

Volatility, Information and Stock Market Crashes

by

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Abstract

In this paper, we examine the evolution of the S&P500 returns volatility around market crashes using a Markov-Switching model. We find that volatility typically switches into the high volatility state well before a crash and remains in the high state for a considerable period of time after the crash. These results do not support the view that crashes are due to the resolution of uncertainty (e.g. Romer, 1993), but are consistent with the model in Frankel (2008) where the adaptive forecasts of volatility by uniformed traders result in a crash.

Key words: Stock Market Crash, Volatility, Markov Switching.

JEL codes: C11, D8, G0

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1 Introduction

Stock prices are subject to sudden and large declines. Although market crashes are still puzzling, several theories have been developed to explain asset price crashes. Yet, empirical evaluation and support of these theories is very limited. The purpose of this paper is to provide an empirical assessment of two specific classes of theories of stock market crashes. We focus on the model by Frankel (2008) where adaptive forecasts of volatility play a key role in generating a crash and on the class of theories where crashes occur due to the resolution of uncertainty (Romer, 1993; Caplin and Leahy, 1994; Lee, 1998; Zeira, 1999; Hong and Stein, 2003). To the latter class of models, we will refer to as ‘information’ theories. In Frankel (2008) uniformed traders extrapolate increases in volatility into the future and due to risk aversion, higher volatility results in lower demand for stocks and ultimately a crash. Informational crashes, in contrast, are the result of changes in the assessment of the economic environment, which occur in a discrete way. That is, at some point all remaining uncertainty is resolved instantaneously and consequently asset prices also adjust in a discrete way.

Note that Frankel (2008) and information theories yield rather distinct predictions concerning volatility around market crashes. While in Frankel (2008) volatility increases shortly before a crash, information theories predict that after a crash uncertainty is lower than before the crash. Hence, to the extent that volatility mirrors uncertainty, we should observe lower volatility after a crash. In this paper, we exploit these differences in the implications for the behavior of volatility to empirically evaluate these two alternative theories.

Previous empirical evidence for theories of stock market crashes is mostly based on anecdotal evidence.¹ In this paper, we provide a first formal empirical comparison of these two classes of theories of stock market crashes. More specifically, we examine how the volatility of stock returns evolves before and after crashes, based on a Markov switching model for the volatility of the return on the S&P 500 index using Bayesian methods. This framework allows us to identify volatility states, and also to explore the transition

¹For instance Frankel (2008) argues that the empirical observations that frenzies occur with much lower frequency than crashes is at odds with the predictions of information-based models of crashes.

between states.

Our analysis shows that volatility typically increases well before a crash. Although this result is essentially consistent with both theories, we also find that volatility remains in the high state for a substantial amount of time after crashes occur, which is at odds with the idea that crashes are due to the resolution of uncertainty as advocated in information based theories. If a crash was indeed due to the revelation of information, then one would expect volatility to decline quickly after a crash. However, in Frankel (2008) crashes are associated with higher volatility. Thus, we conclude that the evidence presented in this paper favors the Frankel (2008) model over information models.

The remainder of the paper is structured as follows: Section 2 reviews the theories that we evaluate empirically. Section 3 briefly discusses the data and our empirical model, while Section 4 presents our results. Section 5 summarizes and concludes the paper.

2 Adaptive Volatility Forecasts, Information and Stock Market Crashes

In this section we discuss the theoretical motivation for our analysis. We discuss, on the one hand, the Frankel (2008) model where forecasts of volatility play a key role in generating crashes and, on the other hand, information-based theories of crashes.

In Frankel (2008) a crash is the outcome of the interaction between informed and uninformed traders. Informed traders observe a signal which leads them to lower their demand for stocks. Consequently the price of the stock falls. Uninformed investors believe that the volatility of stock prices is serially correlated and therefore they extrapolate the observed increase in volatility into the future. Moreover, uninformed traders are risk averse and therefore the higher expected volatility induces them to lower their demand for the stock. This reduction in demand generates a crash. Essentially, the mechanism at work in this model resembles a feedback strategy as in De Long et al. (1990), with the difference that in Frankel (2008) uninformed traders extrapolate volatility and not the price itself.

Note that in Frankel (2008) volatility increases slightly before a crash. Although Frankel (2008) is silent about the level of volatility after the crash, it appears plausible that

volatility may remain higher than before the crash for some time. Otherwise, uniformed traders would lower their expectation of volatility and the crash would soon be followed by a boom. In any case, a crash is associated with an increase in volatility.

In information-based theories of stock market crashes, the sudden revelation of information causes a crash. Romer (1993) argues that asset prices may only gradually incorporate information if, for instance, the quality of the information is uncertain. In this model, investors, observe signals about the true payoff of an asset and these signals are either reliable or not. Over time, as trade occurs, investors learn about the quality of the signal, and at some point they are able to fully deduce the quality of the signal. This change in their perception of the distribution of quality occurs discretely and therefore the asset price also changes in a discrete way. Note that in this model, the crash can occur without any recent releases of news and thus, without an obvious reason, since the crash is the result of information aggregation and not new information.

In Caplin and Leahy (1994), the dissemination of private information is constrained since investors do not properly respond to news until some threshold is triggered. When the threshold is reached, behavior changes and the actions of investors reveal a substantial amount of information which may give rise to a large drop in the price as other agents also react to the newly available information.

