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A New Class of Ageing Distributions

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Abstract

The class of new worse/better than used in expectation on average (NWUEA/NBUEA) distributions is proposed. It is defined by the requirement that for each age t a weighted average of mean residual lives be greater/smaller than the life expectancy at birth. The weights are the shares of ages up to t in the steady state stock which results from cohorts whose density is to be classified. The NWUEA class has a second order stochastic dominance property whose first order counterpart defines the class of new worse than used in expectation (NWUE) distributions.

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Distributions of lifetimes frequently have a distinct feature that characterises the underlying ageing process, such as an increasing (or decreasing) failure rate or an increasing (or decreasing) mean residual life. Accordingly several classes of distributions are common in the literature (cf. Klein (1996)). The present note takes as its starting point the class of new worse/better than used in expectation (NWUE/NBUE) distributions: this requires that the mean residual life of an individual of any age be greater/smaller than the life expectancy at birth.

After some notation and basic relationships are set out in section 1, in section 2 we introduce a new class of distributions, termed new worse/better than used in expectation on average (NWUEA/NBUEA). This is an extension of the NWUE/NBUE classes in that only a weighted average of mean residual lives is required to exceed/ fall short of life expectancy at birth.

There have been previous attempts to extend the NWUE /NBUE classes (cf. Klein (1996)): the HNWUE/ HNBUE (harmonic new worse/ better than used in expectation) class does not require that residual life at every age must be greater/smaller than life expectancy at birth, but only that the integral harmonic value of the residual life of an individual of any age t is greater/smaller than life expectancy at birth. The present proposal differs in the averaging procedure: mean residual lives are weighted by the shares of the respective ages in the steady state stock resulting from cohorts with NWUEA/NBUEA life-times. This weighting scheme seems to be attractive in that it can easily be interpreted; moreover it implies that a stochastic dominance relationship between lifetime distributions in the cohort and the stock, which is of the first degree for the NWUE/NBUE class becomes second degree for the NWUEA/NBUEA class. [Throughout this note I only deal with the NWUE case; analogous results for the dual NBUE can easily be derived.]

1. Definitions and preliminaries

Let $T \in [0, z]$ be the time spent in the state of interest (e.g. unemployment, a job, a labour market program etc.) and denote by $f(t)$ and $F(t)$ the density and cumulative distribution function of t in a cohort of elements entering this state at the same time. Then for a steady state stock, i.e. the stock composed of the survivors of succesive identical cohorts, the density $g(t)$ of age, i.e. the time span between entering the state and the date the stock is observed, is given by (cf. Salant (1977)):

$$g(t) = \frac{1 - F(t)}{m} \qquad [1]$$

with

$$m = \int_0^z (1 - F(t))dt = \int_0^z tf(t)dt \qquad [2]$$

As can easily be seen (cf. Layard (1981)) this same density applies to the future durations in the stock, i.e. the time span between the date the stock is observed and exit from the state. The corresponding cumulative distribution function will be denoted by $G(t)$.

The mean residual life at age t $r(t)$ can be expressed in terms of $G(t)$:

$$r(t) = \frac{\int_t^z (u - t) f(u)du}{1 - F(t)} = \frac{\int_t^z (1 - F(u))du}{1 - F(t)} = \frac{m(1 - G(t))}{1 - F(t)} \qquad [3]$$

2. The NWUEA class of distributions

The class of NWUE distributions is defined by

$$\frac{r(t)}{m} \geq 1 \quad \forall t \qquad [4]$$

The requirement in [4] that every single mean residual life $r(t)$ exceed the expected life of a new element $r(0) = m$ can be relaxed. I propose to take a weighted average $\tilde{r}(t)$ over the mean residual lives with the share of each age in the stock as weights:

$$\tilde{r}(t) = \int_0^t \frac{g(u)}{G(t)} r(u)du \qquad [5]$$

Note that $\tilde{r}(z) = b$ where b denotes the mean future duration of the stock.

The class of NWUEA distributions will be defined as

$$\frac{\tilde{r}(t)}{m} \geq 1 \quad \forall t \qquad [6]$$

Both classes can be characterised by a corresponding stochastic dominance relationship:

<u>Proposition 1</u>
The NWUE (NWUEA) property of a distribution of lifetimes in a cohort is equivalent to first (second) degree stochastic dominance of the distribution of future lifetimes in the steady state stock over the distribution of lifetimes in the cohort.

The proof is as follows:
With [3] [4] can be transformed to:

$$F(t) - G(t) \geq 0 \quad \forall t \qquad [4']$$

Correspondingly equ. [6] by inserting [5] and multiplying by $G(t)$ gives

$$\int_0^t g(u)r(u)du - m\int_0^t g(u)du \geq 0 \quad \forall t \qquad [6']$$

Using [1] and [3] leads to:

$$\int_0^t [F(u) - G(u)]du \geq 0 \quad \forall t \qquad [6'']$$

From the stochastic dominance property it follows immediately that NWUE is a subclass of NWUEA:

$$NWUE \subset NWUEA \qquad [7]$$

Proposition 1 has an obvious geometric interpretation: for NWUE distributions the survivor function of the stock $[1-G(t)]$ will lie above the survivor function of the cohort $[1-F(t)]$ throughout. This spells out in more exact terms the intuitive notion that deteriorating chances of exit will give rise to higher persistence in the stock than in the cohort. Davis- Haltiwanger-Schuh (1996) e.g. report higher persistence for existing jobs vis-à-vis newly created jobs. The protracted process of exit out of a stock of unemployed compared to the survivor process for a typical cohort of unemployed has been widely documented (cf. Clark-Summers (1978), OECD (2002)). The class of NWUEA distributions essentially preserves this notion, but relaxes the strict requirement of non-intersection to the condition that the area under the stock survivor function exceed that under the cohort survivor function for all t.

3. Conclusions

We have shown that the NWUE class of distributions has an interesting interpretation in terms of first degree stochastic dominance between the stock and cohort survivor functions. A less restrictive class which requires that the weighted averages of mean residual lives exceed the life expectancy at birth, termed NWUEA, was introduced. This class leads to second degree stochastic dominance between the stock and cohort survivor functions. The NWUEA class includes the NWUE class. Especially for non-parametric analysis this might be helpful to interpret distributions which otherwise would seem irregular.

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