Borjas, 2003; Borjas, 2017). Other studies come to different conclusions. In a seminal paper, Card (1990) studies a large inflow of unskilled Cuban immigrants into Miami in 1980. He finds no significant consequences for employment and wages of low-skilled non-Cubans. Peri and Yasenov (2019) confirm Card’s results, with the point estimates of log wages even being positive. Winter-Ebmer and Zweimüller

(1996) find positive effects of immigration on the wages of young Austrians; Friedberg (2001) shows no significant impact of immigration from Soviet Union to Israel, where most point estimates are positive. Peri (2007) reports that an increase in average wages of US-born workers is caused by immigration. Furthermore, exploring the consequences of immigration on US workers between 1990 and 2006, Ottaviano and Peri (2012) observe a significantly positive effect on wages of college- and noncollege-educated workers.¹¹

To explain these observations, the literature has mostly focused on heterogeneity in worker skills — in particular between immigrants and native workers — and that native workers are able to switch to jobs with different skill demands (see Peri and Sparber, 2009, Ottaviano and Peri, 2012, or Peri, 2016).¹² Then, immigration generally has positive effects on high-skill and negative effects on low-skill native workers. While such an approach explains the effects of immigration on some wages, recent evidence suggests that there can be a non-negative wage effect even among low-skill workers. For example, Foged and Peri (2016) explore how an exogenous inflow of refugees to Denmark affects native workers over the period 1991-2008. Using Danish administrative data on labor market outcomes of individuals, they find that the wages of native workers significantly went up. This wage increase is not only driven by native workers moving to new employers (there conducting more complex tasks), but also by native workers who do not change occupations and continue working on the same kinds of tasks as before. Furthermore, they do not find crowding out of native unskilled workers (i.e., negative effects on employment) or depressing effects on their wages. Thus, these native workers perform the same kinds of tasks as immigrant workers (who are mostly low-skilled) but still benefit from their entry. Moreover, Tabellini (2019) discovers that immigration across U.S. cities between 1910 and 1930 increased natives’ employment, spurred industrial production, and did not generate losses even among those working in occupations highly exposed to immigrants’ competition. Clemens and Hunt (2019) “conclude that the evidence from refugee waves [...] fails to substantiate claims of large detrimental impacts on workers with less than high school.”

Related to our asymmetric equilibrium result, there is evidence that native workers are treated better than immigrants. For example, Battisti et al. (2018) analyze the consequences of immigration in 20 OECD countries and find that, for each

¹¹Also see Dustmann et al. (2008) and Peri (2016) for surveys.

¹²Alternatively, Dustmann et al. (2012) claim that positive wage effects can follow from a perfectly elastic capital supply.

country and skill level, native workers are paid higher wages than immigrants. They discuss that natives' wage premia can be driven by either productivity differences or higher outside options. Our paper *endogenously* generates both, a higher outside option of native workers and higher effort of immigrants. The latter is consistent with Dustmann et al. (2012) as well, who argue that the extent of positive wage effects of immigration for some skill levels can be explained by productivity differences only if immigrant workers are more productive than natives. Productivity effects of immigration are also examined by Mitaritonna et al. (2017) who explore the consequences of highly-skilled immigration to France on firm outcomes, including productivity and employment. They find a positive effect on productivity (as well as on average wages). Ottaviano et al. (2018) analyze the effects of immigration on industries that trade in services, an area in which we would argue informal incentives are particularly important. They find that the inflow of immigrants substantially increased firm exports.¹³ Jordaan (2018) examines the impact of immigration on productivity in the manufacturing sector of Malaysia, which — at all skill levels — has a positive and significant effect on productivity. Furthermore, Tabellini (2019) finds that immigration increased the value added per establishment, as well as firms' productivity.

3 Model

Setup There are a mass $F > 0$ of firms and a mass $N > 0$ of workers. All workers and firms are risk neutral. There are infinitely many periods $t = 1, 2, \dots$, and all players have a common discount factor $\delta \in (0, 1)$.

Workers and firms either are part of a match or not, and each firm can employ exactly one worker. At the beginning of every period, unmatched players enter the labor market. An unmatched firm is randomly matched with an unemployed worker with probability $\alpha^F(n, f) \in (0, 1)$, where $f > 0$ is the mass of unmatched firms, $n > 0$ is the mass of unemployed workers, $\alpha_f^F < 0$, $\alpha_n^F > 0$, $\lim_{n \rightarrow 0} \alpha^F(n, f) = 0$, and $\lim_{n \rightarrow \infty} \alpha^F(n, f) = 1$. Correspondingly, $\alpha^N(n, f)$ is the probability for an unemployed worker to be matched with a firm, with $\alpha_f^N > 0$, $\alpha_n^N < 0$, $\lim_{f \rightarrow 0} \alpha^N(n, f) = 0$,

¹³Ottaviano et al. (2018) distinguish between three different mechanisms (all of which generate significant and positive results), where a direct cost reduction effect can be explained by our model — more effort to increase efficiency of production. Moreover, they identify a substitution effect of immigration allowing the in-house production of previously outsourced tasks, and that immigrants enable a better access to their regions of origins.

Contracts, Strategies, and Equilibrium Concept We consider situations in which effort as well as per-worker output can be observed by both, the firm and the worker, but not by anyone outside the respective match. Hence, no verifiable measure of the agent’s performance exists, and incentives can only be provided informally, i.e., with relational contracts.

We assume strategies are *contract specific*, in the sense of Board and Meyer-Ter-Vehn (2014): actions of firms and workers do not depend on the identity of the worker, calendar time, or history outside the current relationship.¹⁵ Contract-specific strategies imply that firms’ and workers’ strategies cannot condition on any outcomes of other matches, i.e., no multilateral relational contracts as in Levin (2002) are feasible. We focus on pure strategies.

The equilibrium concept we apply is *social equilibrium*. This concept describes a subgame-perfect equilibrium, which is restricted by the assumptions that strategies are contract-specific.¹⁶ We analyze a social equilibrium that maximizes an unmatched firm’s profits. We focus on the *stationary steady state*, which allows us to omit time subscripts.¹⁷

To conclude, two remarks are in order. First, our setup is based on the model of MacLeod and Malcomson (1998) and extends it by the introduction of labor market frictions and continuous (in contrast to binary) effort. Second, the compensation structure (with an upfront wage and a bonus paid at the end of a period) is assumed for simplicity and does not have to be taken literally. For example, the bonus could also be paid in the form of a salary at the beginning of the next period or correspond to future promotion (adapted to take discounting and the possibility of a termination into account), without changing expected payoffs and any of the constraints derived below. It is only important that its payment is contingent on the worker exerting equilibrium effort, which effectively means that it is tied to the worker keeping his job.

¹⁵In Section 5, we analyze asymmetric equilibria based on a worker’s group identity.

¹⁶The equilibrium concept is called *social* because — although strategies are contract-specific — a player’s strategy will depend on the strategies of all market participants, as the possibility of a re-match determines everyone’s *endogenous* outside option. Hence, (as in Ghosh and Ray, 1996; Kranton, 1996; MacLeod and Malcomson, 1998; Fahn, 2017), subgame perfection not only pertains to individual relationships, but the market as a whole has to be in equilibrium. This is because potential deviations also include the opportunity to terminate a current match and go for a new one.

¹⁷We further discuss this aspect below and show in the proof to Proposition 1 that, for our formulation of the optimization problem, the stationarity assumption is without loss of generality for all periods other than $t = 1$.

4 Analysis

4.1 Social Equilibrium

Now, we determine a profit-maximizing steady-state equilibrium from the perspective of an individual firm (i.e., such an equilibrium does not necessarily maximize industry-wide profits), taking the behavior of other firms as given. We derive a symmetric equilibrium in which outcomes among all firms and workers are identical. We focus on an equilibrium in which *any* deviation from equilibrium behavior would lead to the static Nash equilibrium with zero effort and zero payments; thus, such a match is separated at the end of a period (i.e., both parties choose to leave the current match). This is optimal by Abreu (1988): any observable deviation triggers the highest feasible punishment for the defector.

Equilibrium Payoffs The discounted utility stream of an employed worker in the stationary steady state equals

$$U = u + \gamma\delta U,$$

where $u = w + b - c(e)$ is an employed worker's per-period utility. Note that discounted continuation utilities are multiplied with γ because workers might leave the market for exogenous reasons with probability $1 - \gamma$, then having a utility of zero.

The utility of an unemployed worker is denoted by \bar{U} and equals $\bar{U} = \alpha^N U + \delta\gamma(1 - \alpha^N)\bar{U}$. Rearranging it yields

$$\bar{U} = \frac{\alpha^N}{1 - \delta\gamma(1 - \alpha^N)} U.$$

A matched firm's discounted profit stream is denoted by Π , the expected profits of a firm with an open vacancy are denoted by $\bar{\Pi}$:

$$\begin{aligned}\Pi &= e\theta - b - w + \delta [\gamma\Pi + (1 - \gamma)\bar{\Pi}], \\ \bar{\Pi} &= \alpha^F \Pi + \delta (1 - \alpha^F) \bar{\Pi}.\end{aligned}$$

Rearranging them yields

$$\begin{aligned}\Pi &= \frac{(1 - \delta(1 - \alpha^F))(e\theta - b - w)}{(1 - \delta)(1 - \delta\gamma(1 - \alpha^F))}, \\ \bar{\Pi} &= \frac{\alpha^F(e\theta - b - w)}{(1 - \delta)(1 - \delta\gamma(1 - \alpha^F))}.\end{aligned}$$

4.2 Benchmark: Formal Contracts

We start with the brief analysis of a benchmark in which formal short-term contracts on an agent's effort are feasible. Then, paying $b = c(e)$ and $w = (1 - \delta\gamma)\bar{U}$ maximizes profits. Furthermore, $\bar{U} = 0$ in the symmetric equilibrium. Thus, a firm keeps the full social surplus and maximizes $e\theta - c(e)$, hence e^{FB} is implemented. This holds irrespective of the value of α^F , therefore a change in N has no effect on a worker's compensation. Because firms can find a replacement with a larger probability if a worker leaves for exogenous reasons, their profits increase in N .¹⁸

4.3 Analysis of the Baseline Model

To enforce a certain effort level for a worker, each firm is subject to the following constraints. First, it must be in the worker's interest to exert the agreed-upon effort level. Consider a deviation in which the worker chooses zero effort, which is the optimal deviation due to the argument above. In this case, the worker does not receive the bonus; the respective match splits up, so a worker's continuation utility in this case equals $\delta\gamma\bar{U}$. It follows that equilibrium effort e^* must satisfy the agent's incentive compatibility (IC) constraint:

$$-c(e^*) + b + \delta\gamma U \geq \delta\gamma\bar{U}. \quad (\text{IC})$$

Second, an employed worker's utility must be at least as high as his outside option in this period. This equals \bar{U} for the following reason. At the end of period t , the worker stays only if he expects to receive (at least) \bar{U} in the following period. At the beginning of period $t + 1$, the firm could deviate and instead offer a contract with $U = \delta\gamma\bar{U}$ (which constitutes the worker's outside option from the perspective of period $t + 1$ because he would have to wait until the next period before potentially finding a new match). However, the worker would respond to such a deviation by

¹⁸If workers have positive bargaining power, an increase in N reduces their compensation. We discuss general bargaining power in 6.

collecting the wage and choose $e_{t+1} = 0$, with the match then splitting up at the end of period t . Thus, the following individual rationality (IR) constraint must hold:

$$U \geq \bar{U}. \quad (\text{IR})$$

Note that even though $\bar{U} = \frac{\alpha^N}{1-\delta\gamma(1-\alpha^N)}U$ holds in equilibrium and $\alpha^N < 1$, (IR) cannot be omitted. This is because \bar{U} is constituted by an arrangement the worker (potentially) has with a different firm, hence is regarded as exogenous by individual firms.

Third, a firm must pay a bonus as promised. If the firm reneges and refuses to pay the equilibrium bonus in period t , the match splits up at the end of the period and both parties re-enter the matching market. Therefore, the maximum enforceable bonus payment is given by a dynamic enforcement constraint and equals

$$-b + \delta\gamma\Pi + \delta(1-\gamma)\bar{\Pi} \geq \delta\bar{\Pi}. \quad (\text{DE})$$

There, we also have to take into account that even if a firm pays the bonus, its worker might leave for exogenous reasons (which happens with probability $1-\gamma$). Since

$$\Pi - \bar{\Pi} = \frac{(1-\alpha^F)(e\theta - b - w)}{(1-\delta\gamma(1-\alpha^F))},$$

(DE) becomes

$$b \leq \delta\gamma(1-\alpha^F)(e\theta - w). \quad (\text{DE})$$

(DE) describes the maximum bonus the firm can credibly promise in a relational contract. Intuitively, a high level of bonus may not be self-enforceable because a firm has an incentive to renege and go for a potential new match. Holding other parameters constant, a given bonus is more difficult to sustain as α^F is larger, i.e., if it is easier for a firm to fill a vacancy. Also, sticking to its current match has to be optimal for a firm on the equilibrium path, requiring $\delta\gamma\Pi + (1-\gamma)\delta\bar{\Pi} \geq \delta\bar{\Pi}$ and hence $\Pi \geq \bar{\Pi}$. Given $b \geq 0$, this condition is implied by (DE) and hence can be omitted.

Finally, in a stationary steady state, the mass of newly matched firms must be equivalent to the mass of newly matched workers, the mass of unmatched firms at the beginning of a period must be the same as at the end of a period, and the same must hold for workers. Since these conditions do not explicitly appear in Proposition

1 but will be important for comparative statics, we defer a formal characterization of these conditions to Sections 4.4 and 4.5.

Now, we characterize a profit-maximizing equilibrium where firms maximize $\bar{\Pi}$, subject to the constraints just derived. Note that our results would naturally be the same if the objective was to maximize Π , the profits of a matched firm. However, maximizing $\bar{\Pi}$ allows us to (without loss of generality) focus on stationary arrangements. If we maximized Π instead, it would be optimal to treat workers in the first period of their employment differently than in later ones. Thus, maximizing $\bar{\Pi}$ substantially simplifies our exposition.

Our first proposition states how α^F determines equilibrium effort and the utility of workers.