Lee (1998) argues that transaction costs prevent trades and therefore information is not revealed. Again, a small trigger may cause the accumulated private information to be revealed, resulting in high volatility, even without an accompanying event.

In Zeira (1999) asset prices increase due to a favorable change in the economic environment. However, investors do not know for how long this change will last and they have to use the available information to form a distribution on when the situation ends. In the meantime, dividends increase and so does the stock price. At some point, investors learn that the period of favorable conditions is over and they adjust their expectations of future dividends. Essentially, the distribution over potential end dates for the period of high dividend growth immediately collapses to a point distribution and therefore the stock price crashes.²

²Abreu and Brunnermeier (2003) present a model where traders know that a crash will occur eventually, but not when. In this sense, their model is similar to Zeira (1999). However, in Abreu and

Hong and Stein (2003) present a model where information is not incorporated into prices due to short-sale constraints. So the mechanism that generates a crash in the model in Hong and Stein (2003) shares some similarities with Caplin and Leahy (1994). Due to short-sale constraints, bearish investors do not participate in the market and therefore their information is not reflected in prices. When prices decline, rational arbitrageurs are able to deduce the information of bearish investors, as they may still abstain from buying. Thus, hidden information is suddenly revealed resulting in a crash.

To summarize, the information-based theories surveyed here, suggest that uncertainty is lower after the crash. In fact, the crash is the result of the resolution of uncertainty. To the extent that volatility gives an indication of the degree of uncertainty, we should therefore observe that a stock market crash is followed by a period of low volatility. In Frankel (2008) crashes are associated with an increase in volatility. Thus, the Frankel (2008) model and the information-based theories yield rather different predictions concerning the evolution of volatility before and especially after crashes. In our empirical analysis, we exploit these differences to distinguish between these two classes of theories.

3 Data and Empirical Methodology

In this section we describe the data and our empirical methodology. The purpose of our analysis is to explore volatility states around the dates of crashes and to empirically evaluate the theories of crashes summarized in Section 2.

More specifically, we identify the volatility states by estimating a three-state Markov-switching model for the variance of the daily returns on the S&P500 index. The data used in this paper consist of daily observations of the S&P500 index (closing prices) ranging from January 4, 1928 till March 13, 2009 which give us a total of 20,400 observations. The data are obtained from Bloomberg's online database.

Let y_t be the return on the S&P500 (calculated as the first natural logarithmic differ-

Brunnermeier (2003), traders face an optimal timing problem. They want to ride a bubble as long as it generates returns. A crash finally occurs when sufficiently many traders sell. However, the crash can happen even when uninformative news act as a coordination device. Thus, in Abreu and Brunnermeier (2003) a crash is not necessarily associated with the resolution of uncertainty.

ences):

$$y_t \sim N(0, \sigma_t^2), \quad (1)$$

We assume that the variance σ_t^2 follows a Markov-switching process. More specifically, at any point in time, the variance σ_t^2 is one out of three unobservable states, S_{kt} , where $k = 1, 2, 3$. That is, either in the low state S_{1t} , the medium, S_{2t} of the high state S_{3t} :

$$S_{kt} = 1 \text{ if } S_t = k \text{ and } S_{kt} = 0, \text{ otherwise.} \quad (2)$$

The variance of y_t , is:

$$\sigma_t^2 = \sigma_1^2 S_{1t} + \sigma_2^2 S_{2t} + \sigma_3^2 S_{3t}, \quad (3)$$

where

$$\sigma_1^2 < \sigma_2^2 < \sigma_3^2. \quad (4)$$

The unobserved states, S_t , evolve according to a first-order Markov process with transition probabilities

$$p_{ij} = Pr[S_t = j | S_{t-1} = i], i, j = 1, 2, 3, \text{ where } \sum_{j=1}^3 p_{ij} = 1. \quad (5)$$

To estimate this model, we follow Kim et al. (1998) and Kim and Nelson (1999) and apply a Bayesian framework. Using the Gibbs-sampler we iteratively generate simulated samples from a set of the state variables, $\tilde{S}_T = [S_1, S_2, \dots, S_T]'$, the variances, $\tilde{\sigma}^2 = [\sigma_1^2, \sigma_2^2, \sigma_3^2]$, and the transition probabilities, $\tilde{p} = [p_{11}, p_{12}, p_{21}, p_{22}, p_{31}, p_{32}, p_{33}]'$ by drawing from their joint distributions. To derive the joint posterior density, $p(\tilde{S}_T, \tilde{\sigma}^2, \tilde{p} | \tilde{y}_T)$, where $\tilde{y}_T = [y_1, y_2, \dots, y_T]'$, we assume that the transition probabilities, \tilde{p} , conditional on \tilde{S}_T , are independent of $\tilde{\sigma}^2$ and the data, \tilde{y}_T . This conditioning assumption, allows us to rewrite the joint density as

$$\begin{aligned} p(\tilde{S}_T, \tilde{\sigma}^2, \tilde{p} | \tilde{y}_T) &= p(\tilde{\sigma}^2, \tilde{p} | \tilde{y}_T, \tilde{S}_T) p(\tilde{S}_T | \tilde{y}_T) \\ &= p(\tilde{\sigma}^2 | \tilde{y}_T, \tilde{S}_T) p(\tilde{p} | \tilde{y}_T, \tilde{S}_T) p(\tilde{S}_T | \tilde{y}_T) \\ &= p(\tilde{\sigma}^2 | \tilde{y}_T, \tilde{S}_T) p(\tilde{p} | \tilde{S}_T) p(\tilde{S}_T | \tilde{y}_T). \end{aligned} \quad (6)$$

