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Does digitalization increase labor market efficiency? Job search and effort on the job with asymmetric information and firm learning

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Abstract

This paper analyses the effect of firm learning on labor market efficiency in a frictional labor market with asymmetric information. I consider a model with random matching and wage bargaining a la Pissarides (1985, 2000) where worker ability is unknown to firms at the hiring stage. Firm learning increases relative expected earnings in high-ability jobs and, thereby, enhances imitation incentives of low-ability workers. The net effect on aggregate expected match surplus and unemployment is indeterminate a priori. Numerical results show that firm learning does not increase labor market efficiency.

Keywords: job search; on-the-job effort; asymmetric information; learning.

JEL classification: D82; D83; J64.

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1 Introduction

The inability of the labor market to allocate resources efficiently has been attributed to two important sources of frictions. First, search frictions may impose costs on the formation of suitable worker-firm matches. Second, asymmetric information may result in adverse selection of workers at the stage of hiring. The existing literature shows that these frictions together may seriously hamper efficient resource allocation in the labor market and increase the rate of unemployment.[1] However, there is also evidence for firms to learn fast about workers' types.[2] The ongoing digitalization of the workplace, which measures individual performance ever more precisely, is likely to speed up firm learning even further.[3] This may ameliorate distortions due to imperfect information of firms.

In this paper, I analyze the implications of firm learning for labor market efficiency in a Diamond-Mortensen-Pissarides search and matching framework with asymmetric information about worker ability and endogenous unobservable worker effort on the job. Due to asymmetric information, adverse selection is possible: workers may misreport their type at hiring and receive a wage that exceeds the wage based on their true type. This is possible because firms cannot observe worker effort initially, and so cannot infer a worker's type from observed output. In consequence, effort on the job may be sub-optimal. Within this framework, I address a number of questions, e.g.: How does firm learning affect the search behavior of workers and firms? How does it affect a worker's choice of effort on the job, the surplus of a worker-firm match[4], (relative) wages and unemployment rates, and aggregate labor market efficiency?

[1] I review some of this literature below.

[2] Lange (2007) estimates the speed at which firms learn about the ability of their workers. He finds that firms' initial expectation errors decline by 50% during the first three years of employment.

[3] See, for example, O'Connor (2013), Kantor and Streitfeld (2015), and The Economist (2015) on working conditions at Amazon where worker performance is monitored continuously in real time.

[4] Match surplus is the difference between the expected present value of the future incomes that the two parties to a match earn and the expected present value of income that they forgo by participating in the employment relationship (Mortensen and Nagypal (2007, p. 330)).

If asymmetric information causes distortions in the labor market, then an increase in the rate at which firms learn about the true type of a worker can be expected to improve labor market outcomes. Interestingly, however, I find that in the presence of search frictions, the effect of firm learning on labor market efficiency is not necessarily positive and may, in fact, be negative. This is because, if job offers and wages are based on the expected surplus of a worker-firm match, firm learning increases incentives for low-ability workers to imitate high-ability workers. If this effect is sufficiently large, the average expected match surplus decreases, and the average unemployment rate increases, as firms learn faster about a worker's type.

More specifically, I consider a frictional labor market with random matching and wage bargaining a la Pissarides (1985, 2000), where workers and firms meet randomly according to a matching function, and wages are determined via bargaining between workers and firms. In contrast to this benchmark model, information is asymmetric in my model: workers are perfectly informed about their ability, but firms learn only gradually about a worker's ability upon observing his effort over time. The effort of workers on the job is endogenous; it is unobservable by the firm initially but affects worker output and, thereby, serves as a signal of worker ability. Starting wages are based on the worker's current output (which depends on (true) worker ability and effort on the job) as well as the worker's future expected output (which depends on reported worker ability). After the firm learns the worker's true type, the wage is re-negotiated.

Workers decide to search for jobs based on their option value of search, which varies with reported ability. By misreporting his type, a low-ability worker benefits from a greater job arrival rate and earns a higher starting wage. In turn, he also faces a greater cost of effort. In case of adverse selection, relative arrival rates and expected earnings in high-ability jobs increase in the rate of firm learning, increasing imitation incentives of low-ability workers. Then, firm learning has two important counter-vailing effects on the expected match surplus of high-ability workers in separating equilibrium. On the one hand, given efforts, faster firm learning increases their surplus, since effort is suboptimal before firm learning but optimal thereafter (direct effect). On the other hand, faster firm learning decreases their surplus, because the (suboptimally high) effort before firm learning increases even further in response to greater imitation incentives of low-ability workers (indirect effect). The net effect is indeterminate a priori. In pooling equilibrium, faster firm learning unambiguously increases the average expected match surplus, decreasing the (common) unemployment rate. Existence, just as efficiency, of a separating or pooling equilibrium may depend on the rate of firm learning.

In numerical simulations, I find that a separating equilibrium exists that corresponds to equilibrium under perfect information. In this case, there is no adverse selection, and firm learning has no effect on labor market efficiency.

Related literature

There is a growing literature on the problem of worker-firm matching in the presence of costly search and asymmetric information. For example, Lockwood (1991) suggests that adverse selection increases inefficiency in a frictional labor market where firms test workers prior to hiring and unemployment is used as a signal of productivity. More recently, Inderst (2005) analyzes labor market equilibria in a model with random search and adverse selection where new participants enter the market. He derives conditions for the existence of a unique separating equilibrium. Guerrieri, Shimer and Wright (2010) analyze equilibrium existence and efficiency in labor markets with directed search and adverse selection. They show that there always exists a separating equilibrium, which is not generally efficient. In comparison, I consider equilibria in labor markets with random search and adverse selection where firms are allowed to learn about workers' types. I show that firm learning may affect equilibrium existence and efficiency, and it does not generally increase the latter. In numerical simulations, I find that a separating equilibrium exists that is efficient independently of the rate of firm learning. Camera and Delacroix (2004) and Michelacci and Suarez (2006) consider firms' choice of the wage setting mechanism (wage posting versus wage bargaining) in the presence of search frictions and adverse selection. All of the above consider stationary environments without firm learning or wage dynamics.

Another strand of the literature focuses on wage dynamics in search models with asymmetric information. For example, Carrillo-Tudela and Kaas (2015) determine worker turnover and optimal wage contracts in a model with wage posting and firm learning about workers' types. Moen and Rosen (2006) analyze optimal wage contracts in a random search model where firms do not observe workers' effort nor their type. Similarly, Moen and Rosen (2011) and Tsuyuhara (2016) analyze optimal wage contracts with unobservable worker effort (and type) and directed search. These papers focus on the retention and incentive effects of wages in the presence of adverse selection or moral hazard. They do not, however, address the implications of firm learning for labor market efficiency.

The literature on firm learning typically focuses on implications for worker turnover. Examples for theoretical contributions include Jovanovic (1979), Moscarini (2005) and Papageorgiou (2018), where workers and firms jointly learn about match quality over time. Empirical contributions such as Altonji and Pierret (2001), Lange (2007) and Kahn (2013) provide evidence for the degree of asymmetric information and the speed of firm learning. My paper implements firm learning in a tractable model of random job search and wage bargaining with asymmetric information about both worker ability and worker effort. In this setting, I analyze the effect of firm learning on job search, effort on the job, match surplus, (relative) wages and unemployment rates, and aggregate labor market efficiency.

The rest of the paper is organized as follows. Section 2 describes the framework of the model. Sections 3-4 characterize equilibria in case of perfect and imperfect information, respectively. Section 5 discusses the role of firm learning for equilibrium existence and efficiency. Section 6 simulates the model numerically. Section 7 concludes.

2 Model framework

2.1 Workers and firms

Consider a continuous time economy with a continuum of workers and firms. Workers are either employed, or unemployed and searching for a job. They are one of two types, high-ability or low-ability, with ability p_i, $i \in \{H, L\}$, $p_H > p_L > 0$, and cost of effort e, $c(e)$, with $c(0) = 0$, $\partial c(e)/\partial e > 0$, and $\partial^2 c(e)/\partial e^2 > 0$.[5] The measure of workers of both types per period is assumed constant and equal to $\alpha_H = \alpha$ and $\alpha_L = 1 - \alpha$, $0 < \alpha < 1$, respectively. Employed workers are displaced into unemployment according to a Poisson process with parameter $\delta > 0$ due to job destruction shocks. When unemployed, workers receive a constant payoff of b per period. Workers search for jobs only when unemployed – there is no on-the-job search.

Firms each consist of one job, which is either filled or vacant. They must pay a cost k for keeping an open vacancy. The output of a job that is filled with a worker of type i who exerts effort e is equal to $y_i = p_i e$. That is, for any given level of effort $e > 0$, output of a high-ability worker is greater than output of a low-ability worker, $p_H e > p_L e$.

Both firms and workers are risk-neutral. The objective of workers and firms is to maximize their present discounted value of expected income. Future values are discounted at rate r.

Workers are perfectly informed about their type, but firms do not know a worker's ability at the hiring stage and only learn about it over time at exogenous rate ψ, $0 < \psi < 1$ per period.[6]

[5] The single-crossing condition, which ensures that the indifference curves of high- and low-ability workers in wage-effort space intersect only once, is fulfilled even though cost functions are homogeneous (see below).

[6] The implicit assumption here is that it is too costly for firms to observe the effort (and implied ability) of a worker immediately.

2.2 The matching process

There is an aggregate matching function $m(v, u)$ that gives the number of matches between searching firms and workers (divided by the fixed total labor force equal to 1) each period. The matching function is assumed to be non-negative, increasing and concave in both arguments, v, the number of vacancies, and u, the number of unemployed workers, and homogeneous of degree one. There are no matches, if there are no vacancies or unemployed workers, $m(0, u) = m(v, 0) = 0$. Vacancies are filled at rate $q(\frac{v}{u}) = m(1, \frac{u}{v}) = m(v, u)/v$, with $\partial[q(\frac{v}{u})]/\partial[\frac{v}{u}] < 0$. Unemployed workers find jobs at rate $f(\frac{v}{u}) = \frac{v}{u}q(\frac{v}{u}) = m(v, u)/u$, with $\partial[\frac{v}{u}q(\frac{v}{u})]/\partial[\frac{v}{u}] > 0$. The ratio $\frac{v}{u}$ is a measure of labor market tightness and is denoted in the following with θ.

Firms offer jobs either for high-ability workers or for low-ability workers, so I distinguish between vacancy rates for high- and low-ability workers, v_i, $i \in \{H, L\}$, as well as unemployment rates for high- and low-ability workers, u_i, $i \in \{H, L\}$. Since firms cannot distinguish between different types of workers at the time of hiring, the arrival rate for firms offering a job of type i is given by $q(\theta_i) = q(\frac{v_i}{u_{ii}+u_{ij}})$, $i \neq j$, where u_{ii} and u_{ij} are the numbers of workers of type i and j, respectively, who are searching for job i[7] (to be determined in equilibrium).

2.3 Wage contracts

A wage contract in a job of type i, $i \in \{H, L\}$, is a pair (w_i, y_i), where w_i is the wage and y_i is output. Wage contracts are negotiated between workers and firms at the time of hiring (for the period before firm learning, which we call the probation period), and they are re-negotiated after the firm has learned the worker's type. Wages are determined by the generalized Nash bargaining solution, which gives a fraction β of the match's expected joint surplus to the worker, where β is an exogenous measure of relative worker

[7] Here, in contrast to standard matching models with homogeneous jobs, workers search for jobs either of type i or type j.

bargaining power. The expected match surplus depends on expected output and, in turn, on worker effort, as described below.

2.4 Timing of events

At the beginning, each worker finds out about his type i, $i \in \{H, L\}$. Then, firms with unfilled jobs decide whether or not to post a vacancy (of type i or type j) to recruit unemployed workers, depending on the expected values of vacancies.[8] Unemployed workers of type i decide whether to search for a job of type i or $j \neq i$, or remain unemployed, depending on the expected gain from employment. Employed workers lose their jobs with exogenous probability δ, whereupon the worker becomes unemployed and the job becomes vacant. Among workers and firms in established matches, the timing of events is as follows:

1. Firms and employed workers bargain over the wage contract $\omega_i \equiv (w_i, y_i)$, as described in section 2.3 above.

2. Each employed worker of type i chooses effort, e_i, which is unobservable to the firm.

3. The firm and worker produce observable output $y_i(e_i)$, generating expected match surplus $S_i(e_i, e_i')$, where e_i (e_i') is worker effort before (after) firm learning. The worker is paid the corresponding wage, $w_i(e_i, e_i')$, and firms receive corresponding profits, $\pi_i(e_i, e_i') = y_i(e_i) - w_i(e_i, e_i')$.

4. Firms learn about their worker's type with exogenous probability ψ, whereupon firms and employed workers bargain over the new wage contract $\omega_i' \equiv (w_i', y_i')$ for the next period onward.