Proposition 1 (Optimal Informal Incentives) *There exists a profit-maximizing equilibrium with the following properties. There are $\bar{\alpha}^F, \underline{\alpha}^F \in (0, 1)$ such that*

- *For $\alpha^F \geq \bar{\alpha}^F$, equilibrium effort is characterized by $c'(e^*) = \delta\gamma\theta$. Each (matched and unmatched) worker's utility is positive (and $U^* > \bar{U} > 0$).*
- *For $\underline{\alpha}^F \leq \alpha^F < \bar{\alpha}^F$, equilibrium effort is characterized by $c(e^*)/e^* = \delta\gamma(1 - \alpha^F)\theta$, with $e^* < e^{FB}$. Each worker's utility is zero (i.e., $U^* = \bar{U} = 0$).*
- *For $\alpha^F < \underline{\alpha}^F$, equilibrium effort is characterized by $e^* = e^{FB}$. Each worker's utility is zero (i.e., $U^* = \bar{U} = 0$).*

Because formal contracts are not feasible, a firm's promise to reward a worker for his effort must be credible. As explored above, a worker who does not receive a promised payment responds by not exerting effort anymore. Different from "standard" relational-contracting models with one principal and one agent where only the potential future relationship surplus determines enforceable effort, a reneging firm can replace a worker and start over. Therefore, a firm in our setting can only make credible promises to reward effort if turnover is costly (in addition to the standard requirement that the future relationship surplus is sufficiently high).¹⁹ One form of turnover cost stems from labor market frictions which reduce the chances of finding a replacement. If these frictions are large, i.e., the probability of finding a new worker α^F is small, such exogenous turnover costs are enough for firms to honor their promises. If frictions are small and $\alpha^F > \bar{\alpha}^F$, firms make use of another, endogenous,

¹⁹This manifests in $\underline{\alpha}^F$ increasing in δ and γ , and effort increasing in both arguments if $\alpha^F \geq \underline{\alpha}^F$.

mechanism to make turnover costly by granting new workers a rent as in MacLeod and Malcomson, 1998 or Fahn, 2017; thus, firms do not utilize their wage-setting power to fully extract the relationship surplus. This implies that new workers receive an upfront wage which is costly for firms because — different from payments made later on — the wage paid in the first period of a worker’s employment cannot be used to provide incentives. However, this rent reduces a firm’s profits when starting a new relationship and consequently increases its commitment in the current. This is different from models such as Levin (2003) where outside options are exogenously given and the relationship surplus is orthogonal to transfers. Note that, although the stationarity assumption might appear restrictive here, it is without loss of generality to have the wage in a first period of employment the same as in later ones.²⁰ This also implies that the rent paid to new workers is equally allocated over time, hence a worker’s rent is the same in every period of employment.

Such an equilibrium with endogenous turnover costs particularly makes use of the term “social” in social equilibrium. The productivity of a firm’s *current* relationship depends on the costs of starting a new relationship in the *future* — although potential new workers are not able to observe anything that happens in the firm’s current employment relationships. Such a social equilibrium specifies that workers regard an offer with a lower rent as a deviation, thus firms have an incentive to compensate their workers as promised. Put differently, the social equilibrium requires a *norm* that high wages are paid independent of a worker’s tenure.²¹ Before proceeding, note that, instead of workers receiving a rent, turnover might also be made more costly by a reduction of effort in the first period(s) of a match. In Section 6, we discuss this and other potential forms of endogenous turnover costs.

If α^F is sufficiently small, the presence of the labor-market friction alone is sufficient. Then, firms make use of their bargaining power and leave no rents to their workers. Furthermore, equilibrium effort is equal to e^{FB} . For intermediate α^F , the bonus is as high as feasible given α^F , effort is below e^{FB} and determined by a binding (DE) constraint.

Generally, the *optimal* level of turnover costs for firms would balance higher incentives that can be provided in a current relationship with the costs of starting new relationships later on (which happens on the path of play due to exogenous

²⁰See the proof to Proposition 1; if we maximized $\bar{\Pi}$ instead of $\bar{\Pi}$, it would indeed be optimal to pay a rent only in the first period of employment.

²¹See Ghosh and Ray (1996), Kranton (1996), and MacLeod and Malcomson (1998) for more detailed discussions about the role of norms in related settings.

turnover). For $\alpha^F < \bar{\alpha}^F$, equilibrium turnover costs are “too large” (firms can only increase turnover costs but not reduce them). For $\alpha^F \geq \bar{\alpha}^F$, equilibrium turnover costs are at the optimal level from the perspective of an individual firm. Then, equilibrium effort is below e^{FB} because, at e^{FB} , having marginally smaller costs of turnover would only cause a second-order loss in profits due to lower effort. Moreover, taking the behavior of other firms into account, endogenous turnover costs are more expensive for firms than the exogenous costs stemming from labor-market frictions – because the former also increase an employed worker’s outside option. This aspect is further explored in our next section.

4.4 Comparative Statics with a Fixed Mass of Firms

We now conduct comparative statics with respect to the mass of workers N , holding the mass of firms F constant; we endogenize F and determine its value by a zero-profit condition in Section 4.5. A higher N will increase the mass of unemployed workers n and consequently raise $\alpha^F(n, f)$ but reduce $\alpha^N(n, f)$. To simplify the following analysis, we slightly reduce the generality of the α^F, α^N from now on and assume that $\alpha^F(n, f) = \alpha^F(n - f)$ where $\alpha^F(\cdot)$ is increasing, as well as $\alpha^N(n, f) = \alpha^N(n - f)$ where $\alpha^N(\cdot)$ is decreasing.

As a preliminary step, we formalize the conditions that must hold in the labor market in a stationary steady state, where $f^* > 0$ is the equilibrium mass of unmatched firms and $n^* > 0$ the equilibrium mass of unemployed workers. First, the mass of newly matched firms must be equivalent to the mass of newly matched workers, hence

$$n^* \alpha^N = f^* \alpha^F. \quad (1)$$

Second, the mass of unmatched firms at the beginning of a period must be the same as at the end of a period, i.e., $f^* = (1 - \alpha^F)f^* + (1 - \gamma)(F - f^*)$, or equivalently

$$f^* = \frac{1 - \gamma}{1 - \gamma + \alpha^F} F. \quad (2)$$

The same holds for unemployed workers, hence $(1 - \alpha^N)n^* + (1 - \gamma)(N - n^*) = n^*$, or equivalently

$$n^* = \frac{1 - \gamma}{1 - \gamma + \alpha^N} N. \quad (3)$$

Plugging (2) and (3) into (1) yields

$$\alpha^N = \frac{(1 - \gamma)F}{(1 - \gamma + \alpha^F)N - F\alpha^F} \alpha^F$$

and, again using (3),

$$n^* - f^* = N - F.$$

Thus, $\alpha^F (n^* - f^*) = \alpha^F (N - F)$, and $\partial \alpha^F / \partial N = (\alpha^F)'$. Now, we are ready to present the results of comparative statics with respect to N , which depend on the size of α^F in relation to the thresholds $\bar{\alpha}^F$ and $\underline{\alpha}^F$ derived in Proposition 1.

Corollary 1 (Comparative Statics with Constant F) *Assume F is exogenously given.*

- For $\alpha^F \geq \bar{\alpha}^F$, effort e^* is independent of N , whereas total compensation $w^* + b^*$ and an employed worker's utility U^* may increase or decrease. $w^* + b^*$ and U^* increase in N if F is sufficiently small.
- For $\underline{\alpha}^F \leq \alpha^F < \bar{\alpha}^F$, e^* and $w^* + b^*$ decrease in N , whereas U^* is unaffected by N .
- For $\alpha^F < \underline{\alpha}^F$, e^* , $w^* + b^*$, and U^* are unaffected by N .

We first describe the intuition for $\alpha^F \geq \bar{\alpha}^F$. Note that

$$U^* = \frac{\delta \gamma \bar{U} (1 - (1 - \alpha^F) \delta \gamma) - \delta \gamma (1 - \alpha^F) e^* \theta + c(e^*)}{\delta \gamma \alpha^F} \geq 0.$$

We distinguish between (i) a direct effect of a higher N on α^F holding \bar{U} constant and (ii) an indirect effect incorporating changes in \bar{U} . For (i), an increase in N directly increases α^F , which increases employed worker's utility and compensation. This is because total turnover costs are at their optimal level if $\alpha^F \geq \bar{\alpha}^F$ and determine the trade-off between benefits of commitment and costs when workers leave. Therefore, an increase in α^F lets firms increase a worker's rent to the same extent.

For (ii), the indirect effect captures the effect on an employed worker's outside option, $\bar{U} = \alpha^N U / [1 - \delta \gamma (1 - \alpha^N)]$. There, workers are paid more in case they are re-employed, but the probability of finding an alternative job, α^N , goes down. This indirect effect resembles the well-known efficiency wage effect in the spirit of Shapiro and Stiglitz (1984).

If the indirect effect on the outside option is positive, workers always benefit from a higher N . Even if it is negative but not too large, the positive direct effect can dominate, and wages and utilities increase in N . This holds if F is sufficiently small.

Note that there is no direct impact of N on $\bar{\Pi}$ (total turnover costs and effort remain constant), only an indirect one which is negatively proportional to the effect on \bar{U} . If a higher N decreases workers' outside options, firms profits go up, and vice versa (hence, firms can potentially even benefit from larger labor market frictions). We further pursue this aspect in Section 4.5.

If $\alpha^F < \bar{\alpha}^F$, the labor market friction is larger than the optimal level from a firm's perspective. Therefore, if N goes up, firms fully "utilize" the decreased friction and request lower effort in response to their reduced commitment (unless $e^* = e^{FB}$). Moreover, there is no indirect effect on the outside option because $\bar{U} = 0$. Since effort goes down, the worker's compensation also goes down. If frictions are so high that e^{FB} is implemented, a change in N has no consequences on effort.

4.5 Comparative Statics with an Endogenous Mass of Firms

Now, we analyze the case in which F is endogenously determined by a zero-profit condition. We assume that there exists a sufficient pool of potential entrant firms, and each of them can enter the industry by paying an entry cost $K > 0$. Then, a zero-profit condition implies $-K + \bar{\Pi} = 0$ (in addition, $\partial\bar{\Pi}/\partial F < 0$ needs to hold).

Because $\bar{\Pi}$ stays constant, any change in N must be balanced by a change in F to keep $\bar{\Pi}$ at the initial level, that is,

$$d\bar{\Pi} = \frac{\partial\bar{\Pi}}{\partial N}dN + \frac{\partial\bar{\Pi}}{\partial F}dF = 0,$$

and

$$\frac{dF}{dN} = -\frac{\partial\bar{\Pi}/\partial N}{\partial\bar{\Pi}/\partial F}$$

must hold. This yields the following comparative statics.

Proposition 2 (Comparative Statics with Endogenous F) *Assume that F is endogenously determined to keep $\bar{\Pi}$ constant.*

1. *For $\alpha^F \geq \bar{\alpha}^F$, total compensation $w^* + b^*$ and an employed worker's utility U^* increase in N , whereas effort and \bar{U} are unaffected. F might increase or decrease.*

2. For $\alpha^F < \bar{\alpha}^F$, F increases in N , whereas compensation, effort, U^* and \bar{U} are unaffected.

Recall that, with $\alpha^F \geq \bar{\alpha}^F$, there is no direct effect of N on $\bar{\Pi}$, only an indirect effect via \bar{U} . The endogenous entry or exit keeps \bar{U} constant, and consequently only the positive direct effect of a higher N on U and compensation remains. Because a higher α^F increases both the hiring probability and the rent paid from a firm to a worker, employment effects of an increased N can be either positive or negative when $\alpha^F \geq \bar{\alpha}^F$. For $\alpha^F < \bar{\alpha}^F$, the direct effects of a higher N on effort and thus compensation are eliminated by firm entry. Thus, a higher N increases employment, leaving effort and utilities unaffected. In sum, with endogenous F the consequences of a higher N on wages are never negative in our setting in which firms can set wages, and can be strictly positive (when $\alpha^F \geq \bar{\alpha}^F$).²²

To conclude this section, note that the level of entry costs would determine equilibrium steady-state profits and consequently the values α^F, α^N that are consistent with $\bar{\Pi}$. Proposition 2 implies that profits are increasing in α^F for $\alpha^F < \bar{\alpha}^F$. Moreover, since α^F has no direct effect on profits for $\alpha^F \geq \bar{\alpha}^F$, positive outside options then imply that $\bar{\Pi}$ is maximized at $\bar{\alpha}^F$. Thus, the maximum feasible entry costs for firms to be active are such that $\alpha^F = \bar{\alpha}^F$. For all lower cost levels, α^F can be below or above $\bar{\alpha}^F$, i.e., there potentially is an equilibrium with relatively low and one with relatively high labor market frictions. We do not want to make a stand which we think is more likely, however would argue that high levels of α^F might be observed particularly in markets which experience immigration. This is based on the interpretation in which comparative statics with constant F would describe the short-term, while endogenous F would describe the long-term consequences of immigration. Moreover, firms' adjustment would probably not be immediate (in particular if reduced profits would call for firm exit), thus immigration pushing α^F considerably above $\bar{\alpha}^F$ followed by gradual responses by firms would likely yield a new steady state level of $\alpha^F > \bar{\alpha}^F$ (which exists if $\partial\alpha^F/\partial F$ is – in absolute terms – sufficiently small to guarantee $\partial\bar{\Pi}/\partial F < 0$; see the proof to Proposition 2).

²²The situation would be different if workers had positive bargaining power, an aspect we discuss in Section 6.

5 Asymmetric Equilibria

So far, our model has delivered an alternative explanation for positive wage and employment effects of immigration, however with outcomes for all workers — whether they might be immigrants or native workers — being the same. Motivated by the evidence that native workers are treated better than immigrants (Battisti et al., 2018; Dustmann et al., 2012), this section shows that a persistent wage premium for native workers can arise in a setting where they are otherwise identical to immigrants, i.e., they are equally productive and have the same (exogenous) outside option.

Assume there are two kinds of workers, “insiders” and “outsiders.” These identities can be distinguished by firms but all workers are otherwise identical (below, we discuss the implications of insiders having better exogenous outside options than outsiders). We assume that firms with a vacancy are randomly matched with workers, so targeted search is not possible. Moreover, only firms with an open vacancy are (potentially) matched with workers, hence it is not possible for firms with filled positions to look for another type of worker.

If a firm has an open vacancy, the probability of being matched with an outsider is α^{FO} , the probability of being matched with an insider is α^{FI} , and $\alpha^F = \alpha^{FI} + \alpha^{FO}$. When conducting comparative statics, we will also assume that an increase in the amount of outsiders has no effect on α^{FI} (although one might expect a negative relationship, in particular if many outsiders are present). We argue that this assumption is justified as long as there are not too many outsiders, and will further discuss its implications below. Finally, random matching implies that an unemployed insider has the same chances α^N to find a job as an unemployed outsider.