Based on (6) we iteratively simulate drawings from the joint distribution of all the state variables and the model's parameters, given the data in three, sequential steps: First,

we generate the path of the state variable, \tilde{S}_T , conditional on all the model's unknown parameters, $\tilde{\sigma}^2$ and \tilde{p} , and the data, \tilde{y}_T . Second, we generate the transition probabilities, \tilde{p} , conditional on the path of the state variable. And finally, in a third step, we generate the path of the variances, $\tilde{\sigma}^2$, conditional on \tilde{S}_T and the data, \tilde{y}_T .³ Gibbs-sampling is run such that the first 1000 draws are discarded and the next 10000 draws are recorded. We apply almost uninformative priors for all parameters in the model.⁴

In addition to a simulated set of the state variables, variances, and transition probabilities, we also obtain the so-called 'smoothed probabilities', $Pr[S_t = k | S_{t+1}, \tilde{y}_t]$, $k = 1, 2, 3$, associated with the three states. These smoothed probabilities are the estimated probabilities of the volatility states based on all historical information. Since we will examine the behavior of stock market volatility around crashes, these smoothed probabilities are of particular interest for our analysis.

Table 1 presents the marginal posterior distributions of the parameters. These estimated parameters imply that, on average, the low-, medium- and high-volatility state last 106, 56 and 33 days, respectively.

To investigate the validity of the three-state Markov-switching model, we perform an ARCH LM test on the average of 10000 sets of the standardized S&P500 returns,⁵ to check for remaining heteroscedasticity. The ARCH LM statistic, which tests the null hypothesis of no ARCH effects versus the alternative that the standardized returns follow an ARCH process, is distributed as a χ^2 with q degrees of freedom under the null hypothesis (where q is the number of lags the series is regressed on). We obtain LM statistics of 0.354, 1.237, 2.865 and 3.111 at lags 1 through 4. The associated p-values are 0.24, 0.19, 0.14 and 0.11 respectively. Thus, we cannot reject the null of no ARCH effects in the standardized returns, which suggests that the three-state Markov-switching model provides an appropriate description of the dynamics in the S&P500 return variance.

³For a detailed description of this procedure see Chapter 9 in Kim and Nelson (1999).

⁴The authors thank Chang-Jin Kim and Charles R. Nelson for providing their code.

⁵We standardize returns as $y_t^* = y_t / \sigma_t$, where y_t^* denotes the standardized return.

4 Stock Market Crashes and Volatility States

What we are ultimately interested in, is the behavior of volatility around days characterized by large declines in stock prices. As discussed in Section 2, the Frankel (2008) model suggests that the crashes are accompanied by an increase in volatility. In contrast, information-based theories predict that crashes are the result of the revelation of information. Therefore, one would expect volatility to decline shortly after a crash. Thus, inspecting the volatility dynamics should allow us to evaluate these types of theories empirically.

We focus on the 20 largest declines in stock prices which occurred in the period from January 1, 1928 to March 13, 2009. Table 2 shows the dates as well as the decline of the S&P500 index associated with these crashes. We see from the table that almost all of these declines are of an order magnitude of eight to nine percent. The main exception is October 19, 1987 when the S&P500 declined by approximately 20 percent.⁶ Note also that shortly after the crash on October 19, 1987 we observe another large, albeit less exceptional, decline of 8.28 percent on October 26, 1987.

How does volatility behave around days when stocks decline sharply? Figures 1 to 10 show the smoothed probabilities associated with the three volatility states around the dates of the crashes in our sample. More specifically, the graphs show the smoothed probabilities for periods starting four months prior to a crash and ending four months after a crash. Since some of the crashes occurred within relatively short periods of time, we combine some of the crashes in the same figure.

Figure 1 shows how the probabilities associated with the three states evolve over time around the first three crashes in our sample, which occurred on October 28, 1929, October 29, 1929 and November 6, 1929. We observe a high probability of being in the medium volatility state until about a month prior to the crash on October 28, 1929. Then, we observe a transition to the high volatility state, where the probability of being in the high state increases quickly and substantially. After the crashes at the end of October and beginning of November, the high volatility state persists until January, 1930.

In short, we observe a switch into the high volatility state quite some time before

⁶See Schwert (1990) for a detailed discussion of the 1987 crash and stock price volatility.

the crash, which persists for a substantial period of time after the crash. Note that this pattern does not support information theories as the high volatility state which persists after a crash is hard to interpret as being associated with the resolution of uncertainty. According to Frankel (2008), a crash should be associated with an increase in volatility. Although we see from Figure 1 that volatility started to increase quite some time before the crash, it switched into the high state relatively shortly before the crash. Thus, this pattern is in line with the prediction of the Frankel (2008) model. Overall, the behavior of volatility around the first three crashes in our sample are more in line with adaptive volatility forecasts as the source of the crash than with the resolution of uncertainty.