[8] Due to free entry of firms, the expected values of vacancies of different types of jobs are the same and equal to zero (see below). Therefore, firms are indifferent between posting a vacancy of type i or type j in equilibrium.

5. Each employed worker of type i chooses a level of effort, e_i'. Outputs, expected match surplus, wages, and profits are given by $y_i'(e_i')$, $S_i'(e_i, e_i')$, $w_i'(e_i, e_i')$, and $\pi_i'(e_i, e_i') = y_i'(e_i') - w_i'(e_i, e_i')$, where e_i (e_i') is worker effort before (after) firm learning.

2.5 Bellman equations

Consider a firm with a job of type i (producing output y_i) and a worker with ability of type i, $i \in \{H, L\}$.

The firm's expected value of a filled job is $J_i(\pi_i(e_i, e_i'))$. After the firm learns the worker's type and the wage is renegotiated, the firm receives a continuation value of $J_i'(\pi_i(e_i, e_i'))$. The expected value of a vacant job is V_i. These values are given implicitly by the following Bellman equations:

$$\begin{aligned} rJ_i(\pi_i(e_i, e_i')) &= \pi_i(e_i, e_i') + \delta[V_i - J_i(\pi_i(e_i, e_i'))] + \psi[J_i'(\pi_i(e_i, e_i')) - J_i(\pi_i(e_i, e_i'))] \quad (1) \\ rJ_i'(\pi_i(e_i, e_i')) &= \pi_i(e_i, e_i') + \delta[V_i - J_i'(\pi_i(e_i, e_i'))], \quad (2) \\ rV_i &= -k + q(\theta_i)[J_i(\pi_i(e_i, e_i')) - V_i]. \quad (3) \end{aligned}$$

Equation (1) shows that the expected value for a firm with a filled job i includes the firm profit, $\pi_i(e_i, e_i') = y_i(e_i) - w_i(e_i, e_i')$, plus the expected loss, if the match is destructed and the job becomes vacant, which happens at rate δ, plus the expected change in the job's value after the firm has learned the worker's type and his wage has been renegotiated to $w_i'(e_i, e_i')$, which happens at rate ψ.[9][10] Equation (2) shows that the expected continuation value of a job of type i, after the firm has learned the worker's type and the wage has been renegotiated to $w_i'(e_i, e_i')$, equals the profit, $\pi_i(e_i, e_i') = y_i(e_i') - w_i'(e_i, e_i')$, plus the loss in

[9] Workers never quit, so the only reason for the termination of a match is the exogenous separation process δ.

[10] Workers lose their jobs with strictly positive probability, so their wage after firm learning, $w_i'(e_i, e_i')$, is related to effort both before and after firm learning, e_i and e_i', via the value of unemployment (see below).

case of job destruction. Equation (3) shows that the expected value of a vacant job of type i includes the cost of posting a vacancy (e.g., recruiting costs), k, plus the expected gain of filling the job (with a worker of type i or j), which happens with probability $q(\theta_i)$.

The worker obtains an expected value of $W_{ii}(w_i(e_i, e_i'))$, if employed in a job of type i, and he obtains an expected value of $W_{ij}(w_j(e_j, e_i'))$, if employed in a job of type j, $i \neq j$. In the former case, he chooses effort e_i, and in the latter case he deviates to effort $\frac{p_j}{p_i} e_j$, so that the firm takes him to be a type-j worker, observing output $y_j(e_j) = p_j e_j = p_i(\frac{p_j}{p_i} e_j)$, and pays him the wage $w_j(e_j, e_i')$.[11] After the firm learns the worker's type, the worker exerts effort e_i' and receives a continuation value of $W_i'(w_i'(e_i, e_i'))$. The worker receives an expected value of U_i, if unemployed. The corresponding Bellman equations are as follows:

$$rW_{ii}(w_i(e_i, e_i')) = w_i(e_i, e_i') - c_i(e_i) + \delta[U_i - W_{ii}(w_i(e_i, e_i'))] + \psi[W_i'(w_i'(e_i, e_i')) - W_{ii}(w_i(e_i, e_i'))] \tag{4}$$

$$rW_{ij}(w_j(e_j, e_i')) = w_j(e_j, e_i') - c_i(\frac{p_j}{p_i} e_j) + \delta[U_i - W_{ij}(w_j(e_j, e_i'))] + \psi[W_i'(w_i'(e_i, e_i')) - W_{ij}(w_j(e_j, e_i'))] \tag{5}$$

$$rW_i'(w_i'(e_i, e_i')) = w_i'(e_i, e_i') - c_i(e_i') + \delta[U_i - W_i'(w_i'(e_i, e_i'))] \tag{6}$$

$$rU_i = b + \max\ [\theta_i q(\theta_i)(W_{ii}(w_i(e_i, e_i')) - U_i), \theta_j q(\theta_j)(W_{ij}(w_j(e_j, e_i')) - U_i)]. \tag{7}$$

Equation (4) shows that the expected value of employment of a worker of type i in a job of type i includes the wage $w_i(e_i, e_i')$ minus the cost of effort $c_i(e_i)$ plus the expected loss of a separation to unemployment, which happens at rate δ, plus the expected gain after the firm has learned the worker's type and his wage has been renegotiated to $w_i'(e_i, e_i')$,

[11] The difference in efforts that workers of different abilities are required to undertake in order to earn a given wage ensures that the marginal rate of substitution between effort and the wage (at any given effort and wage) is greater for L- than for H-workers. Therefore, the single-crossing condition is fulfilled, even though effort cost functions are the same.

which happens at rate ψ. The expected value of a worker of type i being employed in a job of type j includes the wage $w_j(e_j, e_i')$ minus the cost of effort $c_i(\frac{p_j}{p_i}e_j)$ instead, while his expected value after firm learning includes the renegotiated wage $w_i'(e_i, e_i')$ minus the cost $c_i(e_i')$, according to equations (5) and (6), respectively. Equation (7) shows that the expected value of unemployment for a worker of type i includes unemployment income b plus the option value of searching. The latter consists of the possibility of meeting a firm with a job i at rate $\theta_i q(\theta_i)$, or the possibility of meeting a firm offering a job j at rate $\theta_j q(\theta_j)$, times the expected increase in value associated with the offers, respectively.[12] The arrival rates for workers of type i depend not only on their own search behavior and that of firms, but also on the search behavior of workers of type $j \neq i$. In particular, depending on whether a worker i's incentive constraint is fulfilled or not (see below), we distinguish between two possibilities: first, workers of type i search for jobs of type $i \neq j$ (separating equilibrium); second, both workers of type i and workers of type j search for jobs of the same type (pooling equilibrium).[13]

The incentive constraint for a worker of type i requires that the option value of searching for a job of type i is at least as large as that of searching for a job of type j, $OVS_{ii}^S(e_i, e_i') \geq OVS_{ij}^S(e_j, e_i')$, that is:

$$\theta_i q(\theta_i)(W_{ii}(w_i(e_i, e_i')) - U_i) \geq \theta_j q(\theta_j)(W_{ij}(w_j(e_j, e_i')) - U_i). \tag{8}$$

It describes the main trade-off faced by a worker of type i when searching for a job. In a job of type j, worker i earns the potentially higher wage w_j with potentially greater probability $\theta_j q(\theta_j)$ but also has to exert a potentially greater level of effort compared

[12]In principle, offers may also be rejected, but we are interested in situations where $W(w) > U$, and $J(y - w) > V$, so that there is something to bargain over.

[13]The third theoretical possibility, that workers of type i search for jobs of type $j \neq i$, is excluded by the single-crossing condition, which implies that, if the incentive constraint is not fulfilled for the high-ability (low-ability) type, then it is fulfilled for the low-ability (high-ability) type. In other words, if it does not (does) pay off for an H-worker (L-worker) to provide extra effort sufficient to earn wage w_H instead of w_L, then it pays off for an L-worker (H-worker) even less (more).

to a job of type i. Worker i will search for a job of type i and self-select into the right contract when the expected gain from searching for a job j (instead of job i) does not exceed the expected extra cost of effort. He will mimick the other type j, choosing effort $\frac{p_j}{p_i}e_j$ to earn wage $w_j(e_j, e_i')$ with probability $\theta_j q(\theta_j)$, otherwise.

3 Perfect information equilibrium

In the following, I determine the equilibrium contracts between workers and firms in the case of perfect information, where both the worker and the firm know the worker's type. This case serves as a benchmark for the case of imperfect information, where firms do not know a worker's true type at the time of hiring and only learn about it gradually over time (to be discussed in the next section).

3.1 Workers and firms

With perfect information, both the worker and the firm know the worker's type. In this case, we have $\psi = 0$, $J_i(\pi_i(e_i, e_i')) = J_i'(\pi_i(e_i, e_i')) \equiv J_i^*(\pi_i(e_i^*))$, $V_i \equiv V_i^*$, $W_{ii}(w_i(e_i, e_i')) = W_i'(w_i'(e_i, e_i')) \equiv W_i^*(w_i^*(e_i^*))$, $U_i \equiv U_i^*$, and $\theta_i \equiv \theta_i^*$.[14] The Bellman equations for firms and workers are given by:

$$\begin{aligned} rJ_i^*(\pi_i(e_i^*)) &= \pi_i(e_i^*) + \delta[V_i^* - J_i^*(\pi_i(e_i^*))], \quad (9) \\ rV_i^* &= -k + q(\theta_i^*)[J_i^*(\pi_i(e_i^*)) - V_i^*], \quad (10) \end{aligned}$$

$$\begin{aligned} rW_i^*(w_i^*(e_i^*)) &= w_i^*(e_i^*) - c_i(e_i^*) + \delta[U_i^* - W_i^*(w_i^*(e_i^*))] \quad (11) \\ rU_i^* &= b + \theta_i^* q(\theta_i^*)(W_i^*(w_i^*(e_i^*)) - U_i^*), \quad (12) \end{aligned}$$

[14] $W_{ij}(w_j(e_j, e_i'))$ is not relevant in case of perfect information, since workers of type i cannot pretend to be a different type.

where $\theta_i^* = \frac{v_i^*}{u_i^*}$, since only workers of type i search for jobs of type i ($u_{ii}^* = u_i^*$ and $u_{ij}^* = 0$).

3.2 Wage bargaining

Job matches produce economic rents (due to search frictions), which are shared between firms and workers according to the generalized Nash bargaining solution:[15]

$$w_i^*(e_i^*) = \arg\max\ [(W_i^*(w_i^*(e_i^*)) - U_i^*)^\beta (J_i^*(\pi_i(e_i^*)) - V_i^*)^{1-\beta}]. \tag{13}$$

This results in equilibrium wages[16][17]

$$w_i^*(e_i^*) = y_i(e_i^*) - (r+\delta)(1-\beta)S_i^*, \tag{14}$$

where

$$S_i^* = \frac{y_i(e_i^*) - c(e_i^*) - b}{r + \delta + \theta_i^* q(\theta_i^*)\beta} \tag{15}$$

and

$$q(\theta_i^*)(1-\beta)S_i^* = k. \tag{16}$$

Equations (15) and (16) determine the equilibrium values S_i^* and θ_i^*.[18] Combining the two equations, the latter can be expressed implicitly by

$$\frac{r + \delta + \theta_i^* q(\theta_i^*)\beta}{(1-\beta)q(\theta_i^*)} = \frac{y_i(e_i^*) - c(e_i^*) - b}{k}. \tag{17}$$

Wage contracts in jobs of type i, $i \in \{H, L\}$, are given by $(w_i^*(e_i^*), y_i(e_i^*))$, according to equation (14).

[15]It is assumed that the constraints $W_i^*(w_i^*(e_i^*)) - U_i^* \geq 0$ and $J_i^*(\pi_i(e_i^*)) - V_i^* \geq 0$ are fulfilled.

[16]See Appendix A for details.

[17]I henceforth write S_i^* and θ_i^* in short-hand notation for $S_i^*(e_i^*)$ and $\theta_i^*(e_i^*)$ to improve readability.

[18]A unique equilibrium exists, given standard regularity conditions $\frac{\partial[q(\theta^*)]}{\partial\theta^*} < 0$, $\frac{\partial[\theta^* q(\theta^*)]}{\partial\theta^*} > 0$, $q(0) \to \infty$, $q(\infty) \to 0$.