Now, a firm’s profits when hiring an insider are Π^I , and Π^O when hiring an outsider. Therefore, an unmatched firm’s expected profits are

$$\begin{aligned}\bar{\Pi} &= \alpha^{FI}\Pi^I + \alpha^{FO}\Pi^O + \delta(1 - \alpha^F)\bar{\Pi} \\ &= \frac{\alpha^{FI}\Pi^I + \alpha^{FO}\Pi^O}{1 - \delta(1 - \alpha^F)}.\end{aligned}$$

In the following, our objective is to maximize $\bar{\Pi}$, taking workers’ outside options as exogenously given and holding F constant. The main mechanism and trade-off as in the previous section still hold. To understand what outcomes can be supported in an asymmetric profit-maximizing social equilibrium, take a situation with no outsiders as a starting point, and with α^{FI} above the threshold $\bar{\alpha}^F$ derived in Proposition

1 (i.e., $\alpha^{FI} \geq \bar{\alpha}^F = 1 - c(e)/\delta\gamma e\theta$, where e is characterized by $\delta\gamma\theta - c'(e) = 0$). Therefore, insiders are paid a rent which is needed to make turnover sufficiently costly and deter firms from reneging. Moreover, profit-maximizing effort is below the first best to optimally balance the trade-off between stronger incentives and lower endogenous turnover costs. We now compare this to a situation in which outsiders are present. Importantly, firms' optimal behavior is not uniquely determined. Recall from our previous analysis that, as long as $\alpha^F > \bar{\alpha}^F$, a change in α^F had no *direct* effect on profits but only indirectly via workers' outside options which are taken as given by individual firms. Thus, there naturally exists a symmetric equilibrium in which outsiders are treated exactly as insiders, receive the same payments and exert the same effort. In addition, profit-maximizing equilibria in which outsiders are treated worse than insiders exist.²³ There, lower payments to outsiders increase a firm's expected profits when starting a new employment relationship, making it necessary to raise insiders' rents to address a higher reneging temptation. The best feasible arrangement for insiders involves firms' profits with outsiders to be as high as feasible. As long as α^{FO} is small, such an equilibrium pushes outsiders' payoffs to their outside option of zero and implements an effort level either at e^{FB} or determined by a firm's binding dynamic enforcement constraint. In both cases, an outsider's effort e^O is strictly larger than an insider's effort e^I (which is still characterized by $\delta\gamma\theta - c'(e^I) = 0$); moreover, e^O is increasing in α^{FO} as long as it is below e^{FB} . In such an equilibrium, outsiders might *work harder but earn less* than insiders. At some point, α^{FO} is so high that it is not optimal to further increase insiders' rents. Instead, profits with outsiders are reduced (until eventually they are the same as profits made with insiders), for example by paying them a rent or decreasing their effort (which still has to be weakly higher than e^I).

We do not want make a stand which of these equilibria, the one in which outsiders are treated equally or in which they are treated worse than insiders we consider more likely. We think that this could depend on social norms and conventions. For example, if outsiders in our setting describe immigrants, a society's preferences might either call for not treating them worse, or for mostly protecting the interests of native workers. Our analysis reveals that, in the latter case, insiders *benefit most* from an inflow of immigrants.

In the following proposition, we describe (partial) comparative statics with respect to α^{FO} (holding α^N constant).

²³Note that, for $\alpha^{FI} \geq \bar{\alpha}^F$, the opposite that insiders are treated worse than outsiders is not feasible.

Proposition 3 (Asymmetric Equilibrium) *Assume $\alpha^{FI} > \bar{\alpha}^F$ and that insiders are employed. Then, $U^I > \bar{U}^I > 0$, and insiders' effort is characterized by $\delta\gamma\theta - c'(e^I) = 0$ for all levels of α^{FO} . Starting from $\alpha^{FO} = 0$, an increase in α^{FO} always increases payments to employed insiders. There are multiple profit-maximizing social equilibria:*

- *There exists a symmetric equilibrium in which $e^O = e^I$, $w^O = w^I$, and $b^I = b^O$. In this equilibrium, outsiders also benefit from an increase in α^{FO} .*
- *For a sufficiently small $\alpha^{FO} > 0$, there exists an asymmetric equilibrium in which $w^O = 0$, $b^O = c(e^O)$, and $e^O > e^I$ (thus, $U^O = \bar{U}^O = 0$). In this equilibrium, $\partial(w^I + b^I)/\partial\alpha^{FO} > 0$ and $\partial e^O/\partial\alpha^{FO} \geq 0$.*

The total effect of an inflow of outsiders would also have to incorporate the reduction in α^N and the (positive or negative) consequences on workers' outside options. However, if we endogenized F (as in Section 4.5), the outside option of insiders would have to stay constant, thus the indirect effect would disappear and only the positive direct effect prevail.

A further important result relates to average effort which could be interpreted as productivity. Then, among equilibria maximizing individual firm profits, the average productivity is highest in the asymmetric equilibrium that maximizes insiders' payoffs. Their effort is the same among all potential profit-maximizing equilibria, whereas this asymmetric equilibrium would involve the highest feasible effort exerted by outsiders. Therefore, immigration can increase firms' productivity in our model, which is supported by evidence provided by Mitaritonna et al. (2017), Ottaviano et al. (2018), and Jordaan (2018).

Note again that we have for simplicity assumed that an inflow of outsiders does not reduce α^{FI} . However, even if we allowed for such an interaction, our results would not change fundamentally as those rely on an increase in α^F , the *total* probability of firms being matched.

In what follows, we show that the positive effect can also extend to the case $\alpha^{FI} \leq \bar{\alpha}^F$. The following Lemma states that the threshold of α^{FI} above which insiders are paid a rent is smaller if outsiders are present, and decreases in α^{FO} .

Lemma 1 *There exists a $\bar{\alpha}^{FI} \leq \bar{\alpha}^F$ above which insiders are paid a rent. $\bar{\alpha}^{FI}$ is strictly decreasing in α^{FO} .*

Moreover, for $\alpha^{FI} \leq \bar{\alpha}^{FI}$, all profit-maximizing equilibria are symmetric and outsiders are treated exactly as insiders (thus, the outcome is equivalent to the one derived in Section 4).

Lemma 2 *Assume $\alpha^{FI} \leq \bar{\alpha}^{FI}$. Then, $e^O = e^I$ is uniquely optimal, as well as $w^O = w^I = 0$ and $b^I = b^O = c(e)$. Moreover, effort is larger than the level characterized by $\delta\gamma\theta - c'(e) = 0$ and decreasing in α^{FO} . Finally, $\partial\bar{\Pi}/\partial\alpha^{FO} > 0$.*

Finally, we discuss implications of insiders having a strictly positive exogenous outside option, thus outsiders would potentially be cheaper to employ. Most of our results then continue to hold as long as α^{FO} is sufficiently small. With high α^{FI} , insiders would still get a rent, and the equilibrium in which insiders are paid more but work less hard could still be sustained. Only with a relatively large mass of outsiders (for example if $\alpha^{FO} \geq \bar{\alpha}^F$), excluding insiders could become optimal.

To summarize, we have demonstrated that a higher outside option and seemingly lower productivity of insiders can emerge *endogenously* in an asymmetric equilibrium. This could explain why immigrants are permanently paid less than natives, even after frictions such as language barriers should have been substantially reduced.

6 Discussion and Conclusion

We have demonstrated how immigration can increase the wages of native workers in a setting where informal incentives are needed to motivate workers, firms have wage-setting power, and immigration increases a firm's chances to fill a vacancy. Moreover, employment and productivity levels can go up. To conclude, we discuss the robustness of our results once we relax some assumptions and suggest additional predictions which can be used to assess the validity of our model and distinguish it from alternative explanations.

Robustness First, we explore the consequences of workers having positive bargaining power in wage negotiations. Then, outcomes would rely on the exact specification of the bargaining process, whether disagreement payoffs are determined by separation or only by non-production (as in Hall and Milgrom, 2008), and to what extent renegotiation would happen. Here, we discuss one particular setting which is motivated by dynamic bargaining approaches such as Ramey and Wat-

son (1997) or Fahn (2017).²⁴ Assume that, at the beginning of a period, firm and worker bargain about how the relationship surplus is shared. The relationship surplus contains the expected discounted sum of payoffs generated in this relationship (i.e., $(e\theta - c(e)) / (1 - \delta\gamma)$) minus disagreement payoffs. Disagreement would cause a termination of the match and let both players enter the matching market in the subsequent period. The bargaining outcome would determine a worker’s *minimum payoff*, which however could unilaterally be increased by a firm; thus in equilibrium utility levels of workers would be higher than their bargaining outcomes if this also increased firms’ profits. Finally, any deviation from equilibrium behavior would lead to a termination of the employment relationship.

Given the above bargaining setting, we now discuss the role of endogenous turnover costs. Endogenous turnover costs increase a firm’s commitment and induce workers to exert higher effort. A positive bargaining power also provides incentives to workers to exert higher effort because they want to remain employed to secure the associated rent in the future. If this rent is sufficiently high, a “voluntary” increase by firms is not profitable. However, if the bargaining outcome is not sufficient to implement firms’ desired effort, it remains optimal to increase the costs of turnover by paying workers an additional rent. The latter case is more likely if workers’ bargaining power is low or if α^F is high, since a high α^F increases a firm’s disagreement payoff and thus reduces the relationship surplus. Then, a higher labor supply caused by immigration will continue to increase an employed worker’s compensation, making his bargaining power effectively irrelevant in determining his payoff. To the contrary, if bargaining outcomes would determine equilibrium payoffs (i.e., if worker bargaining power was large or α^F small), but also if formal contracts on effort were possible, an increase in N would reduce a worker’s compensation via the negative effect on his disagreement payoffs.

Second, we discuss the specific form of endogenous turnover costs. Firms would be indifferent between increasing a worker’s compensation (as in our setting) or using different measures, for example letting workers temporarily reduce their effort or conduct inefficient trainings, or doing anything else that destroys surplus by “money burning.”²⁵ To assess a firm’s credibility, however, it is necessary for workers to

²⁴Those are hybrid models where individual choices are made non-cooperatively but bargaining follows the cooperative Nash-bargaining regime (see Miller and Watson, 2013, for an axiomatic foundation); moreover, deviations cause a termination of the relationship .

²⁵See Carmichael and MacLeod (1997) or McAdams (2011) who demonstrate that inefficiencies in the early periods of repeated interactions with anonymous re-matching might be needed to sustain cooperation later on.

observe the realization of turnover costs. Thus, we would argue that the safest way for firms to ensure this is using options such as wages or effort reductions. Those directly affect workers and are obviously costly for firms without supplying direct benefits to them. It remains to discuss why, to make turnover more costly, firms do not use effort reductions in early periods instead of higher wages. Effort reductions would actually increase industry-wide profits because workers' outside options are zero throughout. However, if we extended the model slightly, for example by introducing a product market where prices decrease in total output and allowing a firm to employ more than one worker, paying higher wages would dominate reducing effort for individual firms. The reason is that a rent paid in firm A increases workers' outside options in firm B. Thus, production becomes more expensive for firm B who would consequently reduce its employment and output, allowing firm A to boost its sales (of course, firm B would do the same, causing adverse effects on firm A).

Third, we assume that a firm's chances to fill a vacancy, α^F , are exogenous to a firm's efforts. One might argue that firms should be able to increase α^F , for example by conducting costly search. Even then, our results continue to hold if firms are able to hide their previous search effort from a newly hired worker. To illustrate this argument, assume that α^F is exactly at $\bar{\alpha}^F$. Now, holding search effort fixed, an increase in N and the resulting higher α^F would make it optimal to increase a worker's compensation to keep effort (and consequently the firm's profits) constant. But then, the firm would be better off by reducing costly search and keeping α^F at $\bar{\alpha}^F$. However, if workers believe that the firm has reduced its search effort but are not able to observe whether this has actually occurred, firms would have an incentive to secretly increase search effort once having an open position (without having to pay a higher wage), reducing their incentives to pay a promised bonus in their current employment relationship.

Additional Predictions Our model is complementary to other approaches that have been used to explain positive wage effects of immigration (e.g., task heterogeneity). In the following, we describe possibilities to further determine the relevance of our setup. The empirical assessments of the following predictions would not only help to evaluate the usefulness of our model, but also generate new insights on the consequences of immigration.

First, our results rely on the unavailability of formal, court-enforceable contracts to adequately motivate workers (if effort was verifiable, immigration would either have zero or negative effects on natives' wages). Thus, we would predict that our

mechanism is more relevant in settings where informal incentives and subjective performance measures are more important to motivate workers. We would argue that this holds in the service industry, where aspects such as friendliness or customer orientation are important but difficult to measure objectively. But also high-skill tasks or those that are R&D-intensive are often difficult to be incentivized with the use of formal contracts alone. Furthermore, firms have to rely on informal incentives if the legal system they operate in is weak. As an example for the latter, Fallah et al. (2019) investigate the impact of the Syrian refugee influx on labor market outcomes in Jordan. They find that employment and unemployment were unaffected, whereas hourly wages went up.

Second, the severity of labor market frictions plays an important role in our setting. Recall that, with endogenous F , the effect of immigration on wages is strictly positive for $\alpha^F \geq \bar{\alpha}^F$ and zero for $\alpha^F < \bar{\alpha}^F$ (in the latter case, it is negative with fixed F); employment effects are ambiguous for $\alpha^F \geq \bar{\alpha}^F$ and strictly positive for $\alpha^F < \bar{\alpha}^F$. Thus, we would predict larger positive wage and smaller (positive or negative) employment effects in markets in which firms can fill a vacancy relatively easily, whereas tighter labor markets would be associated with a smaller (or even negative) wage effect but a larger positive impact on employment.

Finally, our results are stronger whenever firms have more pronounced wage-setting or higher bargaining power. Put differently, we would expect negative wage effects of immigration in markets where workers earn according to their marginal productivity, and non-negative or even positive effects if firms have the power to set the terms of employment.