Qualitatively, the smoothed probabilities of being in one of the three states evolve similarly around the fourth crash on June 16, 1930 as shown in Figure 2, and also around the four crashes between October 5, 1931 and October 10, 1932 displayed in Figure 3. We observe again, that volatility starts to increase some time before the crash and remains in the high regime for a considerable period of time after the crashes.

Figure 4 shows a slightly different pattern before the crashes on June 20, 1933 and June 21, 1933. We see that volatility was in the high state for a considerable period of time, switched into the medium state briefly before the first of these two crashes occurred and switched back into the high state essentially on the day of the crash. Nevertheless, here we also observe that volatility basically remained in the high state after the June 21, 1933.

We see from Figure 5 that in the four months prior to the crash on July 26, 1934, volatility frequently switched between the medium and high states. The last switch into the high state occurred relatively shortly before the crash. After the crash, in September 1934, volatility returned to the medium state. We find again that the crash was associated with higher volatility. Similarly, Figure 6 shows that volatility increased about two months prior to the crash and remained in the high regime after the crash.

Figures 7 and 8 shows again similar patterns around the crashes on May 14, 1940, and September 3, 1946. Volatility switched into the high regime shortly before the respective day the crash occurred. Before the crash on May 14, 1940, volatility switched from the low state into the high state, whereas the increase in volatility was more gradual before

the crash on September 3, 1946. Nevertheless, volatility remained in the high and medium states after the crash.

For the largest crash in our sample, October 19, 1987, Figure 9 shows that volatility increased substantially before the crash. The smoothed probability of being in the low state declined steadily from July 1987 onwards, whereas the smoothed probability of being in the medium volatility state increased. Shortly before the crash, volatility increased again, as we observe an increase in the smoothed probability of being in the high state. Shortly after the October 19 crash, another large decline occurred on October 26, 1987 and volatility remained in the high regime well beyond the second crash in October 1987. In January 1988, volatility declined, but remained in the medium state.

The remaining four crashes in our sample occurred between September 29, 2008 and December 1, 2008. Figure 10 shows that volatility switched in the medium state well before September 29, and switched into the high state at the end of August. After December 1, volatility remained in the high state.

In short, we find that volatility behaves rather similarly around the market crashes in our sample. Volatility increases before crashes and remains relatively high after the crash occurred. In terms of the two classes of theories we evaluate in this paper, our finding that volatility is typically in the high state at the time of a crash is consistent with information-based theories as well as with the adaptive expectations mechanism in Frankel (2008). However, we also find that in almost all cases, volatility remains relatively high for a substantial period of time after crashes occur. Put differently, we find almost no evidence for the hypothesis that a crash accompanies the resolution of uncertainty as suggested by the information view. Nevertheless, the result that volatility increases before a crash and declines only slowly after a crash is essentially in line with Frankel (2008). However, we also find that in many instances, volatility switched into the high state without triggering a crash. This finding is hard to square with the Frankel (2008) model, where the uniformed agents should adapt their expectations also in those instances. Overall, we conclude that although neither of the theories is fully supported by our results, the Frankel (2008) model is more in line with our findings than the information view.

The 20 largest crashes we discussed so far occurred either at the beginning of our sam-

ple, that is mostly in the 1930s, or at the end of the sample, after 1987. Although during the period 1946 to 1986, the S&P500 also declined sharply on several occasions, these declines were simply smaller than the ones associated with the crashes at the beginning and at the end of our sample. Therefore, as an additional analysis, we now analyze the five largest daily declines occurred from January 1, 1946 to December 31, 1986. Table 3 shows the dates and the percentage declines of the five largest crashes during this subsample. As we can see, these five declines in the S&P500 are all of an order of magnitude of five to seven percent and therefore smaller than the crashes described in Table 2.

Figures 11 to 15 show the smoothed probabilities associated with the volatility states around these additional crashes. These figures largely confirm our previous conclusions. Volatility increases before a crash and remains higher then before the crash for an extended period of time. The only exception is the crash on September 11, 1986, when volatility switched from the medium into the high state essentially on the day of the crash, then switched back into the medium state quickly after the crash and then, finally, into the low state. Thus, apart from the crash itself and a brief aftermath, volatility was actually lower after the crash than before. This pattern is consistent with the idea that the crash was associated with a resolution of uncertainty. In this sense, this crash is rather unique in our sample.

To summarize our results, Table 4 shows the number of days before and after the individual crashes for which the variance was in the high state. When we focus on the 20 largest crashes in our sample the upper panel of the table shows that the variance switched into the high state on average 22 days before the crash and remained in the high regime for around 68 days on average after the crash. Looking at the smaller crashes in the lower panel of the table, we see a similar pattern. Here, volatility essentially increases on the day of the crash and remains high for around six days. Thus, although we observe some heterogeneity, especially when taking the smaller crashes during the subsample 1948 - 1986 into account, it appears that crashes are associated with an increase in volatility. Yet, the increase typically occurs already substantially before the crash.

5 Concluding Remarks

Why do stock market crashes occur? That is, why do stock prices decline substantially and unexpectedly during a single day? In this paper we explore this question by empirically comparing two particular theories of stock market crashes.