3.3 Steady state unemployment

Steady state turnover implies that the flow rate into employment equals the flow rate out of employment:

$$\theta_H^* q(\theta_H^*) u_H^* = \delta \left(\alpha - u_H^*\right) \quad \text{and} \quad \theta_L^* q(\theta_L^*) u_L^* = \delta \left(1 - \alpha - u_L^*\right). \tag{18}$$

The steady-state rates of unemployment for both types of workers in case of perfect information are, therefore, given by

$$u_H^* = \frac{\delta\alpha}{\delta + \theta_H^* q(\theta_H^*)} \quad \text{and} \quad u_L^* = \frac{\delta(1-\alpha)}{\delta + \theta_L^* q(\theta_L^*)}, \tag{19}$$

which decrease in θ_H^* and θ_L^* (determined in equation (17)), respectively.
Figure 1 illustrates the equilibrium characterized by equations (15) and (16), depicted in curves SS and ZZ, respectively. An increase in output net of the cost of effort, $y_i(e_i^*) - c(e_i^*)$, increases the value of the match, according to (15), shifting up curve SS in the graph. In consequence, the surplus, S_i^*, increases, and the rate at which workers (firms) contact firms (workers), $\theta_i^* q(\theta_i^*)$ $(q(\theta_i^*))$, increases (decreases). The net wage, $w_i^*(e_i^*) - c(e_i^*)$, increases[19] and the unemployment rate, u_i^*, decreases according to (14) and (19), respectively. And vice versa.

3.4 Effort

Since worker types are observable, wage contracts can be made contingent on a worker's effort, which can be directly inferred from observable output. Furthermore, since the wage bargaining scheme splits any joint surplus between the worker and the firm, the effort that is optimal for both parties is the one that maximizes the joint surplus.[20]

[19] To see this, subtract $c(e_i^*)$ from both sides in (14) and note that $1 - \frac{(r+\delta)(1-\beta)}{r+\delta+\theta_i^* q(\theta_i^*)\beta} > 0$ and $y_i(e_i^*) - c(e_i^*) \geq b$.

[20] This effort serves to maximize both the worker's and the firm's expected values (see Appendix A).

Therefore, under perfect information, wage contracts $(w_i^*(e_i^*), y_i(e_i^*))$ specify outputs $y_i = p_i e_i^*$, where efforts e_i^* are chosen to maximize the joint surplus (see equations (15)-(16)) as a function of (variable) effort e:

$$e_i^* = \arg\max_e \frac{y_i(e) - c(e) - b}{r + \delta + \theta_i q(\theta_i)\beta}. \tag{20}$$

The first-best efforts, e_i^*, are, therefore, implicitly given by

$$p_i = \frac{\partial c(e)}{\partial e}. \tag{21}$$

Proposition 1 ***Perfect information.*** *In the case of perfect information, equilibrium consists of the value functions $J_i^*, V_i^*, W_i^*, U_i^*$, $i \in \{H, L\}$, that satisfy the Bellman equations (9)-(12), the free-entry condition $V_i^* = 0$, efforts e_i^* that satisfy the conditions for optimal effort (21), wages $w_i^*(e_i^*)$ that satisfy the bargaining solution (14), and the unemployment and vacancy rates u_i^* and v_i^* that satisfy the steady-state conditions (19).*

Corollary 1. *With perfect information, the surplus and, in turn, the net wage and the job finding rate of high-ability workers is greater than that of low-ability workers.*
See (15)-(16) together with (21) and the fact that $p_H > p_L$.

4 Imperfect information equilibria

In the case of imperfect information, firms do not know a worker's true type at the time of hiring. They can, however, screen workers during wage negotiations via the choice between different wage contracts, along the lines of the standard screening model by Rothschild and Stiglitz (1976).[21] In the following, I solve for the (pure strategy) subgame perfect Nash equilibria in this case, which are defined as follows.

[21] In the present context, workers choose the optimal contract together with firms, whereas, in the standard model, they choose among the contracts offered by firms.

Definition. A subgame perfect Nash equilibrium (SPNE) is a set of expected value functions

$J_i, J_i', V_i, W_{ii}, W_{ij}, W_i', U_i,\ i, j \neq i \in \{H, L\}$ for firms and workers, effort levels e_i, e_i', wages w_i, w_i', and unemployment and vacancy rates u_i, u_j, v_i, v_j, u_i', u_j', v_i', v_j' such that

- wage contracts satisfy the generalized Nash bargaining rule with relative worker bargaining power β,

- workers' search and efforts on the job are optimal given wage contracts and costs of effort,

- firms' search and profits are optimal given wage contracts and the free-entry conditions $V_i = 0,\ i \in \{H, L\}$, and

- the unemployment rates $u_i,\ i \in \{H, L\}$, are in steady state.

I consider pure strategy equilibria only, distinguishing between

(a) separating equilibrium, where high- and low-ability workers produce different levels of output and earn different wages with different probabilities (as firms can infer a worker's type), and

(b) pooling equilibrium, where high- and low-ability workers produce the same level of output and earn the same wage with the same probability.

4.1 Separating equilibrium

In separating equilibrium, the expected value functions of firms and workers are given by:

$$\begin{aligned} rJ_i^S(\pi_i(e_i^S,e_i'^S)) &= \pi_i(e_i^S,e_i'^S)+\delta[V_i^S-J_i^S(\pi_i(e_i^S,e_i'^S))+\psi[J_i'^S(\pi_i(e_i^S,e_i'^S))-J_i^S(\pi_i(e_i^S,e_i'^S))], \\ & \qquad\qquad (22) \\ rJ_i'^S(\pi_i(e_i^S,e_i'^S)) &= \pi_i(e_i^S,e_i'^S)+\delta[V_i^S-J_i'^S(\pi_i(e_i^S,e_i'^S))], \qquad (23) \\ rV_i^S &= -k+q(\theta_i^S)[J_i^S(\pi_i(e_i^S,e_i'^S))-V_i^S], \qquad (24) \end{aligned}$$

$$\begin{aligned} rW_{ii}^S(w_i^S(e_i^S,e_i'^S)) &= w_i^S(e_i^S,e_i'^S)-c(e_i^S)+\delta[U_i^S-W_{ii}^S(w_i^S(e_i^S,e_i'^S))]+\psi\left[W_i'^S(w_i'^S(e_i^S,e_i'^S))-\right. \\ & \left. W_{ii}^S(w_i^S(e_i^S,e_i'^S))\right], \qquad (25) \\ rW_i'^S(w_i'^S(e_i^S,e_i'^S)) &= w_i'^S(e_i^S,e_i'^S)-c(e_i'^S)+\delta[U_i^S-W_i'^S(w_i'^S(e_i^S,e_i'^S)], \qquad (26) \\ rU_i^S &= b+\theta_i^S q(\theta_i^S)[W_{ii}^S(w_i^S(e_i^S,e_i'^S))-U_i^S], \qquad (27) \end{aligned}$$

where $\theta_i^S=\frac{v_i^S}{u_i^S}$, since only workers of type i search for jobs of type i ($u_{ii}^S=u_i^S$ and $u_{ij}^S=0$).[22]

Note that, even though firms can distinguish between worker types from the start in a separating equilibrium, the values of filled jobs before and after firm learning, $J_i^S(\pi_i(e_i^S,e_i'^S))$ and $J_i'^S(\pi_i(e_i^S,e_i'^S))$, are not necessarily the same. This is because the output that wage contracts must specify to separate workers of a given type from workers of the other type before firm learning may be different from output after firm learning.[23] For the same reason, the values of employment before and after firm learning, $W_{ii}^S(w_i^S(e_i^S,e_i'^S))$ and $W_i'^S(w_i'^S(e_i^S,e_i'^S))$, are not necessarily the same.

[22] I use θ_i^S, and S_i^S, $S_i'^S$ (see below), in short-hand notation for $\theta_i^S(e_i^S,e_i'^S)$, $S_i^S(e_i^S,e_i'^S)$, $S_i'^S(e_i^S,e_i'^S)$.

[23] In consequence, workers' wages and firms' profits may not be the same before and after firm learning.

Wage bargaining results in (expected) wages before and after firm learning given by[24]

$$
\begin{aligned}
w_i^S(e_i^S, e_i'^S) &= y_i(e_i^S) + \psi(1-\beta)S_i'^S - (r+\delta+\psi)(1-\beta)S_i^S, &(28)\\
w_i'^S(e_i^S, e_i'^S) &= y_i(e_i'^S) - (r+\delta)(1-\beta)S_i'^S, &(29)
\end{aligned}
$$

where the (expected) surplus before and after firm learning is given by[25]

$$
\begin{aligned}
S_i^S &= \frac{(r+\delta)(y_i(e_i^S) - c(e_i^S)) + \psi(y_i(e_i'^S) - c(e_i'^S)) - (r+\delta+\psi)b}{(r+\delta+\psi)(r+\delta+\theta_i^S q(\theta_i^S)\beta)}, &(30)\\
S_i'^S &= \frac{y_i(e_i'^S) - c(e_i'^S) - (b + \theta_i^S q(\theta_i^S)\beta S_i^S)}{r+\delta}, &(31)
\end{aligned}
$$

and, combining (30) with the free-entry condition $q(\theta_i^S)(1-\beta)S_i^S = k$, θ_i^S is given implicitly by

$$
\frac{(r+\delta+\psi)(r+\delta+\theta_i^S q(\theta_i^S)\beta)}{(1-\beta)q(\theta_i^S)} = \frac{(r+\delta)(y_i(e_i^S) - c(e_i^S)) + \psi(y_i(e_i'^S) - c(e_i'^S)) - (r+\delta+\psi)b}{k}. \tag{32}
$$

Finally, the steady-state rate of unemployment for the two types of workers is given by

$$
u_H^S = \frac{\delta\alpha}{\delta + \theta_H^S q(\theta_H^S)} \quad \text{and} \quad u_L^S = \frac{\delta(1-\alpha)}{\delta + \theta_L^S q(\theta_L^S)}. \tag{33}
$$

Equilibrium is characterized by equations (28)-(33) for any given levels of effort before and after firm learning, e_i^S and $e_i'^S$.

Lemma 1. *In separating equilibrium, taking effort levels e_i^S and $e_i'^S$, $i \in \{H, L\}$, as given, an increase in the firm learning rate, ψ, increases expected values of workers and firms, if and only if output net of the cost of effort is greater after firm learning than before: $y_i(e_i'^S) - c(e_i'^S) > y_i(e_i^S) - c(e_i^S)$. Then, the surplus, S_i^S, and the job finding rate*

[24]Results are derived analogously to the case of perfect information, see Appendix B.

[25]Note that, for $\psi = 0$, the expressions for S_i^S and θ_i^S collapse to the expressions of the standard model (compare (15) and (17)).

of workers, $\theta_i^S q(\theta_i^S)$, increase, while the unemployment rate, u_i^S, decreases. And vice versa. If $y_i(e_i'^S) - c(e_i'^S) = y_i(e_i^S) - c(e_i^S)$, then the firm learning rate has no effect on labor market outcomes.

Expected values of workers and firms, $W_{ii}^S(w_i^S(e_i^S, e_i'^S))$, U_i^S and $J_i^S(\pi_i(e_i^S, e_i'^S))$, increase in the surplus before firm learning, S_i^S (see Appendix B). For given effort levels e_i^S and $e_i'^S$, the surplus, S_i^S, in turn, increases in the firm learning rate, ψ, if and only if output net of the cost of effort is greater after firm learning than before. And vice versa.

Let us next consider equilibrium efforts before and after firm learning, e_i^S and $e_i'^S$. After firm learning, worker types are observable. Therefore, wage contracts can be conditioned on effort, which, similarly to the case of perfect information, is chosen to maximize the (post-learning) surplus, given by equation (31),

$$e_i'^S = \arg\max_e \frac{y_i(e) - c(e) - (b + \theta_i^S q(\theta_i^S)\beta S_i^S)}{r + \delta}, \tag{34}$$

where S_i^S is given by (30) and θ_i^S is given by (32), with effort before learning, e_i^S, considered as given.

As a result, optimal effort after firm learning, $e_i'^S$, is equal to the first-best and given implicitly by

$$p_i = \frac{\partial c(e)}{\partial e}. \tag{35}$$

Lemma 2. *In separating equilibrium, efforts after firm learning are equal to first-best levels of effort : $e_H'^S = e_H^*$, $e_L'^S = e_L^*$.*

The first-order condition for the maximization problem (34) is $p_i - \frac{\partial c}{\partial e} - \theta_i^S q(\theta_i^S)\beta \frac{\partial S_i^S}{\partial e} - \beta S_i^S \frac{\partial[\theta_i^S q(\theta_i^S)]}{\partial e} = 0$. Note that $\frac{\partial S_i^S}{\partial e} = 0$ and $\frac{\partial[\theta_i^S q(\theta_i^S)]}{\partial e} = 0$, if and only if $p_i = \frac{\partial c}{\partial e}$. The second-order condition is fulfilled as long as $\frac{\partial^2 c(e)}{\partial e^2}$ is sufficiently large, which is assumed.

From Lemmas 1-2, it follows that an increase in the firm learning rate is beneficial for both workers and firms, if (any given) effort before firm learning deviates from the first-best: $e_i^S \neq e_i^*$.