Appendix

Proof of Proposition 1:

Here, we first show that our stationarity assumption is without loss of generality. Standard arguments can be applied to confirm that stationary arrangements are optimal from the second period of an employment relationship. In the first such period, though, wages might be different (if first-period effort or bonus were different from later values, the problem could be transformed into one that is payoff equivalent but in which only wages differ). Denote w_1 as the wage paid in the first, w the wage paid in all later periods of an employment relationship. Note again that we focus an equilibrium in which any deviation triggers the highest feasible punishment for the defector (Abreu, 1988): a deviation from equilibrium behavior leads to the static Nash equilibrium with zero effort, zero payments, and a match is separated at the end of a period (i.e., both parties choose to leave the current match). Then, the optimization problem is to maximize

$$\max \bar{\Pi} = \frac{\alpha^F [e\theta - b - w_1 + \delta\gamma (w_1 - w)]}{(1 - \delta)(1 - \delta\gamma(1 - \alpha^F))}$$

subject to

$$-c(e) + b + \delta\gamma w - \delta\gamma(1 - \delta\gamma)\bar{U} \geq 0 \quad (\text{IC})$$

$$-b + \delta\gamma [(1 - \alpha^F)e\theta + \alpha^F w_1 - w] \geq 0 \quad (\text{DE})$$

$$w_1(1 - \delta\gamma) + w\delta\gamma + b - c(e) - (1 - \delta\gamma)\bar{U} \geq 0 \quad (\text{IR1})$$

$$w + b - c(e) - (1 - \delta\gamma)\bar{U} \geq 0 \quad (\text{IR})$$

To show that it is weakly optimal to set $w_1 = w$, let us to the contrary assume that there is a profit-maximizing social equilibrium with $w_1 > w$. Then, we can reduce b by $\delta\gamma\varepsilon$ and increase w by ε . This operation leaves Π_1 , (IC), (DE), and (IR1) unaffected, but relaxes (IR). The opposite operation can be applied if $w_1 < w$, thus it is weakly optimal to set $w_1 = w$. Note that this holds for all periods besides the very first of the game.

Therefore, (IR) is implied by (IR1) and can be omitted. The Lagrange function becomes

$$\begin{aligned}
\mathcal{L} &= \frac{\alpha^F [e\theta - b - w]}{(1 - \delta)(1 - \delta\gamma(1 - \alpha^F))} \\
&+ \lambda_{IC} [-c(e) + b + \delta\gamma w - \delta\gamma(1 - \delta\gamma)\bar{U}] \\
&+ \lambda_{DE} [-b + \delta\gamma(1 - \alpha^F)(e\theta - w)] \\
&+ \lambda_{IR} [w + b - c(e) - (1 - \delta\gamma)\bar{U}],
\end{aligned}$$

with first-order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial e} &= \frac{\alpha^F \theta}{(1 - \delta)(1 - \delta\gamma(1 - \alpha^F))} - \lambda_{IC} c'(e) + \lambda_{DE} \delta\gamma(1 - \alpha^F) \theta - \lambda_{IR} c'(e) = 0 \\
\frac{\partial \mathcal{L}}{\partial b} &= -\frac{\alpha^F}{(1 - \delta)(1 - \delta\gamma(1 - \alpha^F))} + \lambda_{IC} - \lambda_{DE} + \lambda_{IR} = 0 \\
&\Rightarrow \lambda_{IR1} = \frac{\alpha^F}{(1 - \delta)(1 - \delta\gamma(1 - \alpha^F))} - \lambda_{IC} + \lambda_{DE} \\
\frac{\partial \mathcal{L}}{\partial w} &= -\frac{\alpha^F}{(1 - \delta)(1 - \delta\gamma(1 - \alpha^F))} + \lambda_{IC} \delta\gamma - \lambda_{DE} \delta\gamma(1 - \alpha^F) + \lambda_{IR} = 0 \\
&\Rightarrow \lambda_{DE} = \lambda_{IC} \frac{(1 - \delta\gamma)}{(1 - \delta\gamma(1 - \alpha^F))} \\
&\Rightarrow \lambda_{IR} = \frac{\alpha^F - (1 - \delta) \delta\gamma \alpha^F \lambda_{IC}}{(1 - \delta)(1 - \delta\gamma(1 - \alpha^F))}
\end{aligned}$$

Note also that, if $\lambda_{IC} = \lambda_{DE} = 0$, then $\lambda_{IR} > 0$. Therefore, we have the following three cases: **1)** $\lambda_{IC}, \lambda_{DE} > 0$ and $\lambda_{IR} = 0$, **2)** $\lambda_{IC}, \lambda_{DE} > 0$ and $\lambda_{IR} > 0$, **3)** $\lambda_{IC} = \lambda_{DE} = 0$ and $\lambda_{IR} > 0$. In the following, we will derive the outcomes for all three cases, as well as the conditions for each of them to hold.

Case 1: $\lambda_{IC}, \lambda_{DE} > 0$ and $\lambda_{IR} = 0$.

Now,

$$\begin{aligned}
\lambda_{IR} &= \frac{\alpha^F - (1 - \delta) \delta\gamma \alpha^F \lambda_{IC}}{(1 - \delta)(1 - \delta\gamma(1 - \alpha^F))} = 0 \\
&\Rightarrow \lambda_{IC} = \frac{1}{(1 - \delta) \delta\gamma} \\
&\Rightarrow \lambda_{DE} = \frac{(1 - \delta\gamma)}{(1 - \delta\gamma(1 - \alpha^F))(1 - \delta) \delta\gamma},
\end{aligned}$$

and effort is characterized by

$$\theta - \frac{c'(e)}{\delta\gamma} = 0.$$

Moreover, binding (IC) and (DE) constraints yield

$$\begin{aligned} w &= \frac{\delta\gamma(1-\delta\gamma)\bar{U} - \delta\gamma(1-\alpha^F)e\theta + c(e)}{\delta\gamma\alpha^F} \\ b &= (1-\alpha^F) \frac{\delta\gamma e\theta - c(e) - \delta\gamma(1-\delta\gamma)\bar{U}}{\alpha^F} \\ \Rightarrow w + b &= \frac{\delta\gamma(1-\delta\gamma)\bar{U}(1 - (1-\alpha^F)\delta\gamma)}{\delta\gamma\alpha^F} \\ &\quad + \frac{(1-\delta\gamma(1-\alpha^F))c(e) - \delta\gamma(1-\alpha^F)e\theta(1-\delta\gamma)}{\delta\gamma\alpha^F}, \end{aligned}$$

as well as $\bar{\Pi} = [e\theta\delta\gamma - \delta\gamma(1-\delta\gamma)\bar{U} - c(e)] / [\delta\gamma(1-\delta)]$ and
 $U = [\delta\gamma\bar{U}(1 - (1-\alpha^F)\delta\gamma) - \delta\gamma(1-\alpha^F)e\theta + c(e)] / \delta\gamma\alpha^F$.

Using $\bar{U} = \alpha^N [c(e) - \delta\gamma(1-\alpha^F)e\theta] / [\delta\gamma(1-\delta\gamma)(\alpha^F - \alpha^N)]$ yields

$$\begin{aligned} U &= \frac{[1 - \delta\gamma(1-\alpha^N)](c(e) - \delta\gamma(1-\alpha^F)e\theta)}{\delta\gamma(1-\delta\gamma)(\alpha^F - \alpha^N)} \\ w + b &= \frac{[1 - \delta\gamma(1-\alpha^N)](c(e) - \delta\gamma(1-\alpha^F)e\theta)}{\delta\gamma(\alpha^F - \alpha^N)} + c(e) \\ \Pi &= \frac{(1-\delta(1-\alpha^F)) \frac{\delta\gamma e\theta(1-\alpha^N) - c(e)}{\delta\gamma(\alpha^F - \alpha^N)}}{(1-\delta)} \\ \bar{\Pi} &= \frac{\alpha^F}{1-\delta(1-\alpha^F)} \Pi = \alpha^F \frac{\delta\gamma e\theta(1-\alpha^N) - c(e)}{\delta\gamma(1-\delta)(\alpha^F - \alpha^N)} \end{aligned}$$

Moreover, $w = [c(e) - \delta\gamma(1-\alpha^F)e\theta] / [\delta\gamma(\alpha^F - \alpha^N)]$ and
 $b = (1-\alpha^F) [\delta\gamma e\theta(1-\alpha^N) - c(e)] / (\alpha^F - \alpha^N)$.

The consistency requirement is

$$\begin{aligned} U &\geq \bar{U} \\ \Leftrightarrow \alpha^F &\geq \frac{\delta\gamma(e\theta - \bar{U}(1-\delta\gamma)) - c(e)}{\delta\gamma(e\theta - \bar{U}(1-\delta\gamma))} \end{aligned}$$

Due to symmetry, $\bar{U} = 0$ at the threshold. Hence, this case holds if

$$\alpha^F \geq 1 - \frac{c(e^*)}{\delta\gamma e^*\theta},$$

where e^* is characterized by

$$\theta - \frac{c'(e^*)}{\delta\gamma} = 0.$$

Case 2: $\lambda_{IC}, \lambda_{DE} > 0$ and $\lambda_{IR} > 0$.

Binding (IC) and (DE) constraints yield

$$\begin{aligned} w &= \frac{\delta\gamma(1-\delta\gamma)\bar{U} + c(e) - \delta\gamma(1-\alpha^F)e\theta}{\delta\gamma\alpha^F} \\ b &= (1-\alpha^F) \frac{\delta\gamma e\theta - c(e) - \delta\gamma(1-\delta\gamma)\bar{U}}{\alpha^F} \\ \Rightarrow w + b &= \frac{\delta\gamma(1-\delta\gamma)\bar{U}(1 - (1-\alpha^F)\delta\gamma)}{\delta\gamma\alpha^F} \\ &\quad + \frac{(1-\delta\gamma(1-\alpha^F))c(e) - \delta\gamma(1-\alpha^F)e\theta(1-\delta\gamma)}{\delta\gamma\alpha^F}, \end{aligned}$$

The binding (IR) constraint delivers $U = \bar{U} = 0$ and equilibrium effort which is characterized by

$$\delta\gamma(1-\alpha^F)e^*\theta - c(e^*) = 0. \quad (4)$$

This case holds if the condition from case 1 is not satisfied, and if e^* here is below e^{FB} . Finally, incorporating (4) yields

$$\begin{aligned} w &= \frac{c(e^*) - \delta\gamma(1-\alpha^F)e^*\theta}{\delta\gamma\alpha^F} = 0 \\ b &= c(e^*) \\ \bar{\Pi} &= \frac{\alpha^F(e^*\theta - c(e^*))}{(1-\delta)(1-\delta\gamma(1-\alpha^F))} \end{aligned}$$

Case 3: $\lambda_{IC} = \lambda_{DE} = 0$, $\lambda_{IR} > 0$

Now, $e^* = e^{FB}$, hence $\theta - c'(e) = 0$. This case holds if

$$\alpha^F < 1 - \frac{c(e^{FB})}{\delta\gamma e^{FB}\theta},$$

and

$$\begin{aligned} w &= 0 \\ b &= c(e^{FB}) \\ \bar{\Pi} &= \frac{\alpha^F (e^{FB}\theta - c(e^{FB}))}{(1-\delta)(1-\delta\gamma(1-\alpha^F))} \end{aligned}$$

■

Proof of Corollary 1:

1. $\alpha^F \geq \bar{\alpha}^F$

We first analyze the case in which $\alpha^F \geq \bar{\alpha}^F$. Note that $c'(e^*) = \delta\gamma\theta$ and hence $\frac{de^*}{d\alpha^F} = 0$ in this case.

For the following, note that $1 - \alpha^N = \frac{(1-\gamma+\alpha^F)N - (2-\gamma)F\alpha^F}{(1-\gamma+\alpha^F)N - F\alpha^F}$, $\alpha^F - \alpha^N = \frac{1-\gamma+\alpha^F}{(1-\gamma+\alpha^F)N - F\alpha^F}(N - F)\alpha^F$, $1 - \gamma + \alpha^N = \frac{(1-\gamma)(1-\gamma+\alpha^F)N}{(1-\gamma+\alpha^F)N - F\alpha^F}$, and $\frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N} = \frac{(1-\delta\gamma)(1-\gamma+\alpha^F)N - (1-2\delta\gamma+\delta\gamma^2)F\alpha^F}{(1-\gamma)F\alpha^F}$.

Now,

$$\begin{aligned} \bar{U} &= \frac{[c(e^*) - \delta\gamma(1-\alpha^F)e^*\theta](1-\gamma)F}{\delta\gamma(1-\delta\gamma)(1-\gamma+\alpha^F)(N-F)}, \\ U &= \frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N} \bar{U} \\ &= \frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N} \frac{[c(e^*) - \delta\gamma(1-\alpha^F)e^*\theta](1-\gamma)F}{\delta\gamma(1-\delta\gamma)(1-\gamma+\alpha^F)(N-F)} \\ &= \frac{[c(e^*) - \delta\gamma(1-\alpha^F)e^*\theta][(1-\delta\gamma)(1-\gamma+\alpha^F)N - (1-2\delta\gamma+\delta\gamma^2)F\alpha^F]}{\delta\gamma(1-\delta\gamma)(1-\gamma+\alpha^F)(N-F)\alpha^F}, \end{aligned}$$

$$\begin{aligned} w + b &= (1-\delta\gamma)U + c(e^*) \\ &= \frac{[c(e^*) - \delta\gamma(1-\alpha^F)e^*\theta][(1-\delta\gamma)(1-\gamma+\alpha^F)N - (1-2\delta\gamma+\delta\gamma^2)F\alpha^F]}{\delta\gamma(1-\gamma+\alpha^F)(N-F)\alpha^F} + c(e^*), \\ \bar{\Pi} &= \frac{(1-\gamma+\alpha^F)N(\delta\gamma e^*\theta - c(e^*)) - F\alpha^F[(2-\gamma)\delta\gamma e^*\theta - c(e^*)]}{\delta\gamma(1-\delta)(1-\gamma+\alpha^F)(N-F)}. \end{aligned}$$

Moreover,

$$\begin{aligned}\frac{\partial \bar{U}}{\partial \alpha^F} &= \frac{(1-\gamma)F [\delta\gamma e^*\theta (2-\gamma) - c(e^*)]}{\delta\gamma (1-\delta\gamma) (1-\gamma + \alpha^F)^2 (N-F)} > 0, \\ \frac{\partial \bar{U}}{\partial N} &= -\frac{(1-\gamma)F (c(e^*) - \delta\gamma (1-\alpha^F) e^*\theta)}{\delta\gamma (1-\delta\gamma) (1-\gamma + \alpha^F) (N-F)^2} > 0.\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{dU}{dN} &= \frac{d\left(\frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N}\right)}{dN} \bar{U} + \frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N} \frac{d\bar{U}}{dN} \\ &= (1-\delta\gamma) \left(\frac{(1-\gamma + \alpha^F)}{(1-\gamma)F\alpha^F} - \frac{N(\alpha^F)'}{F(\alpha^F)^2} \right) \bar{U} + \frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N} \frac{d\bar{U}}{dN} \\ &= \frac{(c(e^*) - \delta\gamma (1-\alpha^F) e^*\theta)}{\delta\gamma(N-F)} \left[\frac{1}{\alpha^F} - \frac{[1-\gamma\delta(1-\alpha^N)] (1-\gamma)F}{\alpha^N (1-\delta\gamma) (1-\gamma + \alpha^F) (N-F)} \right] \\ &\quad - N(\alpha^F)' \frac{(1-\gamma) (c(e^*) - \delta\gamma (1-\alpha^F) e^*\theta)}{\delta\gamma (1-\gamma + \alpha^F) (N-F) (\alpha^F)^2} \\ &\quad + \frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N} \frac{(1-\gamma)F [\delta\gamma e^*\theta (2-\gamma) - c(e^*)]}{\delta\gamma (1-\delta\gamma) (1-\gamma + \alpha^F)^2 (N-F)} (\alpha^F)' \\ &= N(\alpha^F)' \frac{(\delta\gamma e^*\theta - c(e^*))}{(\alpha^F)^2 \delta\gamma(N-F)} - F(\alpha^F)' \frac{(1-2\delta\gamma + \delta\gamma^2) [\delta\gamma e^*\theta (2-\gamma) - c(e^*)]}{\delta\gamma (1-\delta\gamma) (1-\gamma + \alpha^F)^2 (N-F)} \\ &\quad - F(1-\gamma) \frac{c(e^*) - \delta\gamma (1-\alpha^F) e^*\theta}{\delta\gamma(N-F)^2 \alpha^F (1-\delta\gamma)} \left(\frac{1-\delta\gamma + \delta\gamma\alpha^F}{1-\gamma + \alpha^F} \right)\end{aligned}$$

For any strictly positive $(\alpha^F)'$ and N , this is strictly positive if F is sufficiently small. Furthermore,

$$\begin{aligned}\frac{d\bar{U}}{dN} &= - (1-\gamma)F \frac{c(e^*) - \delta\gamma (1-\alpha^F) e^*\theta}{\delta\gamma (1-\delta\gamma) (1-\gamma + \alpha^F) (N-F)^2} \\ &\quad + (1-\gamma)F \frac{(2-\gamma) \delta\gamma e^*\theta - c(e^*)}{\delta\gamma (1-\delta\gamma) (1-\gamma + \alpha^F)^2 (N-F)} (\alpha^F)'\end{aligned}$$

which is smaller than dU/dN but can still be positive.