We find that although crashes are somewhat idiosyncratic, volatility typically increases before or on the day a crash occurs and remains in the high state for a considerable period of time. This result is at odds with the information view of stock market crashes, where a crash is the result of the resolution of uncertainty. However, the result is broadly consistent with the view suggested in Frankel (2008), where uninformed traders make adaptive forecasts of volatility and the interaction between uninformed and informed traders eventually leads to a crash.

In addition to providing an evaluation of the two classes of theories, our result that stock market crashes are typically associated with a rather persistent increase in volatility also has implications for potential macroeconomic effects of market crashes. Several papers argue that the adverse macroeconomic implications of stock market crashes are strongly related to uncertainty. According to Romer (1990) it was primarily the higher uncertainty associated with the stock market crash in 1929 which resulted in the decline in consumption spending and aggregate demand and which ultimately led to the Great Depression. Bloom (2009) also stresses that jumps in uncertainty have implications for the business cycle, as firms may reduce their activity levels after increases in uncertainty (see also Bloom et al., 2007; Bloom, 2007). Since we find that market crashes are indeed associated with lasting increases in volatility, these adverse, macroeconomic consequences of stock market volatility may indeed be pronounced after strong declines in stock prices.

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Table 1: Bayesian Gibbs-sampling approach to a three-state Markov-switching model of heteroscedasticity for S&P500 returns, (January 4, 1928 - March 13, 2009)

Parameter	Posterior			
	Mean	SD	MD	98 percent posterior bands
p_{11}	0.9877	0.0015	0.9878	(0.9837, 0.9910)
p_{12}	0.0118	0.0015	0.0117	(0.0084, 0.0159)
p_{21}	0.0144	0.0019	0.0143	(0.1038, 0.0192)
p_{22}	0.9772	0.0023	0.9772	(0.9715, 0.9822)
p_{31}	0.0009	0.0009	0.0006	(0.0000, 0.0039)
p_{32}	0.0370	0.0057	0.0367	(0.0253, 0.0517)
p_{33}	0.9621	0.0058	0.9624	(0.9473, 0.9741)
σ_1^2	0.0000	0.0000	0.0000	(0.0000, 0.0000)
σ_2^2	0.0001	0.0000	0.0001	(0.0001, 0.0001)
σ_3^2	0.0008	0.0000	0.0008	(0.0008, 0.0009)

Notes: SD and MD denotes Standard deviation and median, respectively.

Table 2: Fifteen Largest Daily Percent Declines in the S&P500 Index, 1928 - 2009

Number	Date	Daily Decline in Percent
1	October 28, 1929	12.94
2	October 29, 1929	10.16
3	November 6, 1929	9.92
4	June 16, 1930	7.64
5	October 5, 1931	9.07
6	August 12, 1932	8.02
7	October 5, 1932	8.20
8	October 10, 1932	8.55
9	July 20, 1933	8.88
10	July 21, 1933	8.70
11	July 26, 1934	7.83
12	October 18, 1937	9.12
13	May 14, 1940	7.47
14	September 3, 1946	9.91
15	October 19, 1987	20.47
16	October 26, 1987	8.28
17	September 29, 2008	8.79
18	October 9, 2008	7.62
19	October 15, 2008	9.03
20	December 1, 2008	8.93

Notes: The declines are ordered by date.

Table 3: Five Largest Daily Percent Declines in the S&P500 Index, 1948 - 1986

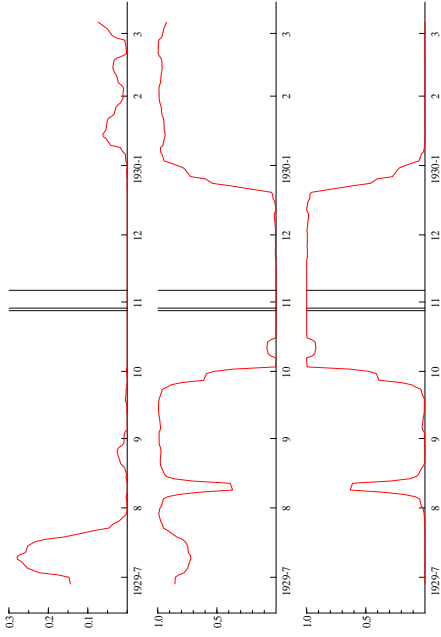
Number	Date	Daily Decline in Percent
1	November 03, 1948	4.61
2	June 26, 1950	5.38
3	September 26, 1955	6.62
4	May 28, 1962	6.68
5	September 11, 1986	4.81

Notes: The declines are ordered by date.