Before firm learning, firms cannot distinguish between worker types. Since the job arrival rate and starting wage are greater in jobs of type H than in jobs of type L at any given effort e, low-ability workers have an incentive to imitate high-ability workers, unless the required cost of effort is sufficiently large. Assume that, at $e_H = e_H^*$, the incentive constraint (8) for low-ability workers is binding. Then, following standard screening models[26], wage contracts are $(w_i^S(e_i^S, e_i'^S), y_i(e_i^S))$ with outputs $y_i = p_i e_i^S$ and efforts $e_L^S = e_L^*$, e_H^S, where effort of high-ability workers before firm learning, e_H^S, is defined to solve the incentive constraint for low-ability workers with equality, given $e_L^S = e_L^*$.[27] That is, the option value of search for low-ability workers when searching for high-ability jobs is equal to their option value of search when searching for low-ability jobs, $OVS_{LH}^S = OVS_{LL}^S$:

$$\theta_H^S q(\theta_H^S)(W_{LH}^S(w_H^S(e_H, e_H'^S)) - U_L^S) = \theta_L^S q(\theta_L^S)(W_{LL}^S(w_L^S(e_L^S, e_L'^S)) - U_L^S) \tag{36}$$

or, equivalently,

$$\frac{\theta_H^S q(\theta_H^S)}{r + \theta_H^S q(\theta_H^S)}(rW_{LH}^S(w_H^S(e_H, e_H'^S)) - b) = \frac{\theta_L^S q(\theta_L^S)}{r + \theta_L^S q(\theta_L^S)}(rW_{LL}^S(w_L^S(e_L^S, e_L'^S)) - b), \tag{37}$$

where

$$W_{LH}^S(w_H^S(e_H, e_H'^S)) = \frac{(r + \theta_H^S q(\theta_H^S))[(r+\delta)(w_H^S(e_H, e_H'^S) - c(\frac{p_H}{p_L} e_H^S)) + \psi(w_L'^S(e_L^S, e_L'^S) - c(e_L'^S))] + b\delta(r + \delta + \psi)}{(r + \delta + \psi) r (r + \delta + \theta_H^S q(\theta_H^S))},$$

$$W_{LL}^S(w_L^S(e_L^S, e_L'^S)) = \frac{(r + \theta_L^S q(\theta_L^S))[(r+\delta)(w_L^S(e_L^S, e_L'^S) - c(e_L^S)) + \psi(w_L'^S(e_L^S, e_L'^S) - c(e_L'^S))] + b\delta(r + \delta + \psi)}{(r + \delta + \psi) r (r + \delta + \theta_L^S q(\theta_L^S))},$$

[26] Screening as a response to the problem of asymmetric information was first studied by Rothschild and Stiglitz (1976) and Wilson (1977) in the context of insurance markets.

[27] Single-crossing ensures that, if one of the two incentive constraints is fulfilled with equality, then the other constraint is fulfilled strictly (see Appendix C).

$e_L^S = e_L'^S = e_L^*$ as given by (21), $w_L^S(e_L^S, e_L'^S) = w_L'^S(e_L^S, e_L'^S) = w_L^*(e_L^*)$ as given by (14), and $\theta_L^S = \theta_L^*$ as given by (17).[28]

That is, e_H^S is chosen so that a low-ability worker is just indifferent between choosing effort $\frac{p_H}{p_L} e_H$ and being paid starting wage $w_H^S(e_H^S, e_H'^S)$ with probability $\theta_H^S q(\theta_H^S)$, and choosing his first-best effort e_L^* and being paid starting wage $w_L^*(e_L^*)$ with probability $\theta_L^* q(\theta_L^*)$.

There are at most two solutions to (36)[29], and only the maximum solution represents a potential equilibrium. This is because, among any two solutions e_{H1} and e_{H2}, $e_{H1} < e_{H2}$, (with corresponding labor market tightness θ_{H1}^S and θ_{H2}^S) the option value of search of high-ability workers at effort e_{H2} is greater than at effort e_{H1} (so at e_{H1}, they have an incentive to deviate to e_{H2}):

$$\theta_{H2}^S q(\theta_{H2}^S)(W_{HH}^S(w_H^S(e_{H2}, e_H'^S)) - U_H^S) > \theta_{H1}^S q(\theta_{H1}^S)(rW_{HH}^S(w_H^S(e_{H1}, e_H'^S)) - U_H^S) \quad (38)$$

or, equivalently,

$$\frac{\theta_{H2}^S q(\theta_{H2}^S)}{r + \theta_{H2}^S q(\theta_{H2}^S)}(rW_{HH}^S(w_H^S(e_{H2}, e_H'^S)) - b) > \frac{\theta_{H1}^S q(\theta_{H1}^S)}{r + \theta_{H1}^S q(\theta_{H1}^S)}(rW_{HH}^S(w_H^S(e_{H1}, e_H'^S)) - b). \quad (39)$$

To see this, note that equation (36) implies that

$$\begin{aligned} \frac{\theta_L^S q(\theta_L^S)}{r + \theta_L^S q(\theta_L^S}(rW_{LL}^S(w_L^S(e_L^S, e_L'^S)) - b) &= \frac{\theta_{H1}^S q(\theta_{H1}^S)}{r + \theta_{H1}^S q(\theta_{H1}^S)}(rW_{LH}^S(w_H^S(e_{H1}, e_L'^S)) - b) \\ &= \frac{\theta_{H2}^S q(\theta_{H2}^S)}{r + \theta_{H2}^S q(\theta_{H2}^S)}(rW_{LH}^S(w_H^S(e_{H2}, e_L'^S)) - b). \end{aligned}$$

Inequality (39) follows from $c(\frac{p_H}{p_L} e_{H2}) - c(e_{H2}) > c(\frac{p_H}{p_L} e_{H1}) - c(e_{H1})$, $\theta_{H2}^S > \theta_{H1}^S$, and

[28]Note that $OVS_{LL}^S(e_L^*, e_L^*) = OVS_L^*(e_L^*)$.

[29]There may be one or two solutions, or none. See Figure 4 for numerical results.

$w_H'^S(e_{H2}, e_H'^S) - c(e_H'^S) > w_H'^S(e_{H1}, e_H'^S) - c(e_H'^S) > w_L'^S(e_L^S, e_L'^S) - c(e_L'^S)$ (compare Appendix C).

Lemma 3. *In separating equilibrium, low-ability workers choose the first-best effort, $e_L^S = e_L^*$, and high-ability workers choose a (sub-)optimal level of effort greater than or equal to the first best, $e_H^S \geq e_H^*$, before firm learning.*

High-ability workers choose the first-best effort, $e_H^S = e_H^*$, if the incentive constraint for low-ability workers (36) is slack at $e_H = e_H^*$. Otherwise, their effort is greater than the first best, $e_H^S > e_H^*$. The greater the benefit for low-ability workers when imitating high-ability workers, the greater must be the output (and implied worker effort) specified in wage contracts for H-jobs to separate high-ability from low-ability workers.

Lemma 4. *In separating equilibrium, faster firm learning does not affect the effort of low-ability workers; it increases the effort of high-ability workers during the probation period, if $e_H^S > e_H^*$.*

If $e_H^S > e_H^*$, faster firm learning increases the option value of searching for high-ability jobs for low-ability workers, OVS_{LH}^S (see the left-hand side of equation (37)), while leaving their option value of searching for low-ability jobs, $OVS_{LL}^S = OVS_L^*$, unchanged. There are three different channels. First, the relative arrival rate of high-ability jobs increases, since $\theta_H^S q(\theta_H^S)$ increases in ψ, while $\theta_L^S q(\theta_L^S) = \theta_L^* q(\theta_L^*)$ remains unchanged (see Lemmas 1-2). Second, relative starting wages in high-ability jobs increase, since w_H^S increases in ψ[30], while $w_L^S = w_L^*$ is constant. Third, an increase in ψ shortens the probation period, and the net wage of low-ability workers increases after probation.[31] In response to an increase in the benefit of imitation for low-ability workers, the effort of high-ability workers, $e_H^S > e_H^*$, has to increase, which decreases their match surplus.

Figure 2 illustrates an example of a separating equilibrium with an effort of high-ability

[30]This follows from $\partial S_H^S/\partial\psi > 0$ and $\partial(\theta_H^S q(\theta_H^S))/\partial\psi > 0$. The simple proof is available upon request.

[31]To see this, note that $\theta_H^S > \theta_L^S$ (see Appendix C). Therefore, for the equality condition (36) to be fulfilled, e_H must such be that $w_H^S(e_H, e_H'^S) - c(\frac{p_H}{p_L} e_H^S) < w_L^S(e_L^S, e_L'^S) - c(e_L^S) = w_L'^S(e_L^S, e_L'^S) - c(e_L'^S)$.

workers greater than the first-best: $e_H^S = e_{H2} > e_H^*$. It depicts the option values of search of high- and low-ability workers, OVS_{HH}^S, OVS_{LL}^S and OVS_{LH}^S, as functions of worker effort before firm learning, e. Note that the functions OVS_{ii}^S, $i \in \{H, L\}$, are concave and increasing (decreasing) in e, if $p_i > \frac{\partial c(e)}{\partial e}$ ($p_i < \frac{\partial c(e)}{\partial e}$). They attain their maximum at first-best levels of effort, e_i^*, where $OVS_{ii}^S = OVS_i^*$, $i \in \{H, L\}$. In separating equilibrium, $e_L^S = e_L^*$ is the effort of low-ability workers, and e_H^S is the effort of high-ability workers. At e_H^S, the option value of search of low-ability workers when searching for H-jobs is equal to their option value when searching for L-jobs. Among any two solutions to (36), e_{H1} and e_{H2}, where $e_{H2} > e_{H1}$ and $e_{H2} > e_H^*$, only the maximum solution, e_{H2}, constitutes an equilibrium (see above).

Proposition 2 ***Imperfect information: Separating Equilibrium.***

In separating equilibrium, low-ability and high-ability workers are unemployed with probabilities $u_L^S = u_L^*$ and u_H^S, given by (33) and (19), respectively. When employed, their wage contracts are $(w_i^S(e_i^S), y_i(e_i^S))$ before firm learning and $(w_i'^S(e_i'^S), y_i(e_i'^S))$, $i \in \{H, L\}$, after firm learning, respectively, where

$w_H^S(e_H^S, e_H'^S)$, $w_H'^S(e_H^S, e_H'^S)$, and $w_L^S(e_L^S, e_L'^S) = w_L'^S(e_L^S, e_L'^S) = w_i^*(e_i^*)$ are given by (28) and (14),

$e_H^S = \max\left[e_H^*,\ e_H : \theta_H^S q(\theta_H^S)(W_{LH}^S(w_H^S(e_H, e_H^*)) - U_L^S) = \theta_L^S q(\theta_L^S)(W_{LL}^S(w_L^S(e_L^S, e_L^*)) - U_L^S)\right]$,

$e_L^S = e_L'^S = e_L^*$ and $e_H'^S = e_H^*$ as given by (21),

θ_i^S is given by (32),

and $(e_i^S, e_i'^S, w_i^S, w_i'^S, \theta_i^S, u_i^S)$ satisfy the value functions $J_i^S, J_i'^S, V_i^S, W_{ii}^S, W_i'^S, U_i^S, i \in \{H, L\}$, given in (22)-(27), with $V_i^S = 0$.

Corollary 2. *In separating equilibrium, an increase in the firm learning rate, ψ, does not affect the expected match surplus of low-ability workers. It does not affect the expected match surplus of high-ability workers and, in turn, has no effect on aggregate efficiency, if $e_H^S = e_H^*$. If $e_H^S > e_H^*$, firm learning increases the effort of high-ability workers, which* *decreases their expected match surplus, and may, in turn, decrease aggregate efficiency in the labor market.*

Follows from Lemmas 1-4. The net effect of firm learning will be discussed in more detail in sections 5-6.