If $dU/dN > 0$, the same holds for $d(w+b)/dN$, and vice versa.

Finally,

$$\begin{aligned} \frac{d\bar{\Pi}}{dN} = & (1-\gamma)F \frac{c(e^*) - (1-\alpha^F)\delta\gamma e^*\theta}{\delta\gamma(1-\delta)(1-\gamma+\alpha^F)(N-F)^2} \\ & - (1-\gamma)F \frac{(2-\gamma)\delta\gamma e^*\theta - c(e^*)}{\delta\gamma(1-\delta)(1-\gamma+\alpha^F)^2(N-F)} (\alpha^F)', \end{aligned}$$

which reveals

$$\frac{d\bar{\Pi}}{dN} = -\frac{d\bar{U}}{dN}.$$

2. $\alpha^F \leq \bar{\alpha}^F$

We second analyze the case in which $\alpha^F \in (\underline{\alpha}^F, \bar{\alpha}^F)$. From the above optimization problem,

$$\begin{aligned} w + b &= \frac{(1-\delta\gamma(1-\alpha^F))c(e) - \delta\gamma(1-\alpha^F)e\theta(1-\delta\gamma)}{\delta\gamma\alpha^F} = c(e) \\ \bar{\Pi} &= \frac{\alpha^F(e^*\theta - c(e^*))}{(1-\delta)(1-\delta\gamma(1-\alpha^F))} \end{aligned}$$

Therefore,

$$\frac{\partial e^*}{\partial N} = \frac{\delta\gamma e^*\theta}{\delta\gamma(1-\alpha^F)\theta - c'(e^*)} (\alpha^F)' < 0$$

$$\frac{\partial(w+b)}{\partial N} = c'(e^*) \frac{\partial e^*}{\partial N} < 0$$

If $\alpha^F \leq \underline{\alpha}^F$ (and $e^* = e^{FB}$) comparative statics are equivalent, only $\partial e^*/\partial N = 0$. ■

Proof of Proposition 2:

Again we distinguish between the $\alpha^F \geq \bar{\alpha}^F$ and $\alpha^F < \bar{\alpha}^F$ and conduct comparative statics for each case separately.

1. $\alpha^F \geq \bar{\alpha}^F$

In the proof to Corollary 1 we have derived

$$\bar{\Pi} = \frac{(1 - \gamma + \alpha^F)N(\delta\gamma e^*\theta - c(e^*)) - F\alpha^F[(2 - \gamma)\delta\gamma e^*\theta - c(e^*)]}{\delta\gamma(1 - \delta)(1 - \gamma + \alpha^F)(N - F)}.$$

Thus,

$$\begin{aligned} \frac{\partial \bar{\Pi}}{\partial N} &= F(1 - \gamma) \frac{c(e^*) - (1 - \alpha^F)\delta\gamma e^*\theta}{\delta\gamma(1 - \delta)(1 - \gamma + \alpha^F)(N - F)^2} \\ &\quad - F(1 - \gamma) \frac{[(2 - \gamma)\delta\gamma e^*\theta - c(e^*)]}{\delta\gamma(1 - \delta)(1 - \gamma + \alpha^F)^2(N - F)} (\alpha^F)' \\ \frac{\partial \bar{\Pi}}{\partial F} &= -N(1 - \gamma) \frac{c(e^*) - (1 - \alpha^F)\delta\gamma e^*\theta}{\delta\gamma(1 - \delta)(1 - \gamma + \alpha^F)(N - F)^2} \\ &\quad + F(1 - \gamma) \frac{[(2 - \gamma)\delta\gamma e^*\theta - c(e^*)]}{\delta\gamma(1 - \delta)(1 - \gamma + \alpha^F)^2(N - F)} (\alpha^F)', \end{aligned}$$

where $\partial \bar{\Pi} / \partial F < 0$ in equilibrium (otherwise, more firms would directly increase profits, causing additional entry).

Moreover,

$$\begin{aligned} \frac{dU}{dN} &= \frac{\partial U}{\partial N} + \frac{\partial U}{\partial F} \frac{dF}{dN} + \frac{\partial U}{\partial \alpha^F} \left((\alpha^F)' - (\alpha^F)' \frac{dF}{dN} \right) \\ &= \frac{[c(e^*) - \delta\gamma(1 - \alpha^F)e^*\theta](1 - \gamma)(1 - \delta\gamma + \delta\gamma\alpha^F)}{\delta\gamma(1 - \delta\gamma)(1 - \gamma + \alpha^F)\alpha^F(N - F)^2} \left(N \frac{dF}{dN} - F \right) \\ &\quad + \left[N \frac{(\delta\gamma e\theta - c(e))}{(\alpha^F)^2} - F \frac{(1 - 2\delta\gamma + \delta\gamma^2)[\delta\gamma e\theta(2 - \gamma) - c(e)]}{(1 - \delta\gamma)(1 - \gamma + \alpha^F)^2} \right] \frac{(1 - \frac{dF}{dN})}{\delta\gamma(N - F)} (\alpha^F)', \end{aligned}$$

with

$$\begin{aligned} \frac{dF}{dN} &= \frac{F \frac{c(e^*) - (1 - \alpha^F)\delta\gamma e^*\theta}{(N - F)} - F \frac{[(2 - \gamma)\delta\gamma e^*\theta - c(e^*)]}{(1 - \gamma + \alpha^F)} (\alpha^F)'}{N \frac{c(e^*) - (1 - \alpha^F)\delta\gamma e^*\theta}{(N - F)} - F \frac{[(2 - \gamma)\delta\gamma e^*\theta - c(e^*)]}{(1 - \gamma + \alpha^F)} (\alpha^F)'} \\ 1 - \frac{dF}{dN} &= \frac{c(e^*) - (1 - \alpha^F)\delta\gamma e^*\theta}{N \frac{c(e^*) - (1 - \alpha^F)\delta\gamma e^*\theta}{(N - F)} - F \frac{[(2 - \gamma)\delta\gamma e^*\theta - c(e^*)]}{(1 - \gamma + \alpha^F)} (\alpha^F)'} > 0 \\ N \frac{dF}{dN} - F &= - \frac{F(N - F) \frac{[(2 - \gamma)\delta\gamma e^*\theta - c(e^*)]}{(1 - \gamma + \alpha^F)} (\alpha^F)'}{N \frac{c(e^*) - (1 - \alpha^F)\delta\gamma e^*\theta}{(N - F)} - F \frac{[(2 - \gamma)\delta\gamma e^*\theta - c(e^*)]}{(1 - \gamma + \alpha^F)} (\alpha^F)'} < 0. \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{dU}{dN} &= \frac{\frac{c(e^*) - (1 - \alpha^F) \delta \gamma e^* \theta}{\alpha^F \delta \gamma (N - F)} \left(\frac{N(\delta \gamma e \theta - c(e))}{\alpha^F} - F \frac{[(2 - \gamma) \delta \gamma e^* \theta - c(e^*)]}{(1 - \gamma + \alpha^F)} \right)}{N \frac{c(e^*) - (1 - \alpha^F) \delta \gamma e^* \theta}{(N - F)} - F \frac{[(2 - \gamma) \delta \gamma e^* \theta - c(e^*)]}{(1 - \gamma + \alpha^F)} (\alpha^F)'} (\alpha^F)' \\ &= \frac{(1 - \delta) [c(e^*) - (1 - \alpha^F) \delta \gamma e^* \theta] \bar{\Pi}}{(\alpha^F)^2 \left[N \frac{c(e^*) - (1 - \alpha^F) \delta \gamma e^* \theta}{(N - F)} - F \frac{[(2 - \gamma) \delta \gamma e^* \theta - c(e^*)]}{(1 - \gamma + \alpha^F)} (\alpha^F)' \right]} > 0, \end{aligned}$$

since $\partial \bar{\Pi} / \partial F < 0$ in equilibrium yields a positive denominator.

Moreover,

$$\begin{aligned} \frac{d\bar{U}}{dN} &= \frac{\partial \bar{U}}{\partial N} + \frac{\partial \bar{U}}{\partial F} \frac{dF}{dN} + \frac{\partial \bar{U}}{\partial \alpha^F} \left(1 - \frac{dF}{dN} \right) (\alpha^F)' \\ &= \left(N \frac{dF}{dN} - F \right) \frac{(1 - \gamma) (c(e^*) - \delta \gamma (1 - \alpha^F) e^* \theta)}{\delta \gamma (1 - \delta \gamma) (1 - \gamma + \alpha^F) (N - F)^2} \\ &\quad + \frac{(1 - \gamma) F (\delta \gamma e^* \theta (2 - \gamma) - c(e^*))}{\delta \gamma (1 - \delta \gamma) (1 - \gamma + \alpha^F)^2 (N - F)} \left(1 - \frac{dF}{dN} \right) (\alpha^F)' \\ &= \frac{(c(e^*) - (1 - \alpha^F) \delta \gamma e^* \theta) (\delta \gamma e^* \theta (2 - \gamma) - c^*(e)) \frac{(1 - \gamma) F - F(1 - \gamma)}{(1 - \gamma + \alpha^F)^2 (N - F)}}{\delta \gamma (1 - \delta \gamma) \left[N \frac{c(e^*) - (1 - \alpha^F) \delta \gamma e^* \theta}{(N - F)} - F \frac{[(2 - \gamma) \delta \gamma e^* \theta - c(e^*)]}{(1 - \gamma + \alpha^F)} (\alpha^F)' \right]} (\alpha^F)' \\ &= 0 \end{aligned}$$

$dU/dN > 0$ implies that $d(w + b)/dN > 0$ as well. Now, we explore how the total effect on compensation is driven by changes in wage and bonus. Recall that

$$\begin{aligned} w &= \frac{\delta \gamma (1 - \delta \gamma) \bar{U} - \delta \gamma (1 - \alpha^F) e \theta + c(e)}{\delta \gamma \alpha^F} \\ b &= (1 - \alpha^F) \frac{\delta \gamma e \theta - c(e) - \delta \gamma (1 - \delta \gamma) \bar{U}}{\alpha^F}. \end{aligned}$$

Because $d\bar{U}/dN = 0$, we can treat \bar{U} as a constant, hence

$$\begin{aligned} \frac{dw}{dN} &= \frac{\delta \gamma e \theta - c(e) - \delta \gamma (1 - \delta \gamma) \bar{U}}{\delta \gamma (\alpha^F)^2} \left(1 - \frac{dF}{dN} \right) (\alpha^F)' > 0 \\ \frac{db}{dN} &= - \frac{\delta \gamma e \theta - c(e) - \delta \gamma (1 - \delta \gamma) \bar{U}}{(\alpha^F)^2} \left(1 - \frac{dF}{dN} \right) (\alpha^F)' < 0 \end{aligned}$$

Finally, note that dF/dN is positive if $\partial\bar{\Pi}/\partial N > 0$, with the sign of $\partial\bar{\Pi}/\partial N$ being identical to the sign of

$$\mathbb{T} \equiv \frac{c(e^*) - (1 - \alpha^F) \delta\gamma e^* \theta}{(N - F)} - \frac{[(2 - \gamma)\delta\gamma e^* \theta - c(e^*)]}{(1 - \gamma + \alpha^F)} (\alpha^F)'.$$

This is positive if $(\alpha^F)'$ is small. To the contrary, at $\alpha^F = (\delta\gamma e^* \theta - c(e^*)) / \delta\gamma e^* \theta$,

$$\mathbb{T} = -\delta\gamma e^* \theta (\alpha^F)' < 0.$$

2. $\alpha^F < \bar{\alpha}^F$

Now, equilibrium effort is given by

$$\delta\gamma (1 - \alpha^F) e^* \theta - c(e^*) = 0 \quad (5)$$

or $e^* = e^{FB}$, whichever is smaller. If $e^* = e^{FB}$, $de^*/dN = 0$. If e^* is determined by (5),

$$\frac{\partial e^*}{\partial \alpha^F} = \frac{\delta\gamma e^* \theta}{\delta\gamma (1 - \alpha^F) \theta - c'(e^*)} < 0.$$

and

$$\bar{\Pi} = \frac{\alpha^F (e\theta - c(e^*))}{(1 - \delta) (1 - \delta\gamma (1 - \alpha^F))},$$

with

$$\begin{aligned} \frac{\partial \bar{\Pi}}{\partial F} &= \left(\frac{(e\theta - c(e^*)) (1 - \delta\gamma)}{(1 - \delta) (1 - \delta\gamma (1 - \alpha^F))^2} + \frac{\alpha^F (\theta - c'(e^*))}{(1 - \delta) (1 - \delta\gamma (1 - \alpha^F))} \frac{\partial e^*}{\partial \alpha^F} \right) (\alpha^F)' \\ &= \frac{e^* \theta}{(1 - \delta) [\delta\gamma (1 - \alpha^F) \theta - c'(e^*)]} (\alpha^F)' < 0 \\ \frac{\partial \bar{\Pi}}{\partial N} &= - \left(\frac{(e\theta - c(e^*)) (1 - \delta\gamma)}{(1 - \delta) (1 - \delta\gamma (1 - \alpha^F))^2} + \frac{\alpha^F (\theta - c'(e^*))}{(1 - \delta) (1 - \delta\gamma (1 - \alpha^F))} \frac{\partial e^*}{\partial \alpha^F} \right) (\alpha^F)' \\ &= - \frac{e^* \theta}{(1 - \delta) [\delta\gamma (1 - \alpha^F) \theta - c'(e^*)]} (\alpha^F)' > 0. \end{aligned}$$