Table 4: Volatility in the High State Before and After Crashes

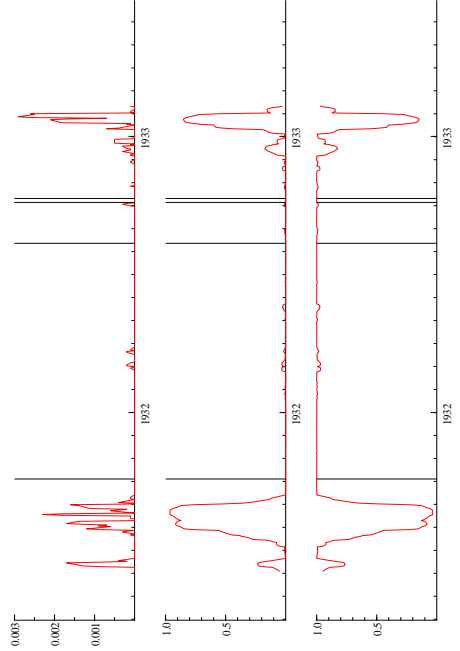
	days in the high state	
	before the crash	after the crash
1928 - 2009		
Oct 28 until Nov 6, 1929	18	31
June 16, 1930	6	41
October 5, 1931 until October 10, 1932	21	62
July 20 and 21, 1933	109	169
July 26, 1934	5	13
October 18, 1937	32	196
May 14, 1940	2	22
September 3, 1946	4	17
October 19 until 26, 1987	4	53
Sept 29, until Dec , 2008	18	74
average	21.9	67.8
1948 - 1986		
November 3, 1948	1	4
June 26, 1950	0	4
September 26, 1955	0	5
May 28, 1962	3	14
September 11, 1986	0	1
average	0.8	5.6

Figure 1: Crashes No. 1 - 3



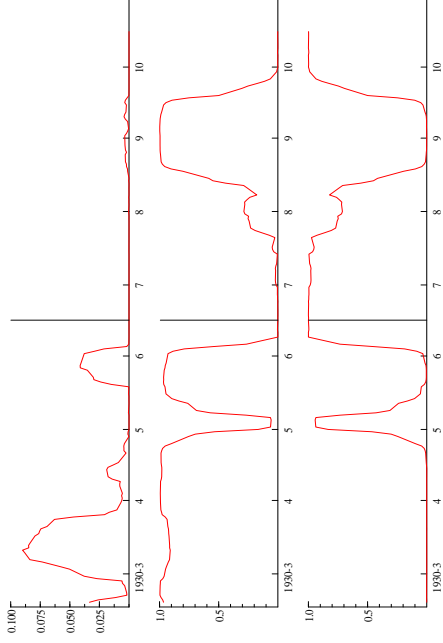
Notes: The top, middle and bottom panels show the smoothed probabilities of the low, medium and high volatility states, respectively. Vertical lines represent crashes on October, 28, 1929, October 29, 1929 and November 6, 1929.

Figure 3: Crashes No. 5 - 8



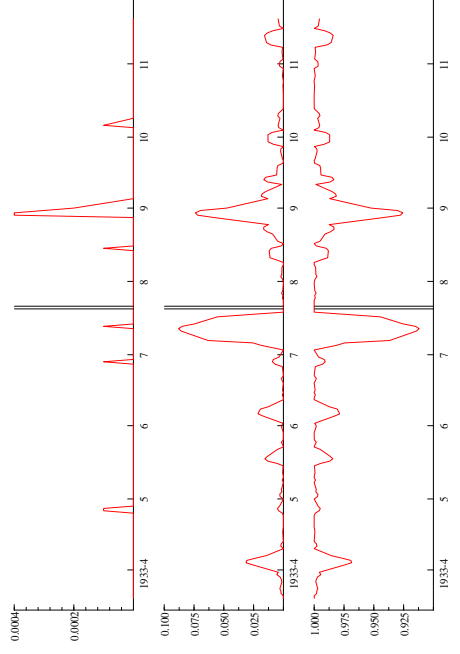
Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical lines represent crashes on October 5, 1931, August 12, 1932, October 5, 1932 and October 10, October 1932.

Figure 2: Crash No. 4



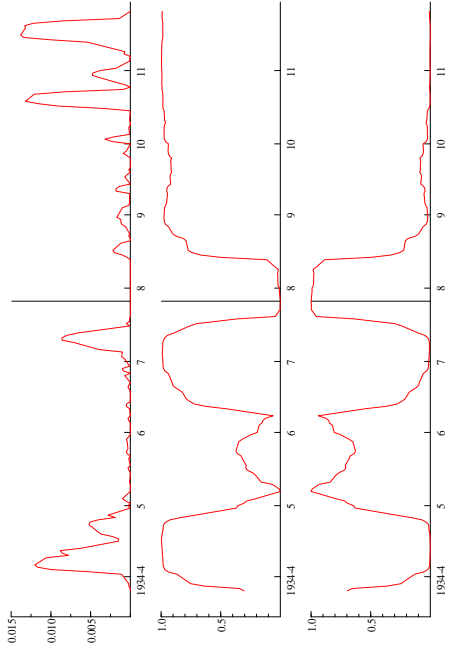
Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical line represents the crash on June 16, 1930.

Figure 4: Crashes No. 9 and 10



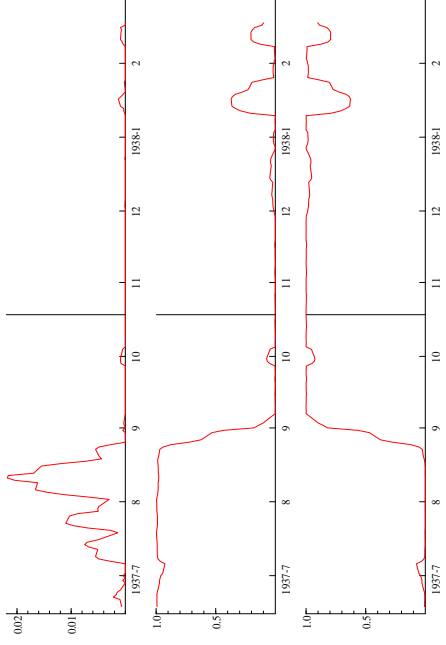
Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical lines represent crashes on June 20, 1933 and June 21, 1933.