4.2 Pooling equilibrium

In pooling equilibrium, the expected value functions of firms and workers are given as follows:

$$\begin{aligned} rJ_i^P(\pi^P(\mathbf{e^P})) &= \pi^P(\mathbf{e^P}) + \delta[V_i^P - J_i^P(\pi^P(\mathbf{e^P}))] + \psi[J_i'^P(\pi_i'^P(\mathbf{e^P})) - J_i^P(\pi^P(\mathbf{e^P}))], && (40)\\ rJ_i'^P(\pi_i'^P(\mathbf{e^P})) &= \pi_i'^P(\mathbf{e^P}) + \delta[V_i^P - J_i^{P'}(\pi_i'^P(\mathbf{e^P}))], && (41)\\ rV_i^P &= -k + q(\theta_i^P)[J_i^P(\pi^P(\mathbf{e^P})) - V_i^P], && (42)\end{aligned}$$

$$\begin{aligned} rW_i^P(w^P(\mathbf{e^P})) &= w^P(\mathbf{e^P}) - c(e_i^P) + \delta[U_i^P - W_i^P(w^P(\mathbf{e^P}))] + \psi[W_i'^P(w_i'^P(\mathbf{e^P})) - W_i^P(w^P(\mathbf{e^P}))], && (43)\\ rW_i'^P(w_i'^P(\mathbf{e^P})) &= w_i'^P(\mathbf{e^P}) - c(e_i'^P) + \delta[U_i^P - W_i'^P(w_i'^P(\mathbf{e^P}))], && (44)\\ rU_i^P &= b + \theta_i^P q(\theta_i^P)(W_i^P(w^P(\mathbf{e^P})) - U_i^P), && (45)\end{aligned}$$

where $\mathbf{e^P} = (e_H^P, e_L^P, e_H'^P, e_L'^P)$, and profits, outputs, and wages during the probation period are the same for both types of workers:

$$\pi^P(\mathbf{e^P}) = y^P(e_L^P) - w^P(\mathbf{e^P}),$$

$$y^P(e_L^P) \equiv y_L(e_L^P) = y_H(e_H^P) = p_L e_L^P = p_H e_H^P.$$

Since both types of workers apply for the same jobs, wages are negotiated based on the expected surplus, which is a function of the expected values of firms and workers,

$S^P = J^P(\pi^P(\mathbf{e^P})) - V^P + W^P(w^P(\mathbf{e^P})) - U^P$, where

$$
\begin{aligned}
J^P(\pi^P(\mathbf{e^P})) &= \alpha J^P_H(\pi^P(\mathbf{e^P})) + (1-\alpha) J^P_L(\pi^P(\mathbf{e^P})), &(46)\\
V^P &= \alpha V^P_H + (1-\alpha) V^P_L, &(47)\\
W^P(w^P(\mathbf{e^P})) &= \alpha W^P_H(w^P(\mathbf{e^P})) + (1-\alpha) W^P_L(w^P(\mathbf{e^P})), &(48)\\
U^P &= \alpha U^P_H + (1-\alpha) U^P_L. &(49)
\end{aligned}
$$

Wage bargaining results in starting wages given by[32][33]

$$w^P(\mathbf{e^P}) = y^P(e^P_L) + \psi(1-\beta)(\alpha S'^P_H + (1-\alpha) S'^P_L) - (r+\delta+\psi)(1-\beta) S^P, \tag{50}$$

where

$$S^P = \frac{(r+\delta)(y^P(e^P_L) - \alpha c(e^P_H) - (1-\alpha)c(e^P_L)) + \psi(\alpha(y_H(e'^P_H) - c(e'^P_H)) + (1-\alpha)(y_L(e'^P_L) - c(e'^P_L))) - (r+\delta+\psi)b}{(r+\delta+\psi)(r+\delta+\theta^P q(\theta^P)\beta)}, \tag{51}$$

and $\theta^P = \theta^P_H = \theta^P_L$ is implicitly defined by

$$
\begin{aligned}
\frac{(r+\delta+\psi)(r+\delta+\theta^P q(\theta^P)\beta)}{(1-\beta)q(\theta^P)} = {} & \frac{1}{k}\big[(r+\delta)(y(e^P_i) - (\alpha c(e^P_H) + (1-\alpha)c(e^P_L)) + \quad (52)\\
& + \psi(\alpha(y_H(e'^P_H) - c(e'^P_H)) + (1-\alpha)(y_L(e'^P_L) - c(e'^P_L))) - (r+\delta+\psi)b\big].
\end{aligned}
$$

After firm learning, output, profit and wages are given by

$$y_i(e'^P_i) = p_i e'^P_i,$$

$$\pi'^P_i(\mathbf{e^P}) = y_i(e'^P_i) - w'^P_i(\mathbf{e^P}),$$

[32] See Appendix D for details.

[33] S^P, S'^P_i and θ^P is short-hand notation for $S^P(\mathbf{e^P})$, $S'^P_i(\mathbf{e^P})$, and $\theta^P(\mathbf{e^P})$.

$$w_i'^P(\mathbf{e^P}) = y_i(e_i'^P) - (r+\delta)(1-\beta)S_i'^P, \tag{53}$$

where

$$S_i'^P = \frac{y_i(e_i'^P) - c(e_i'^P) - (b + \theta^P q(\theta^P)\beta S^P)}{r+\delta}. \tag{54}$$

The steady-state rate of unemployment is the same for both types of workers and given by

$$u^P = \frac{\delta}{\delta + \theta^P q(\theta^P)}. \tag{55}$$

Equilibrium is characterized by (50)-(55) for given levels of effort before and after firm learning, e_L^P, $e_H^P = \frac{p_L}{p_H} e_L^P$, $e_L'^P$ and $e_H'^P$.

Lemma 5. *In pooling equilibrium, taking effort levels e_i^P and $e_i'^P$, $i \in \{H, L\}$, as given, an increase in the firm learning rate, ψ, increases expected values of workers and firms, if and only if (expected) output net of the expected cost of effort is greater after firm learning than before: $\alpha y_H(e_H'^P) + (1-\alpha)y_L(e_L'^P) - \alpha c(e_H'^P) - (1-\alpha)c(e_L'^P) > y^P(e_L^P) - \alpha c(e_H^P) - (1-\alpha)c(e_L^P)$. Then, the expected surplus, S^P, and the job finding rate of workers, $\theta^P q(\theta^P)$, increase, while the unemployment rate, u^P, decreases. And vice versa. If $\alpha y_H(e_H'^P) + (1-\alpha)y_L(e_L'^P) - \alpha c(e_H'^P) - (1-\alpha)c(e_L'^P) = y^P(e_L^P) - \alpha c(e_H^P) - (1-\alpha)c(e_L^P)$, then the firm learning rate has no effect on labor market outcomes.*

Expected values of workers and firms, $W^P(w^P(\mathbf{e^P}))$, U^P and $J^P(\pi^P(\mathbf{e^P}))$, increase in the expected surplus, S^P (see Appendix E). For given effort levels e_i^P and $e_i'^P$, the expected surplus, S^P, in turn, increases in the firm learning rate, ψ, if and only if (expected) output net of the expected cost of effort is greater after firm learning than before. And vice versa.

Next, consider equilibrium efforts in pooling equilibrium before and after firm learning, e_i^P and $e_i'^P$.

Lemma 6. *In pooling equilibrium, efforts after firm learning are equal to first-best levels of effort: $e_H'^P = e_H^*$, $e_L'^P = e_L^*$.*

After firm learning, effort is equal to the first-best, $e_i'^P = e_i^*$, analogously to the case of separating equilibrium, see section 4.1.[34]

Lemma 7. *In pooling equilibrium, low-ability workers choose a suboptimal level of effort greater than the first-best, $e_L^P > e_L^*$, and high-ability workers choose a suboptimal level of effort smaller than the first-best, $e_H^P < e_H^*$, before firm learning.*

The effort of low-ability workers before firm learning, e_L^P, is the effort that maximizes the expected match surplus in pooling equilibrium before firm learning:

$$e_L^P = \arg\max_e S^P = \alpha S_H^P(e) + (1-\alpha) S_L^P(e), \tag{56}$$

where $S_H^P(e) = J_H^P(\pi^P(\mathbf{e^P})) - V_H^P + W_H^P(w^P(\mathbf{e^P})) - U_H^P$, $S_L^P(e) = J_L^P(\pi^P(\mathbf{e^P})) - V_L^P + W_L^P(w^P(\mathbf{e^P})) - U_L^P$, $e_H'^P = e_H^*$, $e_L'^P = e_L^*$, $e_H^P = \frac{p_L}{p_H} e$, and $e_L^P = e$. The expected surplus S^P is a weighted average of the expected surplus of low- and high-ability workers. It follows that $e_L^P > e_L^*$ and $e_H^P < e_H^*$, or, using $e_H^P = \frac{p_L}{p_H} e_L^P$, $e_L^* < e_L^P < \frac{p_H}{p_L} e_H^*$ and $\frac{p_L}{p_H} e_L^* < e_H^P < e_H^*$.

Lemma 8. *In pooling equilibrium, faster firm learning does not affect worker effort.*

In pooling equilibrium, worker efforts e_i^P and $e_i'^P$ are chosen to maximize the respective expected surplus, S^P and $S_i'^P$, $i \in \{H, L\}$, both before and after firm learning. They are independent of the rate of firm learning.

Proposition 3 ***Imperfect information: Pooling Equilibrium.***

In pooling equilbrium, low- and high-ability workers are unemployed with probability u^P, given by (55). When employed, their wage contract is $(w^P(\mathbf{e^P}), y^P(e_L^P))$ before firm learning, and $(w_i'^P(\mathbf{e^P}), y_i(e_i'^P))$, $i \in \{H, L\}$, after firm learning, where

$w^P(\mathbf{e^P}) = y^P(e_i^P) - (r+\delta)(1-\beta)S^P$ is given by (50),

[34] The corresponding maximization problem is $e_i'^P = \arg\max_e S_i'^P(e)$, where effort before learning, e_i^P, is considered as given.

$w_i'^P(\mathbf{e^P})$ *is given by (53),*

e_L^P *is given by (56),*

$e_H^P = \frac{p_L}{p_H} e_L^P$,

$e_i'^P = e_i^*$ *as given by (21),*

θ^P *is given by (52),*

and $(e_i^P, e_i'^P, w^P, w_i'^P, \theta^P, u^P)$ *satisfy the value functions* $J^P, J_i'^P, V^P, W^P, W_i'^P, U^P$, $i, j \neq i \in \{H, L\}$ *given in (46)-(49), with* $V^P = 0$.

Corollary 3. *In pooling equilibrium, an increase in the firm learning rate increases the expected match surplus of low- and high-ability workers and, in turn, increases aggregate efficiency in the labor market.*

Follows from Lemmas 5-8.

4.3 Equilibrium existence

Any equilibrium, separating or pooling, exists, if there are no incentives of (high- or low-ability) workers or firms to deviate. Consider a separating equilibrium with job finding rates $\theta_i^S q(\theta_i^S)$ and wage contracts (w_i^S, y_i^S), $(w_i'^S, y_i'^S)$[35], $i \in \{H, L\}$. There is no profitable deviation to a pooling equilibrium with a common job finding rate $\theta^P q(\theta^P)$ and wage contracts (w^P, y^P), $(w_i'^P, y_i'^P)$[36], if $S_H^S \geq S^P$. Then, a separating equilibrium exists. However, if $S_H^S < S^P$ (and, therefore, $S_L^S < S^P$, which follows from $S_L^S < S_H^S$ due to $\theta_L^S < \theta_H^S$[37]), deviation to a pooling contract increases expected values for both types of workers as well as firms, and only a pooling equilibrium exists.[38]

[35] y_i^S and $y_i'^S$ is short-hand notation for $y_i(e_i^S)$ and $y_i(e_i'^S)$.

[36] y^P and $y_i'^P$ is short-hand notation for $y^P(e_L^P)$ and $y_i(e_i'^P)$.

[37] See Appendix C.

[38] Here, unlike in basic screening games without wage bargaining a la Rothschild and Stiglitz (1976), a pooling equilibrium can exist. This is because, with wage bargaining, workers always get a fixed share of the match surplus, and an incentive of high-ability workers to deviate to a separating contract only exists, if $S_H^S \geq S^P$.

5 The role of firm learning

Let us summarize the above results. In separating equilibrium, an increase in the rate of firm learning, ψ, does not affect the expected match surplus of low-ability workers, who choose their first-best effort both before and after firm learning. Similarly, there is no effect on the expected match surplus of high-ability workers, if $e_H^S = e_H^*$ (see Lemmas 1-4):

$$\frac{\partial S_L^S}{\partial \psi} = 0, \qquad\qquad \frac{\partial S_H^S}{\partial \psi}|_{e_H^S = e_H^*} = 0.$$

However, if $e_H^S \neq e_H^*$, firm learning has two countervailing effects on the match surplus of high-ability workers before firm learning, S_H^S. On the one hand, taking e_H^S as given, S_H^S increases in ψ (direct effect, see Lemma 1). On the other hand, e_H^S increases and, therefore, S_H^S decreases in ψ, as imitation incentives of low-ability workers are enhanced (indirect effect, see Lemma 3). In sum, the match surplus, S_H^S (and, therefore, aggregate efficiency in the labor market, $S_H^S + S_L^S$) may increase or decrease, as firms learn faster:

$$\frac{\partial S_H^S}{\partial \psi}|_{e_H^S \neq e_H^*} = \underbrace{\frac{\partial S_H^S}{\partial \psi}}_{>0} + \underbrace{\frac{\partial S_H^S}{\partial e_H^S}}_{<0} \underbrace{\frac{\partial e_H^S}{\partial \psi}}_{>0} \gtreqless 0.$$

In pooling equilibrium, an increase in firm learning unambiguously increases the expected match surplus. This is because both low- and high-ability workers exert suboptimal levels of effort during the probation period but optimal efforts thereafter, and efforts are independent of the rate of firm learning (see Lemmas 5-8):

$$\frac{\partial S^P}{\partial \psi} = \underbrace{\frac{\partial S^P}{\partial \psi}}_{>0} + \underbrace{\frac{\partial S^P}{\partial e_L^P} \frac{\partial e_L^P}{\partial \psi}}_{0} > 0.$$

In addition, firm learning may change whether the expected match surplus of high-ability workers is greater in separating or in pooling equilibrium, $S_H^S \gtreqless S^P$. Then, the existence of a separating (pooling) equilibrium depends on the rate of firm learning.