Therefore,

$$\frac{dF}{dN} = 1 > 0,$$

i.e., employment effects are positive, and

$$\frac{de^*}{dN} = \frac{\partial e^*}{\partial \alpha^F} \left((\alpha^F)' - \frac{dF}{dN} (\alpha^F)' \right) = 0.$$

It follows that $d(w + b)/dN = 0$. ■

Proof of Proposition 3:

The set of constraints is

$$-b^I + \delta\gamma (\Pi^I - \bar{\Pi}) \geq 0 \quad (\text{DEI})$$

$$-b^O + \delta\gamma (\Pi^O - \bar{\Pi}) \geq 0 \quad (\text{DEO})$$

$$U^I - \bar{U}^I \geq 0 \quad (\text{IRI})$$

$$U^O - \bar{U}^O \geq 0 \quad (\text{IRO})$$

$$-c(e^I) + b^I + \delta\gamma [U^I - \bar{U}^I] \geq 0 \quad (\text{ICI})$$

$$-c(e^O) + b^O + \delta\gamma [U^O - \bar{U}^O] \geq 0 \quad (\text{ICO})$$

Now, $\Pi^I = \pi^I + \delta [\gamma\Pi^I + (1 - \gamma)\bar{\Pi}]$ and

$\Pi^O = \pi^O + \delta [\gamma\Pi^O + (1 - \gamma)\bar{\Pi}]$, where $\pi^I = e^I\theta - w^I - b^I$ and $\pi^O = e^O\theta - w^O - b^O$.

Thus,

$$\Pi^I = \frac{\pi^I (1 - \delta + \delta\alpha^{FI}) (1 - \delta\gamma) + \delta\alpha^{FO} [(1 - \delta)\gamma\pi^I + (1 - \gamma)\pi^O]}{(1 - \delta)(1 - \delta\gamma)(1 - \delta\gamma + \alpha^F\delta\gamma)}$$

$$\Pi^O = \frac{\pi^O (1 - \delta + \delta\alpha^{FO}) (1 - \delta\gamma) + \delta\alpha^{FI} [(1 - \delta)\gamma\pi^O + (1 - \gamma)\pi^I]}{(1 - \delta)(1 - \delta\gamma)(1 - \delta\gamma + \alpha^F\delta\gamma)}$$

$$\bar{\Pi} = \frac{\alpha^{FI}\pi^I + \alpha^{FO}\pi^O}{(1 - \delta)(1 - \delta\gamma + \alpha^F\delta\gamma)}$$

$$\Pi^I - \bar{\Pi} = \frac{(1 - \alpha^{FI} - \delta\gamma(1 - \alpha^F))\pi^I - \alpha^{FO}\pi^O}{(1 - \delta\gamma)(1 - \delta\gamma + \alpha^F\delta\gamma)}$$

$$\Pi^O - \bar{\Pi} = \frac{(1 - \alpha^{FO} - \delta\gamma(1 - \alpha^F))\pi^O - \alpha^{FI}\pi^I}{(1 - \delta\gamma)(1 - \delta\gamma + \alpha^F\delta\gamma)}$$

This allows us to rewrite the optimization problem, which becomes to maximize $\alpha^{FI}\pi^I + \alpha^{FO}\pi^O$, subject to

$$-b^I + \delta\gamma \frac{(1 - \alpha^{FI} - \delta\gamma(1 - \alpha^F))\pi^I - \alpha^{FO}\pi^O}{(1 - \delta\gamma)(1 - \delta\gamma + \alpha^F\delta\gamma)} \geq 0$$

(DEI)

$$\Leftrightarrow \frac{-b^I(1 - \delta\gamma + \delta\gamma\alpha^{FO}) + \delta\gamma(e^I\theta - w^I)[1 - \alpha^{FI} - \delta\gamma(1 - \alpha^F)] - \delta\gamma\alpha^{FO}\pi^O}{(1 - \delta\gamma)(1 - \delta\gamma + \alpha^F\delta\gamma)} \geq 0$$

(DEI)

$$-b^O + \delta\gamma \frac{(1 - \alpha^{FO} - \delta\gamma(1 - \alpha^F))\pi^O - \alpha^{FI}\pi^I}{(1 - \delta\gamma)(1 - \delta\gamma + \alpha^F\delta\gamma)} \geq 0$$

(DEO)

$$\Leftrightarrow \frac{-b^O(1 - \delta\gamma + \delta\gamma\alpha^{FI}) + \delta\gamma(1 - \alpha^{FO} - \delta\gamma(1 - \alpha^F))(e^O\theta - w^O) - \delta\gamma\alpha^{FI}\pi^I}{(1 - \delta\gamma)(1 - \delta\gamma + \alpha^F\delta\gamma)} \geq 0$$

(DEO)

$$-c(e^I) + b^I + \delta\gamma(w^I - (1 - \delta\gamma)\bar{U}^I) \geq 0$$

(ICI)

$$w^I + b^I - c(e^I) - (1 - \delta\gamma)\bar{U}^I \geq 0$$

(IRI)

$$-c(e^O) + b^O + \delta\gamma(w^O - (1 - \delta\gamma)\bar{U}^O) \geq 0$$

(ICO)

$$w^O + b^O - c(e^O) - (1 - \delta\gamma)\bar{U}^O \geq 0$$

(IRO)

There, $\bar{U}^I = \alpha^N U^I + (1 - \alpha^N)\delta\gamma\bar{U}^I$ and $\bar{U}^O = \alpha^N U^O + (1 - \alpha^N)\delta\gamma\bar{U}^O$ are taken as given by firms. Also note that b^I and b^O cannot be negative. We omit these conditions for now and will later check whether they are satisfied.

Setting up the Lagrange function and obtaining first-order conditions yields

$$\begin{aligned}\frac{\partial L}{\partial e^I} = & \alpha^{FI}\theta + \lambda_{DEI}\delta\gamma \frac{\theta(1-\alpha^{FI})(1-\delta\gamma) + \alpha^{FO}\delta\gamma\theta}{(1-\delta\gamma)(1-\delta\gamma + \alpha^F\delta\gamma)} \\ & - \lambda_{DEO}\delta\gamma \frac{\alpha^{FI}\theta}{(1-\delta\gamma)(1-\delta\gamma + \alpha^F\delta\gamma)} - c'(e^I)(\lambda_{ICI} + \lambda_{IRI}) = 0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial e^O} = & \alpha^{FO}\theta - \lambda_{DEI}\delta\gamma \frac{\alpha^{FO}\theta}{(1-\delta\gamma)(1-\delta\gamma + \alpha^F\delta\gamma)} \\ & + \lambda_{DEO}\delta\gamma \frac{\theta(1-\alpha^{FO})(1-\delta\gamma) + \alpha^{FI}\delta\gamma\theta}{(1-\delta\gamma)(1-\delta\gamma + \alpha^F\delta\gamma)} - c'(e^O)(\lambda_{ICO} + \lambda_{IRO}) = 0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial b^I} = & -\alpha^{FI} - \lambda_{DEI} \frac{1-\delta\gamma + \alpha^{FO}\delta\gamma}{(1-\delta\gamma)(1-\delta\gamma + \alpha^F\delta\gamma)} \\ & + \lambda_{DEO} \frac{\delta\gamma\alpha^{FI}}{(1-\delta\gamma)(1-\delta\gamma + \alpha^F\delta\gamma)} + \lambda_{ICI} + \lambda_{IRI} = 0 \\ \Rightarrow \lambda_{ICI} = & \alpha^{FI} + \frac{\lambda_{DEI}(1-\delta\gamma + \alpha^{FO}\delta\gamma) - \lambda_{DEO}\delta\gamma\alpha^{FI}}{(1-\delta\gamma)(1-\delta\gamma + \alpha^F\delta\gamma)} - \lambda_{IRI}\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial w^I} = & -\alpha^{FI} - \lambda_{DEI}\delta\gamma \frac{(1-\alpha^{FI})(1-\delta\gamma) + \alpha^{FO}\delta\gamma}{(1-\delta\gamma)(1-\delta\gamma + \alpha^F\delta\gamma)} \\ & + \lambda_{DEO} \frac{\delta\gamma\alpha^{FI}}{(1-\delta\gamma)(1-\delta\gamma + \alpha^F\delta\gamma)} + \lambda_{ICI}\delta\gamma + \lambda_{IRI} = 0\end{aligned}$$

$$\Rightarrow \lambda_{IRI} = \alpha^{FI} \left(1 - \frac{\delta\gamma(\lambda_{DEI} + \lambda_{DEO})}{(1-\delta\gamma)(1-\delta\gamma + \alpha^F\delta\gamma)} \right)$$

$$\Rightarrow \lambda_{ICI} = \frac{\lambda_{DEI}}{(1-\delta\gamma)}$$

and

$$\begin{aligned}\frac{\partial L}{\partial b^O} &= -\alpha^{FO} + \lambda_{DEI} \frac{\delta\gamma\alpha^{FO}}{(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)} \\ &\quad - \lambda_{DEO} \frac{1-\delta\gamma+\delta\gamma\alpha^{FI}}{(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)} + \lambda_{ICO} + \lambda_{IRO} = 0 \\ \Rightarrow \lambda_{ICO} &= \alpha^{FO} - \frac{\lambda_{DEI}\delta\gamma\alpha^{FO} - \lambda_{DEO}(1-\delta\gamma+\delta\gamma\alpha^{FI})}{(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)} - \lambda_{IRO}\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial w^O} &= -\alpha^{FO} + \delta\gamma \frac{\lambda_{DEI}\alpha^{FO} - \lambda_{DEO}[(1-\alpha^{FO})(1-\delta\gamma) + \alpha^{FI}\delta\gamma]}{(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)} \\ &\quad + \lambda_{ICO}\delta\gamma + \lambda_{IRO} = 0 \\ \Rightarrow \lambda_{IRO} &= \alpha^{FO} \left(1 - \frac{\delta\gamma(\lambda_{DEI} + \lambda_{DEO})}{(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)} \right) \\ \Rightarrow \lambda_{ICO} &= \frac{\lambda_{DEO}}{(1-\delta\gamma)}\end{aligned}$$

Thus, effort levels are characterized by

$$\begin{aligned}\alpha^{FI}(\theta - c'(e^I)) \left(1 - \frac{\delta\gamma(\lambda_{DEI} + \lambda_{DEO})}{(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)} \right) + \frac{\delta\gamma\theta - c'(e^I)}{(1-\delta\gamma)}\lambda_{DEI} &= 0 \\ \alpha^{FO}(\theta - c'(e^O)) \left(1 - \frac{\delta\gamma(\lambda_{DEO} + \lambda_{DEI})}{(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)} \right) + \frac{\delta\gamma\theta - c'(e^O)}{(1-\delta\gamma)}\lambda_{DEO} &= 0\end{aligned}$$

Results

The analysis yields the following outcomes: Either, $\lambda_{IRI} = \lambda_{IRO} = 0$ or $\lambda_{IRI}, \lambda_{IRO} > 0$. Moreover, either $\lambda_{DEI}, \lambda_{ICI} > 0$ or $\lambda_{DEI} = \lambda_{ICI} = 0$, and equivalently for λ_{DEO} and λ_{ICO} .

In the following, we analyze these cases separately, starting with $\lambda_{IRI} = \lambda_{IRO} = 0$ and differentiating with respect to λ_{DEI} and λ_{DEO} .

A) $\lambda_{IRO} = \lambda_{IRI} = 0$

This implies $\lambda_{DEI} + \lambda_{DEO} = \frac{(1-\delta\gamma)(1-\delta\gamma+\alpha^F\delta\gamma)}{\delta\gamma}$, thus effort levels are characterized by

$$\begin{aligned}
(\delta\gamma\theta - c'(e^I)) \frac{\lambda_{DEI}}{(1 - \delta\gamma)} &= 0 \\
(\delta\gamma\theta - c'(e^O)) \frac{\lambda_{DEO}}{(1 - \delta\gamma)} &= 0.
\end{aligned}$$

First, we show that we can ignore the cases $\lambda_{ICO}, \lambda_{DEO} > 0, \lambda_{ICI} = \lambda_{DEI} = 0$ and $\lambda_{ICO}, \lambda_{DEO} > 0, \lambda_{ICI}, \lambda_{DEI} > 0$.