Figure 5: Crash No. 11



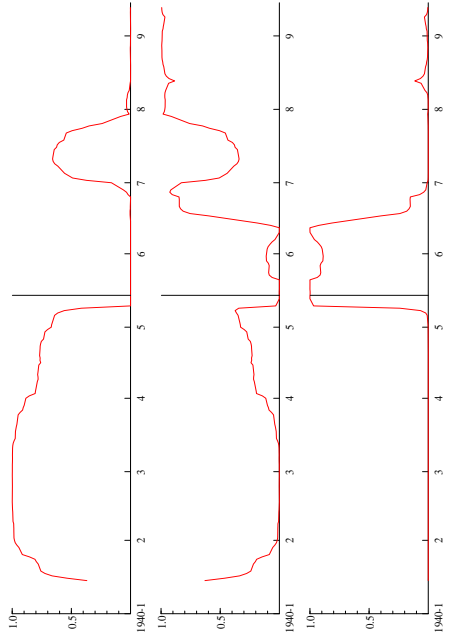
Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical line represents the crash on June 26, 1934.

Figure 6: Crash No. 12



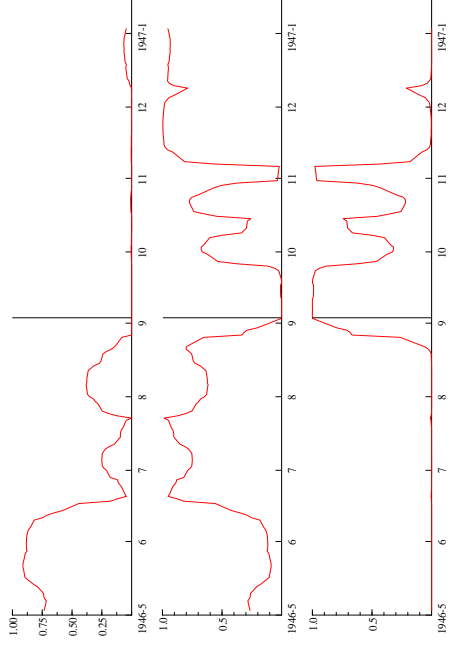
Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical line represents the crash on October 18, 1937.

Figure 7: Crash No. 13



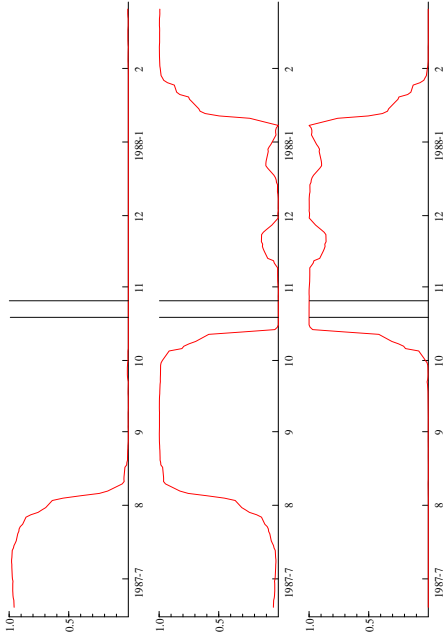
Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical line represents the crash on May 14, 1940.

Figure 8: Crash No. 14



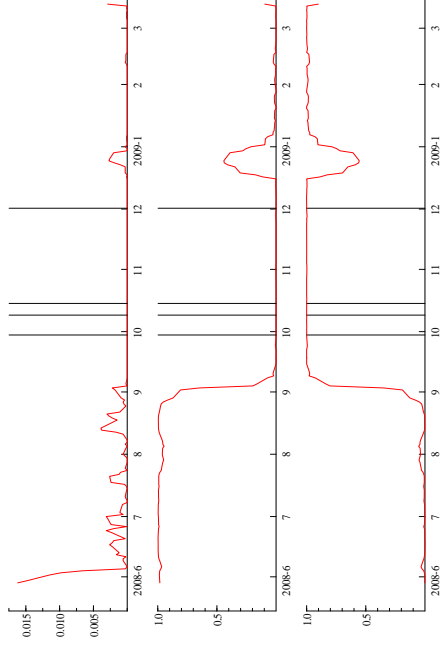
Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical line represents the crash on September 3, 1946.