These results indicate that firm learning may have a non-monotonous effect on aggregate labor market efficiency. The net effect depends on parameter values as well as the functional forms of the matching function and the cost-of-effort function. In section 6 below, I simulate the effect of firm learning on effort and, in turn, on the match surplus of low- and high-ability workers, numerically. I find that the effort of high-ability workers during the probation period is optimal, $e_H^S = e_H^*$, and, therefore, $S_H^S = S_H^* > S^P$, for all values of $\psi \in [0, 1]$. In consequence, the match surplus of both high- and low-ability workers during probation equals the surplus that would obtain, if firms had perfect information about workers' ability from the start, and faster firm learning does not affect labor market efficiency.

6 Quantitative analysis

In this section, I calibrate the model above. I first calculate the option values of search for high- and low-ability workers under perfect and imperfect information. Then, I numerically derive the respective effort levels of high- and low-ability workers, and their corresponding match surplus. This allows me to determine equilibrium existence, and to compare equilibrium values in a labor market with imperfect information to their respective first-best values. Next, I perform quantitative comparative statics exercises by simulating the effects of an increase in the rate of firm learning. Finally, I test the sensitivity of results to the choice of parameter values.

6.1 Parameter values

In order to calculate the impact of the rate at which firms learn about the productivity of workers on efficiency, I use parameter values to match U.S. labor market facts (see Table 1). The model period is chosen to be one year and the discount rate $r = 0.02$

is set at the current annual real interest rate in the U.S. The matching function is assumed to be Cobb-Douglas, $m(u,v) = m_0 u^{\xi} v^{1-\xi}$, where m is the number of jobs formed during one period, m_0 is the matching constant, u is the number of unemployed workers looking for a job and v is the number of vacant jobs; $\xi = 0.5$ is the matching elasticity with respect to the number of unemployed workers.[39] For the cost of effort, I use a quadratic functional form, $c(e) = e^2$. The parameter values and their respective source, as well as the functional forms of the arrival rate for firms (as implied by the matching function), $m_0 q(\theta)$, and the cost of effort, $c(e)$, are summarized in Table 1. Given these, I target the current average unemployment rate in the U.S. of 4%. I also target the unemployment benefit to be 40% of the average wage of employed workers after firm learning, following Shimer (2005). Lastly, I target an average v-u ratio of 0.72 based on Pissarides (2009). I choose the parameter values for the productivities of high- and low-ability workers, p_H and p_L, the unemployment benefit b, the vacancy posting cost k, and the matching constant m_0 that most closely match the three target moments as well as the condition that the expected match surplus of high- and low-ability workers, respectively, (and, therefore, their option value of search) is strictly positive for a non-empty set of effort levels $e > 0$.[40] The firm learning rate $0 < \psi < 1$ is set at 0.4 in the baseline scenario, implying that a firm learns a worker's true type after 2.5 years on average. The next section 6.2 derives the corresponding labor market equilibria under perfect and imperfect information. Changes in the firm learning rate, and their effects on the existence and efficiency of labor market equilibria, are evaluated in section 6.3. The relative productivity of high- and low-ability workers as well as the functional form of the effort cost function are subject to a sensitivity analysis in section 6.4.

[39] Therefore, the arrival rate is $\theta q(\theta) = m_0\theta^{1-\xi} = m_0\theta^{0.5}$ for workers and $q(\theta) = m_0\theta^{-\xi} = m_0\theta^{-0.5}$ for firms.

[40] This results in endogenous variable values close to or equal to their target values: an average unemployment of 4%, an average replacement rate of 0.35, and an average v-u ratio of 0.7.

6.2 Baseline results

Figure 3 plots the option values of search of high- and low-ability workers under imperfect information, $OVS^S_{HH}(e)$, $OVS^S_{LL}(e)$ and $OVS^S_{LH}(e)$, as functions of worker effort before firm learning, e, replicating Figure 2.[41] Note that $OVS^S_{HH}(e)$ and $OVS^S_{LL}(e)$ are concave functions with maximum values at efforts e^*_H and e^*_L, respectively, where $OVS^S_{HH}(e^*_H) = OVS^*_H(e^*_H)$ and $OVS^S_{LL}(e^*_L) = OVS^*_L(e^*_L)$. It can be seen that $e^*_H > e^*_L$, as implied by the output and cost-of-effort functions. The figure also shows that low-ability workers benefit from imitating high-ability workers, $OVS^S_{LH}(e) > OVS^S_{LL}(e)$, only if effort and, therefore, the cost of imitation, $c(\frac{p_H}{p_L}e) - c(e)$, is sufficiently small. As e increases, the cost of imitation increases, such that $OVS^S_{LH}(e)$ is smaller than $OVS^S_{LL}(e)$ for sufficiently large values of e. The maximum level of effort at which imitation pays off for low-ability workers – e_H as defined in the incentive compatibility constraint (36) – turns out to be slightly smaller than e^*_H. In consequence, the incentive contraint of low-ability workers is slack at e^*_H, and high-ability workers do not have to deviate from their first-best in order to separate themselves from low-ability workers. Thus, $e^S_H = e^*_H$ and $S^S_H = S^*_H$. Since $S^*_H > S^P$, it follows that only the separating equilibrium exists. Table 2 summarizes equilibrium values under perfect and imperfect information, showing the pooling equilibrium values in brackets for comparison.

6.3 Firm learning and the labor market

I now analyse equilibrium responses to changes in the rate of firm learning, ψ. The blue line in Figure 4 shows the effort of high-ability workers, e_H, at which the incentive constraint for low-ability workers in separating equilibrium (IC_L) is binding (see equation (36)), as a function of ψ. Note that, if ψ is sufficiently small, low-ability workers never benefit from imitation, so the set of e_H that solve (36) is empty. At a threshold value of ψ of around 0.06, there is exactly one solution for e_H equal to about 0.7. For values

[41] Effort after learning is given and equal to its first-best value, $e'^S_i = e^*_i$.

of ψ greater than that, there are two solutions, of which only the maximum solution constitutes a potential equilibrium (see section 4.1). The figure shows that, for values of ψ above the threshold value, e_H increases in ψ. However, it remains below the first-best effort e_H^* (equal to 1.5), which implies that IC_L is slack at e_H^*. In consequence, high-ability workers do not have to increase their effort to a suboptimally high level to prevent low-ability workers from imitating them. The effort of both high- and low-ability workers, and their respective match surplus, is equal to the first best independently of the rate of firm learning: $e_i^S = e_i^*$ and $S_i^S = S_i^*$, $i \in \{H, L\}$.

6.4 Sensitivity analysis

The above results may be sensitive to the relative productivity of high- and low-ability workers, p_H/p_L, and the functional form of the effort cost function, $c(e)$. An increase in the relative productivity of workers, p_H/p_L, increases the gap between the efforts of high- and low-ability workers in the first best, which increases the difference in the expected match surplus in high- and low-ability jobs after firm learning and, therefore, the benefit of imitation for low-ability workers. However, it also increases the cost of imitation. Assuming an increase in the relative productivity of H- and L-workers from 1.5 to 2, I find that the cost of imitation for low-ability workers increases by more than its benefit (not shown). In consequence, high-ability workers are still not required to increase their effort to a suboptimally high level, and efforts of both types of workers are optimal. Since $S_H^S = S_H^* > S^P$, the separating equilibrium is the only equilibrium, and the labor market is efficient.

Similarly, a decrease in the curvature of the effort cost function increases the gap between the efforts of high- and low-ability workers in the first best and, therefore, the benefit of imitation for low-ability workers. It also decreases the cost of imitation for given levels of effort. However, using a cost function of $c(e) = e^{1.2}$, I find that the cost of imitation outweighs its benefit at the first-best effort of high-ability workers. Therefore, the separating equilibrium is equal to the first best, and firm learning does not affect labor market efficiency.

7 Conclusion

This paper studies the impact of firm learning on labor market efficiency in the presence of both search frictions and information frictions. Firms do not know worker ability at the time of hiring and only gradually learn about it over time. I show that faster firm learning does not necessarily increase labor market efficiency, for two reasons. First, low-ability workers may not have an incentive to imitate high-ability workers despite the fact that information about worker ability is asymmetric. In this case, effort (of both types of workers) is the same as if firms had perfect information about a worker's type from the time of hiring, and the labor market is efficient independently of the rate of firm learning. Second, in case of adverse selection, (high-ability) workers choose an inefficiently high level of effort on the job during probation. After firm learning, they choose the optimal level of effort. This (direct) effect of firm learning increases the expected surplus of worker-firm matches. However, in separating equilibrium, firm learning also enhances imitation incentives of low-ability workers, in turn increasing the initial effort of high-ability workers. This is because firm learning increases the relative arrival rate and expected earnings in high-ability jobs, which are based on both the current and the future expected match surplus. This (indirect) effect of firm learning decreases the expected match surplus in high-ability jobs. Depending on the relative size of effects, faster firm learning may potentially harm labor market efficiency, decreasing the average expected match surplus and increasing unemployment.

Numerical results show that, in the current setting, imitation is too costly for low-ability workers at first-best levels of effort, so firm learning does not affect labor market efficiency. It should be interesting to consider extensions that make imitation feasible.[42] Then, firm learning can be expected to put upward pressure on effort during probation in high-ability jobs, which diminishes efficiency.

[42]Imitation may become feasible, if individual output cannot be perfectly observed, or if individual output is not fully determined by worker ability and effort (but also, for example, by a random element such as 'luck').

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Figures

Figure 1: Perfect information equilibrium.

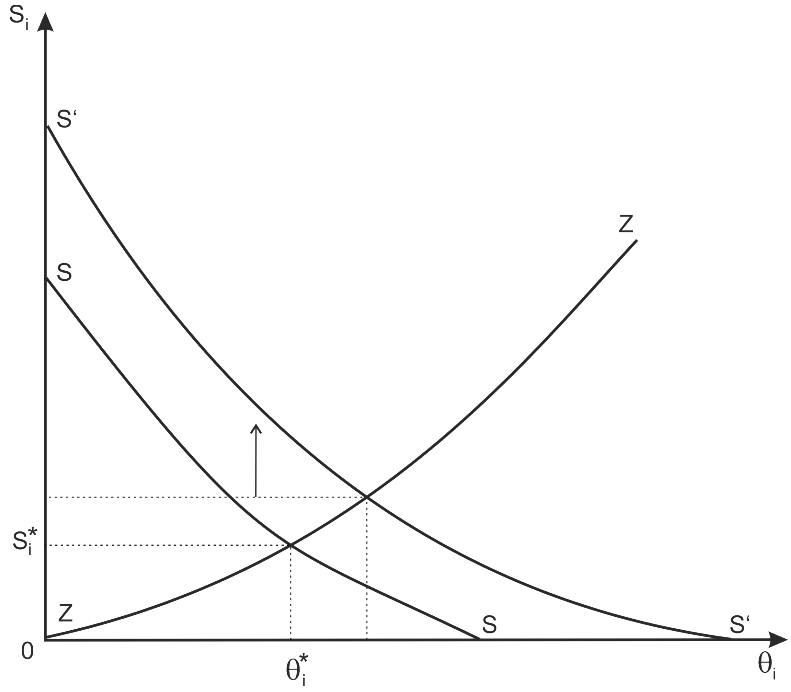


Figure 2: Separating equilibrium.