To the contrary, assume

I) $\lambda_{ICO}, \lambda_{DEO} > 0, \lambda_{ICI} = \lambda_{DEI} = 0$

Then, e^O is characterized by $\delta\gamma\theta - c'(e^O) = 0$, e^I is not uniquely identified. Binding (ICO) and (DEO) constraints yield

$$\begin{aligned}
w^O &= \frac{\delta\gamma\alpha^{FI}\pi^I + \delta\gamma e^O\theta\alpha^{FO}(1 - \delta\gamma) - (\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1 - \delta\gamma)\bar{U}^O)(1 - \delta\gamma + \alpha^{FI}\delta\gamma)}{\delta\gamma(1 - \delta\gamma)\alpha^{FO}} \\
b^O &= \delta\gamma \frac{-\delta\gamma\alpha^{FI}\pi^I + (\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1 - \delta\gamma)\bar{U}^O)(1 - \delta\gamma + \alpha^F\delta\gamma - \alpha^{FO})}{\delta\gamma(1 - \delta\gamma)\alpha^{FO}} \\
w^O + b^O &= \frac{\delta\gamma\alpha^{FI}\pi^I + \delta\gamma e^O\theta\alpha^{FO} - (\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1 - \delta\gamma)\bar{U}^O)(1 - \delta\gamma + \alpha^F\delta\gamma)}{\delta\gamma\alpha^{FO}},
\end{aligned}$$

and

$$\bar{\Pi} = \frac{(\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1 - \delta\gamma)\bar{U}^O)}{\delta\gamma(1 - \delta)}$$

Consistency requires that these values satisfy (DEI) and (ICI), which become

$$b^I \leq \delta\gamma(e^I\theta - w^I) - (\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1 - \delta\gamma)\bar{U}^O) \quad (\text{DEI})$$

$$b^I \geq c(e^I) - \delta\gamma(w^I - (1 - \delta\gamma)\bar{U}^I), \quad (\text{ICI})$$

thus the following condition is necessary:

$$\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1 - \delta\gamma)\bar{U}^O \leq \delta\gamma e^I\theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I$$

This condition requires $\bar{U}^I \leq \bar{U}^O$ because e^O is characterized by $\delta\gamma\theta - c'(e^O) = 0$ and thus maximizes $\delta\gamma e\theta - c(e)$. If $\bar{U}^I = \bar{U}^O$, both types are treated the same, a situation we will analyze in case III below. If $\bar{U}^I < \bar{U}^O$, it is optimal for firms to deviate and only employ insiders. Since $\alpha^{FI} \geq 1 - \frac{c(e)}{\delta\gamma e\theta}$ and the right hand side

decreasing in a worker's outside option, a firm's expected profits then would amount to

$$\bar{\Pi} = \frac{(\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I)}{\delta\gamma(1 - \delta)},$$

with e^I also characterized by $\delta\gamma\theta - c'(e^I) = 0$. Therefore, $\bar{U}^I < \bar{U}^O$ is not possible because only insiders would be hired in this case, driving down \bar{U}^O .

II) $\lambda_{ICO}, \lambda_{DEO} > 0, \lambda_{ICI}, \lambda_{DEI} > 0$

Then, e^O is characterized by $\delta\gamma\theta - c'(e^O) = 0$, e^I is characterized by $\delta\gamma\theta - c'(e^I) = 0$, hence

$$e^I = e^O.$$

Binding (IC) and (DE) constraints yield

$$\begin{aligned} w^I &= \frac{\delta\gamma\alpha^{FO}\pi^O + \delta\gamma e^I\theta\alpha^{FI}(1 - \delta\gamma) - (\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I)(1 - \delta\gamma + \alpha^{FO}\delta\gamma)}{\delta\gamma(1 - \delta\gamma)\alpha^{FI}} \\ b^I &= \delta\gamma \frac{-\delta\gamma\alpha^{FO}\pi^O + (\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I)(1 - \delta\gamma + \alpha^F\delta\gamma - \alpha^{FI})}{\delta\gamma(1 - \delta\gamma)\alpha^{FI}} \\ w^I + b^I &= \frac{\delta\gamma\alpha^{FO}\pi^O + \delta\gamma e^I\theta\alpha^{FI} - (\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I)[1 - \delta\gamma + \alpha^F\delta\gamma]}{\delta\gamma\alpha^{FI}} \\ w^O + b^O &= \frac{\delta\gamma\alpha^{FI}\pi^I + \delta\gamma e^O\theta\alpha^{FO} - (\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1 - \delta\gamma)\bar{U}^O)(1 - \delta\gamma + \alpha^F\delta\gamma)}{\delta\gamma\alpha^{FO}} \end{aligned}$$

Plugging this into the profit functions yields

$$\begin{aligned} \pi^O &= e^O\theta - w^O - b^O = \frac{(\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1 - \delta\gamma)\bar{U}^O)(1 - \delta\gamma + \alpha^F\delta\gamma) - \delta\gamma\alpha^{FI}\pi^I}{\delta\gamma\alpha^{FO}} \\ \pi^I &= e^I\theta - w^I - b^I = \frac{(\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I)[1 - \delta\gamma + \alpha^F\delta\gamma] - \delta\gamma\alpha^{FO}\pi^O}{\delta\gamma\alpha^{FI}}, \end{aligned}$$

and

$$\begin{aligned} (\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1 - \delta\gamma)\bar{U}^O) &= (\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I) \\ (\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I) &= (\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1 - \delta\gamma)\bar{U}^O). \end{aligned}$$

Given $e^I = e^O$, this case can consequently only hold if $\bar{U}^I = \bar{U}^O$ and outsiders and

insiders are treated identically, a case we will analyze below.

III) $\lambda_{ICO}, \lambda_{DEO} = 0, \lambda_{ICI} = \lambda_{DEI} > 0$

Then, e^I is characterized by $\delta\gamma\theta - c'(e^I) = 0$, e^O is not uniquely identified. Binding (ICI) and (DEI) constraints yield

$$\begin{aligned} b^I &= c(e^I) - \delta\gamma(w^I - (1 - \delta\gamma)\bar{U}^I) \\ w^I &= \frac{\delta\gamma e^I \theta (1 - \delta\gamma) \alpha^{FI} + \delta\gamma \alpha^{FO} \pi^O - [\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I] (1 - \delta\gamma + \delta\gamma \alpha^{FO})}{\delta\gamma(1 - \delta\gamma) \alpha^{FI}} \\ w^I + b^I &= \frac{\delta\gamma \alpha^{FO} \pi^O + \alpha^{FI} \delta\gamma e^I \theta - [\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I] (1 - \delta\gamma + \delta\gamma \alpha^F)}{\delta\gamma \alpha^{FI}} \end{aligned}$$

Also note that expected profits are

$$\bar{\Pi} = \frac{[\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I]}{\delta\gamma(1 - \delta\gamma)},$$

hence π^O has no direct effect on an individual firm's expected profits.

Consistency requires that these values satisfy (DEO), (IRO) and (ICO), which become

$$b^O \leq \delta\gamma(e^O \theta - w^O) - (\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I) \quad (\text{DEO})$$

$$b^O \geq c(e^O) - \delta\gamma(w^O - (1 - \delta\gamma)\bar{U}^O) \quad (\text{ICO})$$

$$b^O \geq c(e^O) - (w^O - (1 - \delta\gamma)\bar{U}^O). \quad (\text{IRO})$$

(DEO) and (ICO) yields the necessary condition

$$\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I \leq \delta\gamma e^O \theta - c(e^O) - \delta\gamma(1 - \delta\gamma)\bar{U}^O. \quad (6)$$

Because e^I maximizes $\delta\gamma e\theta - c(e)$, this is only possible if $\bar{U}^I \geq \bar{U}^O$. In the following we separately analyze the cases $\bar{U}^I > \bar{U}^O$ and $\bar{U}^I = \bar{U}^O$.

A) $\bar{U}^I = \bar{U}^O$ Now, condition (6) can only hold if $e^O = e^I$. Moreover, since matching probabilities are the same for insiders and outsiders, consistency (i.e., $\bar{U}^I = \bar{U}^O$) requires $w^O + b^O = w^I + b^I$, which is achieved by setting $w^O = w^I$ and

$b^O = b^I$. This also implies that

$$\pi^I = \pi^O = \frac{[\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I](1 - \delta\gamma + \delta\gamma\alpha^F)}{\delta\gamma\alpha^F}.$$

Then, (IR) constraints of insiders and outsiders are identical, and satisfied for

$$\alpha^F \geq 1 - \frac{c(e^I)}{\delta\gamma(e^I\theta - (1 - \delta\gamma)\bar{U}^I)}.$$

In a symmetric social equilibrium in which all firms' actions are identical, this condition becomes

$$\alpha^F \geq 1 - \frac{c(e^I)}{\delta\gamma e^I \theta}.$$

B) $\bar{U}^I > \bar{U}^O$

Since $\bar{\Pi} = \frac{[\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I]}{\delta\gamma(1 - \delta\gamma)}$ and \bar{U}^I is taken as given by firms, several profit-maximizing arrangements with outsiders exist. For example, setting $w^O = (1 - \delta\gamma)\bar{U}^O$ and $b^O = c(e^O)$ satisfies (ICO) and (IRO), then (DEO) becomes $\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I \leq \delta\gamma e^O \theta - c(e^O) - \delta\gamma(1 - \delta\gamma)\bar{U}^O$, and values of e^O exist that satisfy this condition.

Moreover, the following consistency requirements must hold. The first is (IRI), i.e.,

$$\begin{aligned} & \delta\gamma\alpha^{FO}\pi^O + (\alpha^F - \alpha^{FO})(1 - \delta\gamma)c(e^I) \\ & - [\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I]((1 - \delta\gamma)(1 - \alpha^F) + \alpha^{FO}) \geq 0. \end{aligned} \quad (7)$$

Since $\alpha^{FI} \geq 1 - \frac{c(e)}{\delta\gamma e\theta}$, condition (7) would hold for any $\pi^O \geq \pi^I$. Rewriting this condition yields

$$\alpha^{FI} \geq \bar{\alpha}^{FI} = \frac{(\delta\gamma e^I \theta - c(e^I))(1 - \delta\gamma(1 - \alpha^{FO})) - \alpha^{FO}\delta\gamma\pi^O}{\delta\gamma e^I \theta (1 - \delta\gamma)},$$

where we also take into account that (IRI) binds at $\bar{\alpha}^{FI}$, in which case $\bar{U}^I = 0$.

Additionally, $b^I \geq 0$ (and consequently $\Pi^I \geq \bar{\Pi}$) must be satisfied, which yields

$$\pi^O \leq \frac{[\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I] ((1 - \alpha^F)(1 - \delta\gamma) + \alpha^{FO})}{\delta\gamma\alpha^{FO}}, \quad (8)$$

and is equivalent to $\pi^O \leq \pi^I \frac{((1 - \alpha^F)(1 - \delta\gamma) + \alpha^{FO})}{\alpha^{FO}}$, thus $\pi^O > \pi^I$ is indeed feasible.²⁶

Also note that, if condition (8) binds, the consistency requirement (7) becomes

$$\alpha^{FI}(1 - \delta\gamma)c(e^I) \geq 0,$$

thus holds for all levels α^{FI} .

It follows that any π^O that satisfies conditions (7) and (8) can be supported by a profit maximizing social equilibrium, and levels of e^O , w^O and b^O exist that generate such a π^O , with (DEO), (ICO) and (IRO) holding as well.

Then,

$$w^I + b^I = \frac{\delta\gamma\alpha^{FO}\pi^O + \alpha^{FI}\delta\gamma e^I \theta - [\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I] (1 - \delta\gamma + \delta\gamma\alpha^F)}{\delta\gamma\alpha^{FI}}$$

is increasing in π^O . Since $U^I = \frac{w^I + b^I - c(e^I)}{1 - \delta\gamma}$ and e^I independent of π^O , (employed and unemployed) insiders benefit from a higher π^O .

In the following, we compute an equilibrium in which π^O and consequently the payoffs of insiders are maximized. An arrangement maximizing π^O subject to (ICO), (IRO) and (DEO) would involve setting $w^O = (1 - \delta\gamma)\bar{U}^O$ and $b^O = c(e^O)$, yielding $w^O = \bar{U}^O = 0$. Thus, $\pi^O = e^O \theta - c(e^O)$, and e^O is constrained by

$$\delta\gamma e^O \theta - c(e^O) \geq \delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I. \quad (9)$$

If e^{FB} (characterized by $\theta - c'(e) = 0$) satisfies (9), then $e^O = e^{FB}$. Otherwise, e^O is characterized by the binding (9), and

$$c(e^O) = \delta\gamma e^O \theta - (\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I).$$

Note that condition (9) implies that $e^O = e^I$ at $\bar{\alpha}^{FI}$ (where $\bar{U}^I = 0$). For higher values of $\bar{\alpha}^{FI}$, $\bar{U}^I > 0$, then $e^O > e^I$ is indeed possible.

²⁶Here, we take into account that (DEI) and (ICI) continue to bind if $b^I = 0$. This can easily be confirmed: with $b^I = 0$, the problem is to maximize $\alpha^{FI}(e^I \theta - w^I) + \alpha^{FO}\pi^O$ subject to (DEI) and (ICI). Then, if either (DEI) or (ICI) did not bind, either e^I or π^O could be increased without violating a constraint, thereby increasing expected profits.

The maximized π^O must satisfy condition (8), i.e.,

$$\alpha^{FI} \leq \tilde{\alpha}^{FI} = \frac{[\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I] (1 - \delta\gamma(1 - \alpha^{FO})) - \pi^O \delta\gamma \alpha^{FO}}{(1 - \delta\gamma) [\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I]},$$

or, as a constraint on α^{FO} ,

$$\alpha^{FO} \leq \tilde{\alpha}^{FO} = \frac{(1 - \alpha^{FI})(1 - \delta\gamma) [\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I]}{\delta\gamma [\pi^O - (\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I)]}.$$

Otherwise, the binding condition (8) determines π^O . Then, either e^O is reduced or outsiders are also paid a rent. In any case, $\pi^O > \pi^I$, and the consistency requirement (7) is satisfied given $\alpha^{FI} \geq 1 - \frac{c(e)}{\delta\gamma e\theta}$. Also note that $\tilde{\alpha}^{FI} = 1$ at $\alpha^{FO} = 0$ and $\tilde{\alpha}^{FI} < 1$ for $\alpha^{FO} > 0$. Finally, if $\alpha^{FI} > \tilde{\alpha}^{FI}$ (or, equivalently, $\alpha^{FO} > \tilde{\alpha}^{FO}$), $b^I = 0$, and

$$\pi^O = \frac{[\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I] [(1 - \alpha^F)(1 - \delta\gamma) + \alpha^{FO}]}{\delta\gamma \alpha^{FO}},$$

or

$$\pi^O = \pi^I \frac{[(1 - \alpha^F)(1 - \delta\gamma) + \alpha^{FO}]}{\alpha^{FO}}.$$

Outcomes and Comparative Statics

In the following we conduct partial comparative statics with respect to α^F (holding α^N constant) in a market equilibrium that maximizes insiders' payoffs. Now, we also have to take effects on outside options into account. Recall that

$$\bar{U}^I = \frac{\alpha^N U^I}{1 - \delta\gamma(1 - \alpha^N)} = \frac{\alpha^N}{1 - \delta\gamma(1 - \alpha^N)} \frac{w^I + b^I - c(e^I)}{1 - \delta\gamma}.$$

Moreover,

$$w^I + b^I = \frac{\delta\gamma \alpha^{FO} \pi^O + \alpha^{FI} \delta\gamma e^I \theta - [\delta\gamma e^I \theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I] (1 - \delta\gamma + \delta\gamma \alpha^F)}{\delta\gamma \alpha^{FI}}.$$

We analyze all three possible cases separately:

1. $\alpha^{FI} < \tilde{\alpha}^{FI}$ and $e^O = e^{FB}$

Then, $\pi^O = e^{FB}\theta - c(e^{FB})$ and

$$\begin{aligned} & w^I + b^I \\ &= (1 - \delta\gamma(1 - \alpha^N)) \frac{\delta\gamma\alpha^{FO}\pi^O - (1 - \delta\gamma + \delta\gamma\alpha^{FO})\delta\gamma e^I\theta + \frac{c(e^I)(1-\delta\gamma)(1-\delta\gamma+\delta\gamma\alpha^F)}{[1-\delta\gamma(1-\alpha^N)]}}{\delta\gamma[(\alpha^{FI} - \alpha^N)(1 - \delta\gamma) - \alpha^N\delta\gamma\alpha^{FO}]}, \end{aligned}$$

hence

$$\begin{aligned} & \frac{\partial(w^I + b^I)}{\partial\alpha^{FO}} \\ &= (1 - \delta\gamma(1 - \alpha^N)) (1 - \delta\gamma) \frac{\pi^O (\alpha^{FI} - \alpha^N) - \alpha^{FI} (\delta\gamma e^I\theta - c(e^I))}{[(\alpha^{FI} - \alpha^N)(1 - \delta\gamma) - \alpha^N\delta\gamma\alpha^{FO}]^2} \end{aligned}$$

This term is positive because the (DEO) constraint,

$$\delta\gamma e^{FB}\theta - c(e^{FB}) \geq \delta\gamma e^I\theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I,$$

becomes

$$\pi^O (\alpha^{FI} - \alpha^N) \geq \alpha^{FI} (\delta\gamma e^I\theta - c(e^I)) + e^{FB}\theta [(\alpha^{FI} - \alpha^N)(1 - \delta\gamma) - \alpha^N\delta\gamma\alpha^{FO}],$$

where $[(\alpha^{FI} - \alpha^N)(1 - \delta\gamma) - \alpha^N\delta\gamma\alpha^{FO}] > 0$ because $w^I + b^I > 0$.