Figure 9: Crashes No. 15 and 16



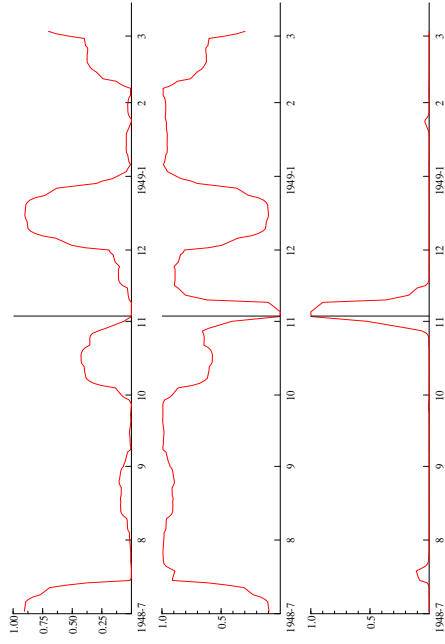
Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical lines represent the crashes on October 19, 1987 and October 26, 1987.

Figure 10: Crashes No. 17 - 20



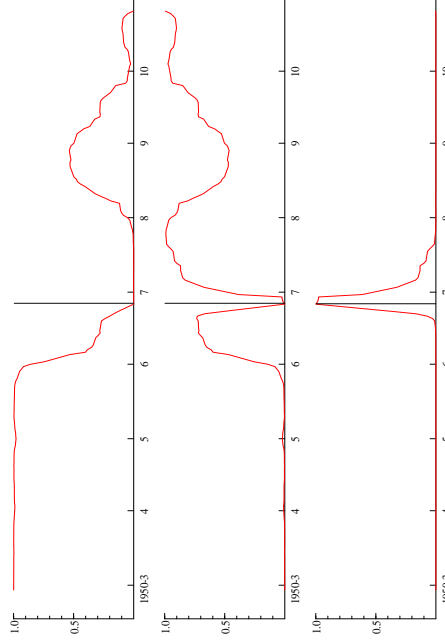
Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical lines represent crashes on September 29, 2008, October 9, 2008, October 15, 2008 and December 1, 2008.

Figure 11: Crash No. 1 during 1948 - 1986



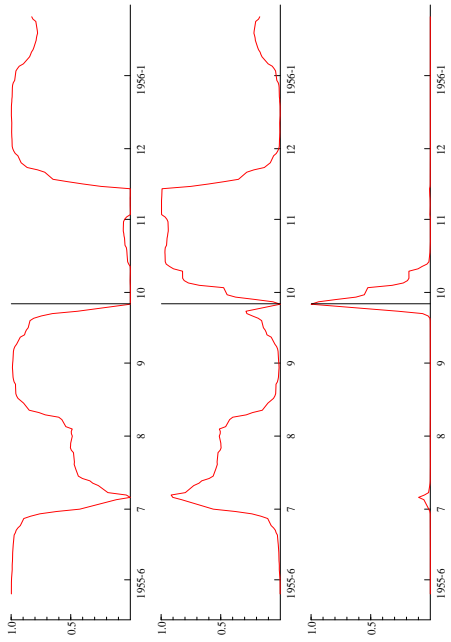
Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical line represents crash on November 3, 1948

Figure 12: Crash No. 2 during 1948 - 1986



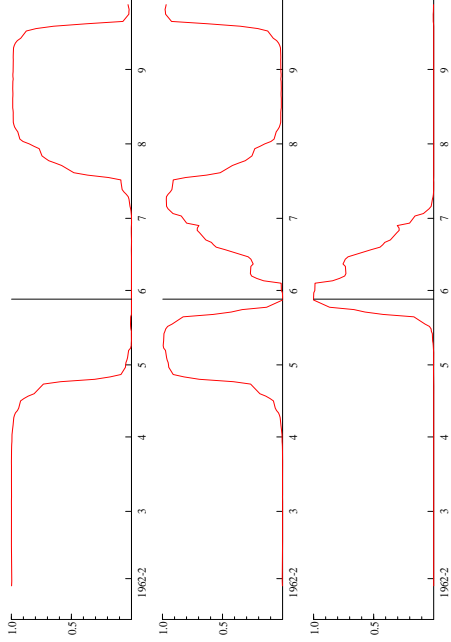
Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical line represents crash on June 26, 1950

Figure 13: Crash No. 3 during 1948 - 1986



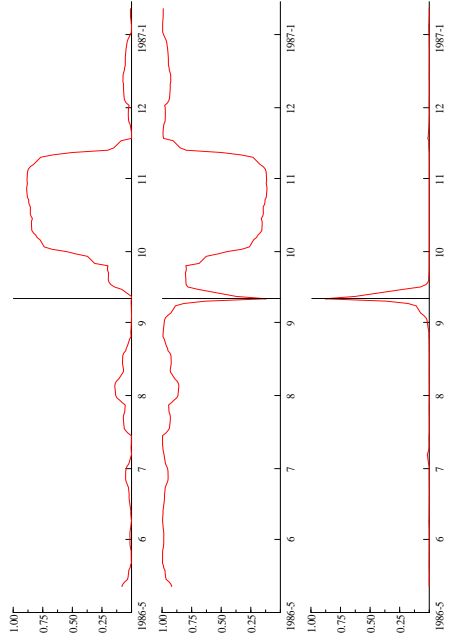
Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical line represents crash on September 26, 1955.

Figure 14: Crash No. 4 during 1948 - 1986



Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical line represents crash on May 28, 1962.

Figure 15: Crash No. 5 during 1948 - 1986



Notes: Top, middle and bottom panels: same as in notes under Figure 1. Vertical line represents crash on September 11, 1986.