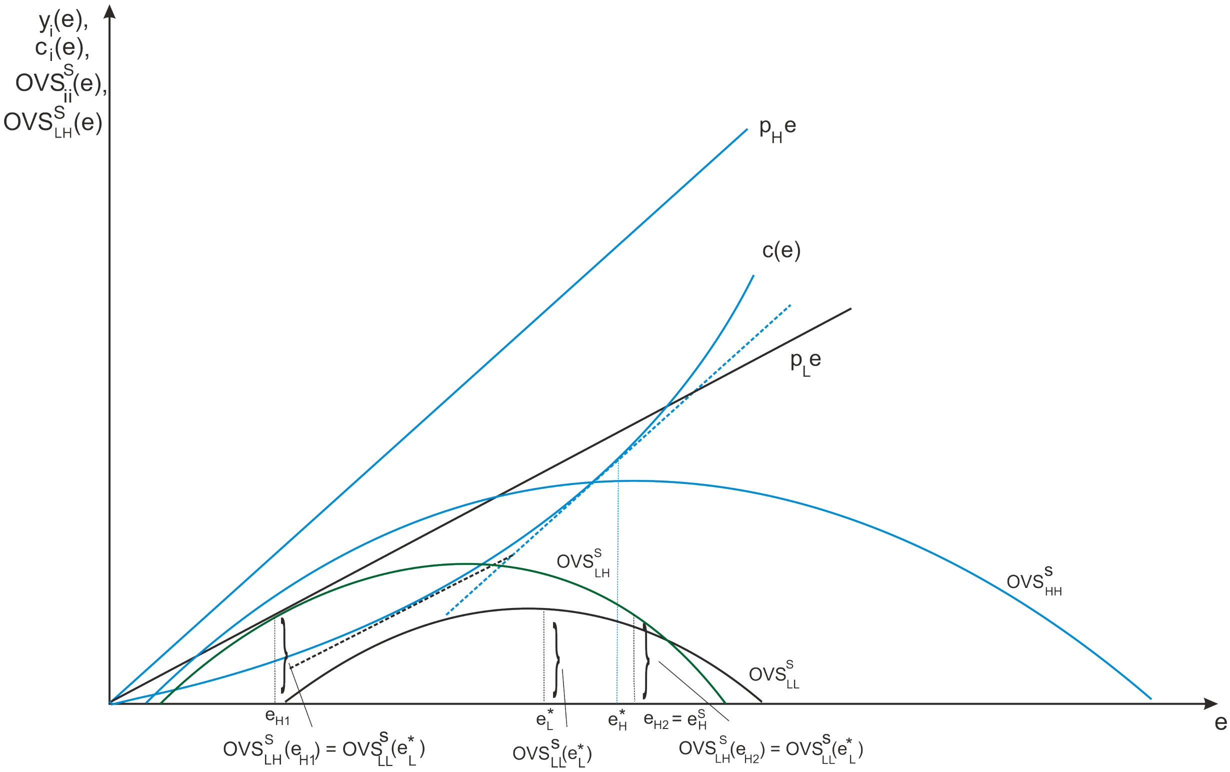


Figure 3: Option values of search

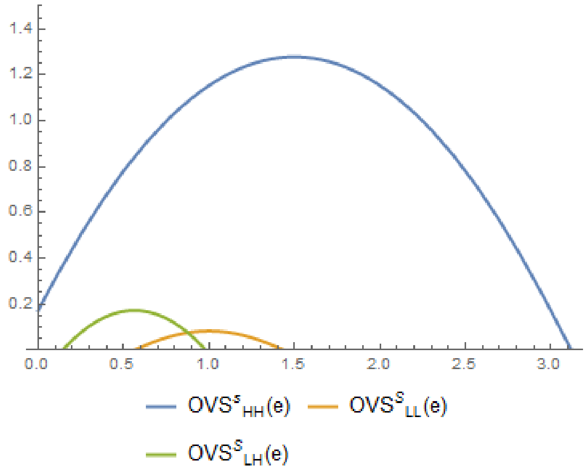
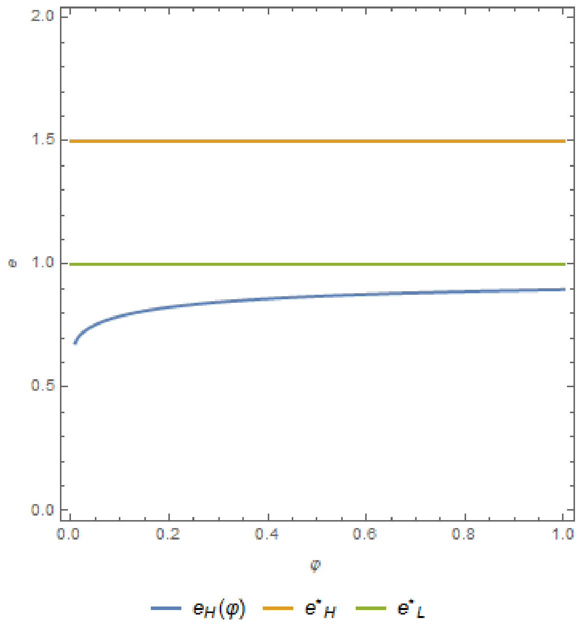


Figure 4: Firm learning and effort on the job



Tables

Table 1: Parameter values and functional forms.

Symbol	Description	Value	Source/Target
$q(\theta)$	Arrival rate for firms	$m_0\theta^{-\xi}$	
$c(e)$	Worker's effort cost function	e^2	
ψ	Firm learning rate	0.4	
r	Discount rate	0.02	U.S. Federal Reserve (2018)
δ	Job separation rate	0.4	Shimer (2005)
α	Share of high-ability workers	0.4	U.S. Labor Statistics (2016)
ξ	Matching function elasticity	0.5	Mortensen Nagypal (2007)
β	Worker's bargaining power	0.5	Mortensen Nagypal (2007)
p_H	Productivity of a high-ability worker	3	Match targets:
p_L	Productivity of a low-ability worker	2	Unemployment rate: 0.04
b	Unemployment benefit	0.9	Replacement rate: 0.4
k	Vacancy posting cost	0.8	v-u ratio: 0.72
m_0	Matching constant	12	$S_H^* > 0$, $S_L^* > 0$

NOTES: (1) An annual separation rate of 0.4 corresponds to the quarterly separation rate of 0.1 in Shimer (2005). It implies that jobs last for about 2.5 years on average. (2) A matching function elasticity of 0.5 is well within the empirically-supported range reported by Petrongolo and Pissarides (2001). (3) The Hosios (1990) condition for socially efficient vacancy posting in the decentralized equilibrium requires that $\beta = \xi$.

Table 2: Labor market equilibria under perfect and imperfect information: $\psi = 0.4$.

Variable	High-ability workers: first-best	separating equilibrium	pooling	Low-ability workers: first-best	separating equilibrium	pooling
Effort on the job: e_i^*, e_i^S, e_i^P	1.5	1.5	(0.85)	1	1	(1.28)
Labor market tightness: θ_i^*, θ_i^S, θ^P	1.59	1.59	(0.56)	0.10	0.10	(0.56)
Match surplus: S_i^*, S_i^S, S^P	0.16	0.16	(0.10)	0.04	0.04	(0.10)
Wage: w_i^*, w_i^S, w^P	4.46	4.46	(2.60)	1.99	1.99	(2.60)
Unemployment rate: u_i^*, u_i^S, u^P	0.01	0.01	(0.04)	0.06	0.06	(0.04)

NOTE: (1) Values of efforts, match surplus, and wages in separating and in pooling equilibrium denote values during probation, respectively.

Appendix

A. Perfect information: Wage bargaining.

Wages are negotiated according to the generalized Nash bargaining solution:[43]

$$w_i^*(e_i^*) = \arg\max[(W_i^*(w_i^*(e_i^*)) - U_i^*)^{\beta}(J_i^*(\pi_i(e_i^*)) - V_i)^{1-\beta}].$$

The corresponding first-order condition is

$$\beta(J_i^*(\pi_i(e_i^*)) - V_i)\frac{\partial W_i^*(w_i^*(e_i^*))}{\partial w_i^*(e_i^*)} = -(1-\beta)(W_i^*(w_i^*(e_i^*)) - U_i^*)\frac{\partial J_i^*(\pi_i(e_i^*))}{\partial w_i(e_i^*)}.$$

Substituting for $\frac{\partial W_i^*(w_i^*(e_i^*))}{\partial w_i^*(e_i^*)} = \frac{1}{r+\delta}$ and $\frac{\partial J_i^*(\pi_i(e_i^*))}{\partial w_i(e_i^*)} = -\frac{1}{r+\delta}$, we derive

$$W_i^*(w_i^*(e_i^*)) = U_i^* + \beta S_i^*,$$

where

$$S_i^* = J_i^*(\pi_i(e_i^*)) - V_i + W_i^*(w_i^*(e_i^*)) - U_i^*,$$

or, using (9) and (11) to substitute for $J_i^*(\pi_i(e_i^*)) = \frac{\pi_i(e_i^*)+\delta V_i}{r+\delta}$ and $W_i^*(w_i^*(e_i^*)) = \frac{w_i^*(e_i^*))-c(e_i^*)+\delta U_i^*}{r+\delta}$,

$$S_i^* = \frac{y_i(e_i^*) - c(e_i^*) - rV_i - rU_i^*}{r+\delta}.$$

Substituting for $rU_i^* = b + \theta_i q(\theta_i)(W_i^*(w_i^*(e_i^*)) - U_i^*) = b + \theta_i q(\theta_i)\beta S_i^*$ and $V_i = 0$ in (7), the surplus can be re-written as

$$S_i^* = \frac{y_i(e_i^*) - c(e_i^*) - b}{r + \delta + \theta_i q(\theta_i)\beta}. \quad (57)$$

[43] It is assumed that the constraints $W_i^*(w_i^*(e_i^*)) - U_i^* \geq 0$ and $J_i^*(\pi_i(e_i^*)) - V_i \geq 0$ are fulfilled.

Using $J_i^*(\pi_i(e_i^*)) = (1-\beta)S_i^*$, the zero-profit condition, $V_i = 0$, can be re-written as

$$q(\theta_i)(1-\beta)S_i^* = k, \tag{58}$$

Equations (57) and (58) determine the equilibrium values S_i^* and θ_i^*. The latter is given implicitly by

$$\frac{r+\delta+\theta_i^* q(\theta_i^*)\beta}{(1-\beta)q(\theta_i^*)} = \frac{y_i(e_i^*) - c(e_i^*) - b}{k}.$$

Using $J_i^*(\pi_i(e_i^*)) = (1-\beta)S_i^*$ in (9) together with $V_i = 0$, we find the equilibrium wage in the case of perfect information:

$$w_i^*(e_i^*) = y_i(e_i^*) - (r+\delta)(1-\beta)S_i^*.$$

B. Separating equilibrium: Wage bargaining.

Wages in separating equilibrium, $w_i^S(e_i^S)$, are negotiated based on the generalized Nash bargaining solution:

$$w_i^S(e_i) = \arg\max[(W_{ii}^S(w_i^S(e_i^S)) - U_i^S)^\beta (J_i^S(\pi_i^S(e_i^S)) - V_i^S)^{1-\beta}],$$

where β is the relative measure of worker bargaining strength.[44]

The corresponding first-order condition is

$$\beta(J_i^S(\pi_i^S(e_i^S)) - V_i^S)\frac{\partial W_{ii}^S(w_i^S(e_i^S))}{\partial w_i^S(e_i^S)} = (1-\beta)(W_{ii}^S(w_i^S(e_i^S)) - U_i^S)\frac{\partial J_i^S(\pi_i^S(e_i^S))}{\partial w_i^S(e_i^S)}. \tag{59}$$

Re-writing (1) as

$$r\frac{\partial J_i^S(\pi_i^S(e_i^S))}{\partial w_i^S(e_i^S)} = -1 - (\delta+\psi)\frac{\partial J_i^S(\pi_i^S(e_i^S))}{\partial w_i^S(e_i^S)}$$

[44] It is assumed that the constraints $W_{ii}^S(w_i^S(e_i^S)) - U_i^S \geq 0$ and $J_i^S(\pi_i^S(e_i^S)) - V_i^S \geq 0$ are fulfilled.

and (4) as

$$r\frac{\partial W_{ii}^S(w_i^S(e_i^S))}{\partial w_i^S(e_i^S)} = 1 - (\delta+\psi)\frac{\partial W_{ii}^S(w_i^S(e_i^S))}{\partial w_i^S(e_i^S)},$$

we can solve for $\frac{\partial W_{ii}^S(w_i^S(e_i^S))}{\partial w_i^S(e_i^S)} = \frac{1}{r+\delta+\psi}$ and $\frac{\partial J_i^S(\pi_i^S(e_i^S))}{\partial w_i^S(e_i^S)} = -\frac{1}{r+\delta+\psi}$. Substituting these in (59), and re-arranging, we derive

$$W_{ii}^S(w_i^S(e_i^S)) = U_i^S + \beta\left[J_i^S(\pi_i^S(e_i^S)) - V_i^S + W_{ii}^S(w_i^S(e_i^S)) - U_i^S\right]. \tag{60}$$