$\partial(w^I + b^I)/\partial\alpha^{FO} > 0$ also implies $\partial U^I/\partial\alpha^{FO} > 0$ and $\partial\bar{U}^I/\partial\alpha^{FO} > 0$, since $U^I = (w^I + b^I - c(e^I))/(1 - \delta\gamma)$ and e^I is independent of α^{FO} .

Finally,

$$\bar{\Pi} = \frac{[\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I]}{(1 - \delta)\delta\gamma},$$

thus

$$\frac{\partial\bar{\Pi}}{\partial\alpha^{FO}} = -\frac{(1 - \delta\gamma)\frac{\partial\bar{U}^I}{\partial\alpha^{FO}}}{(1 - \delta)} < 0.$$

2. $\alpha^{FI} < \tilde{\alpha}^{FI}$ and $e^O < e^{FB}$, characterized by binding (DEO)

Now, $\delta\gamma e^O\theta - c(e^O) = \delta\gamma e^I\theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I$, hence

$$\pi^O = e^O\theta - c(e^O) = (1 - \delta\gamma)e^O\theta + \delta\gamma e^I\theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I \text{ and}$$

$$w^I + b^I = \frac{[\delta\gamma\alpha^{FO}(1 - \delta\gamma)e^O\theta - (1 - \alpha^{FI})\delta\gamma e^I\theta](1 - \delta\gamma(1 - \alpha^N)) + c(e^I)(1 - \delta\gamma)}{\delta\gamma(1 - \delta\gamma)(\alpha^{FI} - \alpha^N)}.$$

Therefore, the binding (DEO) constraint becomes

$$\begin{aligned} & \delta\gamma e^O\theta(\alpha^{FI} - \alpha^N + \alpha^N\alpha^{FO}) - c(e^O)(\alpha^{FI} - \alpha^N) \\ &= \delta\gamma e^I\theta \left(\frac{(1 - \alpha^N)\alpha^{FI} - \delta\gamma(\alpha^{FI} - \alpha^N)}{(1 - \delta\gamma)} \right) - \frac{c(e^I)((1 - \delta\gamma)\alpha^{FI} + \delta\gamma\alpha^N)}{(1 - \delta\gamma(1 - \alpha^N))}, \end{aligned}$$

with

$$\frac{\partial e^O}{\partial \alpha^{FO}} = -\frac{\delta\gamma e^O\theta\alpha^N}{\delta\gamma\theta(\alpha^{FI} - \alpha^N + \alpha^N\alpha^{FO}) - c'(e^O)(\alpha^{FI} - \alpha^N)} > 0.$$

Thus,

$$\frac{\partial(w^I + b^I)}{\partial \alpha^{FO}} = \frac{\left(e^O + \alpha^{FO}\frac{\partial e^O}{\partial \alpha^{FO}}\right)\theta(1 - \delta\gamma(1 - \alpha^N))}{(\alpha^{FI} - \alpha^N)} > 0.$$

Again, $\partial U^I/\partial \alpha^{FO}$ and $\partial \bar{U}^I/\partial \alpha^{FO} > 0$, and

$$\frac{\partial \bar{\Pi}}{\partial \alpha^{FO}} = -\frac{(1 - \delta\gamma)\frac{\partial \bar{U}^I}{\partial \alpha^{FO}}}{(1 - \delta)} < 0.$$

3. $\alpha^{FI} \geq \tilde{\alpha}^{FI}$

Now, the binding condition (8) yields

$$\pi^O = \frac{[\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1 - \delta\gamma)\bar{U}^I]((1 - \alpha^F)(1 - \delta\gamma) + \alpha^{FO})}{\delta\gamma\alpha^{FO}},$$

thus

$$w^I + b^I = \frac{c(e^I) + \delta\gamma(1 - \delta\gamma)\bar{U}^I}{\delta\gamma}$$

and

$$w^I + b^I = \frac{c(e^I)}{\delta\gamma(1 - \alpha^N)}.$$

This yields $\partial U^I / \partial \alpha^{FO} = \partial \bar{U}^I / \partial \alpha^{FO} = \partial \bar{\Pi} / \partial \alpha^{FO} = 0$, whereas

$$\begin{aligned} \pi^O &= \frac{\left[\delta\gamma e^I \theta - \frac{c(e^I)}{(1 - \alpha^N)} \right] \left((1 - \alpha^F)(1 - \delta\gamma) + \alpha^{FO} \right)}{\delta\gamma \alpha^{FO}} \\ \frac{\partial \pi^O}{\partial \alpha^{FO}} &= - \frac{\left[\delta\gamma e^I \theta - \frac{c(e^I)}{(1 - \alpha^N)} \right] (1 - \delta\gamma) (1 - \alpha^{FI})}{\delta\gamma (\alpha^{FO})^2} < 0. \end{aligned}$$

■

Proof of Lemma 1:

Recall from the proof to Proposition 3 that the (IRI) constraint, condition (7), equals

$$\alpha^{FI} \geq \bar{\alpha}^{FI} = \frac{(\delta\gamma e^I \theta - c(e^I)) (1 - \delta\gamma (1 - \alpha^{FO})) - \alpha^{FO} \delta\gamma \pi^O}{\delta\gamma e^I \theta (1 - \delta\gamma)}. \quad (10)$$

It follows that $\bar{\alpha}^{FI}$ is larger with $\alpha^{FO} = 0$ than with $\bar{\alpha}^{FO} > 0$.

At $\bar{\alpha}^{FI}$, (DEO) reveals that $e^O = e^I$ and both, (IRI) and (IRO), just bind. Thus, the optimal π^O is uniquely determined at $\bar{\alpha}^{FI}$, and

$$\frac{\partial \bar{\alpha}^{FI}}{\partial \alpha^{FO}} < 0.$$

■

Proof of Lemma 2:

If $\alpha^{FI} < \bar{\alpha}^{FI}$, (IRI) binds which implies that (IRO) binds as well (see the proof to Proposition 3). Thus, $w^I = c(e^I) - b^I$ and $w^O = c(e^O) - b^O$. Plugging these values into the respective (DE) functions, taking into account (IC) constraints, and that our objective is to maximize

$$\bar{\Pi} = \frac{\alpha^{FI} (e^I \theta - w^I - b^I) + \alpha^{FO} (e^O \theta - w^O - b^O)}{(1 - \delta)(1 - \delta\gamma + \alpha^F \delta\gamma)},$$

reveals that it is weakly optimal to set $b^I = c(e^I)$ and $b^O = c(e^O)$. Thus, for $\alpha^{FI} < \bar{\alpha}^{FI}$ the problem becomes to maximize

$$\frac{\alpha^{FI} (e^I \theta - c(e^I)) + \alpha^{FO} (e^O \theta - c(e^O))}{(1 - \delta)(1 - \delta\gamma + \alpha^F \delta\gamma)},$$

subject to

$$-c(e^I) + \delta\gamma \frac{[(1 - \alpha^{FI})(1 - \delta\gamma) + \delta\gamma\alpha^{FO}] e^I \theta - \alpha^{FO} (e^O \theta - c(e^O))}{(1 - \delta\gamma + \delta\gamma\alpha^{FO})} \geq 0 \quad (\text{DEI})$$

$$-c(e^O) + \delta\gamma \frac{[(1 - \alpha^{FO})(1 - \delta\gamma) + \delta\gamma\alpha^{FI}] e^O \theta - \alpha^{FI} (e^I \theta - c(e^I))}{(1 - \delta\gamma + \delta\gamma\alpha^{FI})} \geq 0. \quad (\text{DEO})$$

It is immediate that $e^I = e^O = e^{FB}$ if these values satisfy (DEI) and (DEO). We now show that $e^I = e^O$ must always hold. To do so, we first rewrite constraints to

$$\begin{aligned} & -c(e^I)(1 - \delta\gamma) - \delta\gamma\alpha^{FO} (c(e^I) - c(e^O)) \\ & + \delta\gamma [(1 - \delta\gamma + \delta\gamma\alpha^F) e^I \theta - \alpha^{FI} e^I \theta - \alpha^{FO} e^O \theta] \geq 0 \end{aligned} \quad (\text{DEI})$$

$$\begin{aligned} & -c(e^O)(1 - \delta\gamma) - \delta\gamma\alpha^{FI} (c(e^O) - c(e^I)) \\ & + \delta\gamma [(1 - \delta\gamma + \delta\gamma\alpha^F) e^O \theta - \alpha^{FI} e^I \theta - \alpha^{FO} e^O \theta] \geq 0, \end{aligned} \quad (\text{DEO})$$

and define Δ as the difference between the left-hand side of (DEI) and the left-hand side of (DEO):

$$\Delta = [(\delta\gamma e^I \theta - c(e^I)) - (\delta\gamma e^O \theta - c(e^O))] (1 - \delta\gamma + \delta\gamma\alpha^F)$$

By the definition of Δ , (DEI) is slack if $\Delta > 0$, whereas (DEO) is slack if $\Delta < 0$. For the following, we also define \bar{e} is the effort level characterized by $\delta\gamma\theta - c'(\bar{e}) = 0$.

Now, assume $e^I > e^O$, hence at least one of the (DE) constraints binds and restricts profits. We show that we can increase e^O and e^I in a way that does not directly affect profits but relaxes the binding constraint, thus allows firms to eventually increase their profits.

Applying the total differential, a marginal change in e^O , by de^O , does not change profits if

$$de^I = -\frac{\alpha^{FO} (\theta - c'(e^O))}{\alpha^{FI} (\theta - c'(e^I))} de^O.$$

This operation changes the left-hand side of (DEI) by

$$\frac{\alpha^{FO} (\theta - c'(e^O))}{\alpha^{FI} (\theta - c'(e^I))} de^O \left(\frac{-(\delta\gamma\theta - c'(e^I)) (1 - \delta\gamma + \delta\gamma\alpha^F)}{(1 - \delta\gamma + \delta\gamma\alpha^{FO})} \right),$$

and the right-hand side of (DEO) by

$$\frac{(\delta\gamma\theta - c'(e^O)) (1 - \delta\gamma + \delta\gamma\alpha^F)}{(1 - \delta\gamma + \delta\gamma\alpha^{FI})} de^O$$

First, assume $e^O < e^I < \bar{e}$, hence $\Delta > 0$ and (DEI) is slack whereas (DEO) binds. Then, this operation with $de^O > 0$ tightens (DEI) and relaxes (DEO), which allows firms to increase profits.

Second, assume $e^O < \bar{e} \leq e^I$. Then, this operation with $de^O > 0$ relaxes both constraints, which allows firms to increase profits.

Third, assume $e^I > e^O \geq \bar{e}$, hence $\Delta < 0$ and (DEO) is slack whereas (DEI) binds. Then, this operation with $de^O > 0$ tightens (DEO) and relaxes (DEI), which allows firms to increase profits.

Summing up, $e^I > e^O$ is not optimal. Equivalently, we can show that $e^I < e^O$ cannot be optimal as well, allowing us conclude that $e^I = e^O = e$ in a profit-maximizing social equilibrium with $\alpha^{FI} < \bar{\alpha}^{FI}$.

Therefore, (DEI) and (DEO) coincide, and the optimization problem becomes to maximize

$$\frac{\alpha^F (e\theta - c(e))}{(1 - \delta) (1 - \delta\gamma + \alpha^F \delta\gamma)},$$

subject to

$$-c(e) + \delta\gamma (1 - \alpha^F) e\theta \geq 0. \quad (\text{DE})$$

Naturally, $e = e^{FB}$ if it satisfies the (DE) constraint. Otherwise, the binding (DE) constraint determines equilibrium effort, with

$$\frac{de}{d\alpha^{FO}} = \frac{\delta\gamma e\theta}{-c'(e) + \delta\gamma (1 - \alpha^F) \theta} < 0.$$

Since $e = \bar{e}$ at $\alpha^{FI} = \bar{\alpha}^{FI}$, this implies that $e > \bar{e}$ for $\alpha^{FI} < \bar{\alpha}^{FI}$.

Moreover,

$$\begin{aligned}\frac{\partial \bar{\Pi}}{\partial \alpha^{FO}} &= \frac{(e\theta - c(e))(1 - \delta\gamma)}{(1 - \delta)(1 - \delta\gamma + \alpha^F \delta\gamma)^2} + \frac{\alpha^F(\theta - c'(e))}{(1 - \delta)(1 - \delta\gamma + \alpha^F \delta\gamma)} \frac{de}{d\alpha^{FO}} \\ &= \frac{e\theta}{(1 - \delta)} \frac{\delta\gamma\theta - c'(e)}{[-c'(e) + \delta\gamma(1 - \alpha^F)\theta]} > 0,\end{aligned}$$

where the denominator – the partial derivative of the left-hand side of (DE) with respect to e – must be negative if (DE) binds. ■

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