In terms of total expected utility, the worker receives his threat point, U_i^S, plus a share β of the surplus S_i^S, which is defined as

$$S_i^S = J_i^S(\pi_i^S(e_i^S)) - V_i^S + W_{ii}^S(w_i^S(e_i^S)) - U_i^S, \tag{61}$$

or, using (22)-(23), and (25)-(26), to substitute for $J_i^S(\pi_i^S(e_i^S)) = \frac{(r+\delta)\pi_i^S(e_i^S)+\psi\pi_i'^S(e_i'^S))+(r+\delta+\psi)\delta V_i^S}{(r+\delta)(r+\delta+\psi)}$ and $W_{ii}^S(w_i^S(e_i^S)) = \frac{(r+\delta)(w_i^S(e_i^S)-c(e_i^S))+\psi(w_i'^S(e_i'^S)-c(e_i'^S))+(r+\delta+\psi)\delta U_i^S}{(r+\delta)(r+\delta+\psi)}$,

$$S_i^S = \frac{(r+\delta)(y_i(e_i^S)-c(e_i^S)) + \psi(y_i(e_i'^S)-c(e_i'^S)) - r(r+\delta+\psi)(U_i^S+V_i^S)}{(r+\delta)(r+\delta+\psi)}. \tag{62}$$

Using (60), we can re-write (27) as

$$rU_i^S = b + \theta_i^S q(\theta_i^S)\beta S_i^S. \tag{63}$$

Using (63) and the zero-profit condition $V_i^S = 0$ in (62), the surplus can be re-written as

$$S_i^S = \frac{(r+\delta)(y_i(e_i^S)-c(e_i^S)) + \psi(y_i(e_i'^S)-c(e_i'^S)) - (r+\delta+\psi)b}{(r+\delta+\psi)(r+\delta+\theta_i^S q(\theta_i^S)\beta)}. \tag{64}$$

Furthermore, (60) and (61) imply

$$J_i^S(\pi_i^S(e_i^S)) = (1-\beta)S_i^S. \tag{65}$$

Using the free-entry condition, $V_i^S = 0$, in (24), we have

$$q(\theta_i^S)J_i(\pi_i^S(e_i^S)) = k. \tag{66}$$

Substituting (65) in (66), we derive

$$q(\theta_i^S)(1-\beta)S_i^S = k. \tag{67}$$

Equations (64) and (67) are two equations in two unknowns, θ_i^S and S_i^S. Combining them, we get

$$\frac{(r+\delta+\psi)(r+\delta+\theta_i^S q(\theta_i^S)\beta)}{(1-\beta)q(\theta_i^S)} = \frac{(r+\delta)(y_i(e_i^S)-c(e_i^S))+\psi(y_i(e_i'^S)-c(e_i'^S))-(r+\delta+\psi)b}{k}.$$

Using (65) to substitute for $J_i^S(\pi_i^S)$ in (22) together with $V_i^S = 0$, we find the equilibrium wage:

$$w_i^S(e_i^S) = y_i(e_i^S) + \psi\frac{y_i(e_i'^S) - w_i'^S(e_i'^S)}{r+\delta} - (r+\delta+\psi)(1-\beta)S_i^S,$$

where, analogously,

$$w_i'^S(e_i'^S) = y_i(e_i'^S) - (r+\delta)(1-\beta)S_i'^S,$$

so

$$w_i^S(e_i^S) = y_i(e_i^S) + \psi(1-\beta)S_i'^S - (r+\delta+\psi)(1-\beta)S_i^S.$$

C. Separating equilibrium: Incentive constraint of high-ability workers.

In the following, I show that the incentive constraint for high-ability workers is slack at effort e_H^S as described in section 4.1. That is, high-ability workers do not find it profitable to deviate from e_H^S and choose effort $\frac{p_L}{p_H}e_L^S$ to earn starting wages $w_L^S(e_L^S, e_H'^S)$ with probability $\theta_L^S q(\theta_L^S)$ instead of earning starting wages $w_H^S(e_H^S, e_H'^S)$ with probability

$\theta_H^S q(\theta_H^S)$:

$$\theta_H^S q(\theta_H^S)(W_{HH}^S(w_H^S(e_H^S, e_H'^S)) - U_H^S) > \theta_L^S q(\theta_L^S)(W_{HL}^S(w_L^S(e_L^S, e_H'^S)) - U_H^S)$$

or, equivalently,

$$\frac{\theta_H^S q(\theta_H^S)}{r + \theta_H^S q(\theta_H^S)}(rW_{HH}^S(w_H^S(e_H^S, e_H'^S)) - b) > \frac{\theta_L^S q(\theta_L^S)}{r + \theta_L^S q(\theta_L^S)}(rW_{HL}^S(w_L^S(e_L^S, e_H'^S)) - b). \quad (68)$$

To see this, note that incentive constraint for low-ability workers (36) implies that

$$\begin{aligned} &\frac{(\theta_H^S q(\theta_H^S))(r+\delta)}{(r+\delta+\psi)(r+\delta+\theta_H^S q(\theta_H^S))} w_H^S(e_H^S, e_L'^S) - \frac{(\theta_L^S q(\theta_L^S))(r+\delta)}{(r+\delta+\psi)(r+\delta+\theta_L^S q(\theta_L^S))} w_L^S(e_L^S, e_L'^S) = \qquad (69) \\ &-\frac{\theta_H^S q(\theta_H^S)}{r+\theta_H^S q(\theta_H^S)}\left[\frac{(r+\theta_H^S q(\theta_H^S))[(r+\delta)(-c(\frac{p_H}{p_L}e_H^S)) + \psi(w_L'^S(e_L^S, e_L'^S) - c(e_L'^S))] + b\delta(r+\delta+\psi)}{(r+\delta+\psi)(r+\delta+\theta_H^S q(\theta_H^S))} - b\right] \\ &+\frac{\theta_L^S q(\theta_L^S)}{r+\theta_L^S q(\theta_L^S)}\left[\frac{(r+\theta_L^S q(\theta_L^S))[(r+\delta)(-c(e_L^S)) + \psi(w_L'^S(e_L^S, e_L'^S) - c(e_L'^S))] + b\delta(r+\delta+\psi)}{(r+\delta+\psi)(r+\delta+\theta_L^S q(\theta_L^S))} - b\right]. \end{aligned}$$

Using (69), we can re-write (68) as follows:

$$\begin{aligned} &\frac{\theta_H^S q(\theta_H^S)}{(r+\delta+\psi)(r+\delta+\theta_H^S q(\theta_H^S))}\left[(r+\delta)(c(\frac{p_H}{p_L}e_H^S) - c(e_H^S)) + \psi(w_H'^S - c(e_H'^S) - (w_L'^S - c(e_L'^S)))\right] + \\ &-\frac{\theta_L^S q(\theta_L^S)}{(r+\delta+\psi)(r+\delta+\theta_L^S q(\theta_L^S))}\left[(r+\delta)(c(e_L^S) - c(\frac{p_L}{p_H}e_L^S)) + \psi(w_H'^S - c(e_H'^S) - (w_L'^S - c(e_L'^S)))\right] > 0. \end{aligned}$$

This condition holds because $e_H^S \geq e_H^* > e_L^* = e_L^S$, so $c(\frac{p_H}{p_L}e_H^S) - c(e_H^S) > c(e_L^S) - c(\frac{p_L}{p_H}e_L^S)$, $\theta_H^S > \theta_L^S$, and $w_H'^S(e_H^S, e_H'^S) - c(e_H'^S) > w_L'^S(e_L^S, e_L'^S) - c(e_L'^S)$.[45]

D. Pooling equilibrium: Wage bargaining.

Wages in pooling equilibrium, $w^P(e_i^P)$, are negotiated based on the generalized Nash

[45]The latter two follow from the fact that, if $\theta_H^S = \theta_L^S$, then $(r+\delta)(p_H e_H^S - c(e_H^S)) + \psi(p_H e_H'^S - c(e_H'^S)) = (r+\delta)(p_L e_L^S - c(e_L^S)) + \psi(p_L e_L'^S - c(e_L'^S))$ and, therefore, $w_L^S(e_L^S, e_L'^S) - c(e_L^S) = w_H^S(e_H^S, e_H'^S) - c(e_H^S) > w_H^S(e_H^S, e_H'^S) - c(\frac{p_H}{p_L}e_H^S)$. So, for the equality condition (36) to be fulfilled, e_H^S must adjust until $(r+\delta)(p_H e_H^S - c(e_H^S)) + \psi(p_H e_H'^S - c(e_H'^S)) > (r+\delta)(p_L e_L^S - c(e_L^S)) + \psi(p_L e_L'^S - c(e_L'^S))$, which implies that $\theta_H^S > \theta_L^S$ and $w_H'^S(e_H^S, e_H'^S) - c(e_H'^S) > w_L'^S(e_L^S, e_L'^S) - c(e_L'^S)$.

bargaining solution:

$$w^P(e_i^P) = \arg\max[(W^P(w^P(e_i^P)) - U^P)^\beta (J^P(\pi^P(e_i^P)) - V^P)^{1-\beta}].$$

where β is the relative measure of worker bargaining strength.[46]

The corresponding first-order condition is

$$\beta(J^P(\pi^P(e_i^P)) - V^P)\frac{\partial W^P(w^P(e_i^P))}{\partial w^P(e_i^P)} = -(1-\beta)(W^P(w^P(e_i^P)) - U^P)\frac{\partial J^P(\pi^P(e_i^P))}{\partial w^P(e_i^P)}.$$

Substituting for $\frac{\partial W^P(w^P(e_i^P))}{\partial w^P(e_i^P)} = \frac{1}{r+\delta+\psi}$ and $\frac{\partial J^P(\pi^P(e_i^P))}{\partial w^P(e_i^P)} = -\frac{1}{r+\delta+\psi}$ and re-arranging, we derive

$$W^P(w^P(e_i^P)) = U^P + \beta S^P,$$

where

$$S^P = J^P(\pi^P(e_i^P)) - V^P + W^P(w^P(e_i^P)) - U^P,$$

Using (40)-(41), (43)-(44), (46) and (48) to substitute for
$J^P(\pi^P(e_i^P)) = \frac{(r+\delta)\pi^P(e_i^P)+\psi(\alpha\pi_H'^P(e_H'^P)+(1-\alpha)\pi_L'^P(e_L'^P))+(r+\delta+\psi)\delta V^P}{(r+\delta)(r+\delta+\psi)}$ and
$W^P(w^P(e_i^P)) = \frac{(r+\delta)w^P(e_i^P)-(r+\delta)(\alpha c(e_H^P)+(1-\alpha)c(e_L^P))+\psi(\alpha(w_H'^P(e_H'^P)-c(e_H'^P))+(1-\alpha)(w_L'^P(e_L'^P)-c(e_L'^P)))+(r+\delta+\psi)\delta U^P}{(r+\delta)(r+\delta+\psi)}$,
and using $rU^P = b + \theta^P q(\theta^P)\beta S^P$ and the zero-profit condition $V^P = 0$, the surplus is

$$S^P = \frac{(r+\delta)(y(e_i^P) - \alpha c^P - (1-\alpha)c^P) + \psi(\alpha(y_H'^P - c'^P) + (1-\alpha)(y_L'^P - c'^P)) - (r+\delta+\psi)b}{(r+\delta+\psi)(r+\delta+\theta^P q(\theta^P)\beta)}. \tag{70}$$

Furthermore, the zero-profit condition implies

$$q(\theta^P)(1-\beta)S^P = k. \tag{71}$$

[46] It is assumed that the constraints $W^P(w^P(e_i^P)) - U^P \geq 0$ and $J^P(\pi^P(e_i^P)) - V^P \geq 0$ are fulfilled.

Equations (70) and (71) are two equations in two unknowns, θ^P and S^P. Combining them, we get an implicit expression for θ^P:

$$\frac{(r+\delta+\psi)(r+\delta+\theta^P q(\theta^P)\beta)}{(1-\beta)q(\theta^P)} = \\ \frac{(r+\delta)(y(e_i^P)-\alpha c^P-(1-\alpha)c^P)+\psi(\alpha(y_H'^P-c'^P)+(1-\alpha)(y_L'^P-c'^P))-(r+\delta+\psi)b}{k}.$$

Combining $J^P(\pi^P(e_i^P))=(1-\beta)S^P$ and $J^P(\pi^P(e_i^P))=\alpha J_H^P+(1-\alpha)J_L^P=\frac{(r+\delta)\pi^P(e_i^P)+\psi(\alpha\pi'_H(e'^P_H)+(1-\alpha)\pi'_L(e'^{P}_{L}}{(r+\delta)(r+\delta+\psi)}$ we find the equilibrium wage:

$$w^P(e_i^P)=y(e_i^P)+\psi\frac{\alpha(y_H(e'^P_H)-w'^P_H(e'^P_H))+(1-\alpha)(y_L(e'^P_L)-w'^P_L(e'^P_L))}{r+\delta}-(r+\delta+\psi)(1-\beta)S^P,$$

where

$$w'^P_i(e'^P_i)=y_i(e'^P_i)-(r+\delta)(1-\beta)S'^P_i,$$

so

$$w^P(e_i^P)=y(e_i^P)+\psi(1-\beta)(\alpha S'^P_H+(1-\alpha)S'^P_L)-(r+\delta+\psi)(1-\beta)S^P.$$