Tax incentives for private life annuities and the social security reform: effects on consumption and on adverse selection

by

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Abstract

In a two-period model with uncertainty about life expectancy, we analyze several measures which are typically included in a social security reform: tax incentives for private life annuities, a cut in the social security benefits and an increase in the social security tax. First, we look at the demand side and study the effects on old-age provision for a given annuity price. It is shown that tax incentives for life annuities indeed stimulate annuity demand, when a partial-equilibrium approach is chosen, where a cut in the supply of public goods to finance the tax incentives does not influence the private consumption choice. In this case, they counteract the negative effects on old-age consumption of the other two reform instruments adopted to maintain long-run solvency of the social security system. However, when considering an increase in the income tax to finance the tax incentives, the positive effect on annuity demand is smaller and may even turn negative for some individuals. Second, we assess the effects of the reform measures on the equilibrium price, in view of an adverse-selection problem in the private annuity market. We find that a cut in the social security benefit rate reduces the adverse selection and consequently the equilibrium price, while an increase in the social security tax raises the equilibrium price. The effect of a tax incentive for life annuities is ambiguous and depends on the degree of risk aversion of the individuals. Adverse selection is mitigated, if the coefficient of relative risk aversion does not exceed a critical value, which is shown to be higher in case that the tax incentives are financed by a reduction in public goods compared to the case when they are financed by an increase in the income tax.

Keywords: annuity market, uncertain lifetime, adverse selection, tax incentives, social security.

JEL codes: D82, D91, G22, H24, H55.

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1. Introduction

In many industrialised countries the social security systems, which are organised according to the pay-as-you-go method, are confronted with the ageing of the population, that is, with the increasing ratio of older to younger people, due to a decrease in fertility and to an increase in life expectancy. As a consequence, many countries around the world have recently undertaken a reform, are in the middle of the reform process or are debating reform options. In the first place, these reforms aim at changing parameters of the social security system in order to maintain long-run solvency of the system. The main policy instruments are an increase of the retirement age, in order to stabilise the dependency ratio, and a cut in the social security benefits. In addition or alternatively, there may be a need to increase the contribution rates.

As a further element, many reforms include tax incentives for the purchase of private life annuities. For example, in Germany a state subsidy for the purchase of life annuities was introduced in 2002 and in Austria an even higher grant exists since 2000. The common argument of politicians for the stimulation of the demand for life annuities trough tax incentives is to prevent a so-called “gap in old-age provision” and old-age poverty, which might otherwise arise as a result of the planned reduction of the social security benefits.[1] This reasoning is in line with the merit-good argument that individuals are myopic, and by this, discount future consumption too much.

In spite of the rapid increase in popularity, there have been no attempts so far to systematically analyse the economic implications of tax incentives for the purchase of life annuities. This study attempts to partially overcome this deficiency by addressing the issue, whether such tax incentives actually produce the desired effect of increasing self-provision for retirement. The answer is not that straightforward as it may appear at first glance due to the following two considerations: First, state subsidies are accompanied by opportunity costs, because somehow they have to be financed out of public budget which in turn may affect individual old-age provision negatively. Second, it is well known that the private market of life annuities fails to be efficient due to the phenomenon of adverse selection, which arises from asymmetric information and leads to poor demand for life annuities. As a result we may see little additional annuity demand due to tax incentives, because they may aggravate the adverse-selection problem. This paper addresses both issues and shows that they may indeed hamper the achievement of the reform objective to increase private old-age provision.

[1] The second policy option to increase self-provision for retirement is mandatory annuitization, which was adopted e.g. in Switzerland and Australia for occupational pensions or is in discussion e.g. in the United States. For a discussion of mandatory retirement plans see e.g. Bateman et. al. (2001).

We formulate a two-period model, where provision for future consumption is guaranteed by a social security system, but individuals can make additional old-age provision voluntarily through private life annuities, and focus on a social security reform, which includes the following potential measures: a cut in social security benefits, an increase in the contribution rate and tax incentives for the purchase of life annuities. Two different methods to finance the tax incentives are considered: The government can either reduce spending for public goods or increase the income tax to keep public budget in balance. Due to the assumption that the reduction in public goods does not influence the private consumption decision of the individuals, the first method of financing allows partial-equilibrium analysis. We regard this as the benchmark scenario, which is compared to the second method of financing, where an increase in the income tax reduces disposable lifetime income, which in turn affects private consumption decisions.

First, we look at the demand side only and study the effects of the reform measures on old-age provision for a given annuity price. It is shown that tax incentives for the purchase of life annuities indeed stimulate annuity demand, when they are financed by a reduction of public goods. By this, they counteract the negative effects on old-age consumption of the other two reform instruments, adopted to assure future financing of the social security system. However, when considering an increase in the income tax to finance the tax incentives, the positive effect on annuity demand is reduced and may even turn negative for some individuals. Thus, only a partial-equilibrium analysis, which neglects the opportunity costs of tax incentives for private old-age provision, supplies evidence that tax incentives are effective.

However, the analysis described so far is based on the assumption of a constant annuity price. The second and more complex issue addressed in this paper concerns the effect of the three reform instruments on the equilibrium price, in view of the adverse-selection problem already mentioned. The fact that the annuity companies cannot distinguish individuals according to their life expectancy induces higher annuity demand of persons with a long life expectancy. As a consequence of the over-representation of annuities bought by high-risk individuals, insurance companies, in order to avoid losses, offer a price which is higher than the actuarially fair price based on the average survival probability of the population.[2] The inefficiently high price induces individuals, especially those with a low life expectancy, to decrease their demand or to drop out of the market. Thus the impact of a social security reform, which aims at a shift towards private old-age provision, on adverse selection is of great relevance: If it aggravates the problem of adverse selection, annuities will become an even less suitable strategy to provide for old-age, i.e. even less annuities will be traded.

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[2] Empirical studies for the well developed US annuity market give evidence that prices are about 7 – 15 % above the fair price due to adverse selection (Walliser, 2000; Mitchell et al., 1999; Friedman and Warshawsky, 1988, 1990). Finkelstein and Poterba (2002) find that adverse selection exists to some similar extent in the voluntary annuity market of the United Kingdom.

Previous work by Walliser (1998, 2000), Abel (1986) and Eckstein, Eichenbaum and Peled (1985) address the problem of whether or not the existence of a social security system improves efficiency of the private annuity market affected by adverse selection. The first two assume price competition in the annuity market, while the latter assumes price and quantity competition. Walliser computes the effects of a privatisation, i.e. the elimination, of the social security system, on the equilibrium price in a calibrated 75-period life-cycle model for a characteristic US cohort.[3] He shows that the elimination of the social security system reduces adverse selection by some small proportion. On the other hand, both the papers of Abel and of Eckstein et al. investigate the effect of a public fully-funded system, which can offer an actuarially fair rate of return, based on population average mortality. Abel shows that the introduction of a fully-funded social security exacerbates adverse selection. In contrast, Eckstein et al. find that in the presence of price and quantity competition in the annuity market, the introduction of a social security system can lead to a Pareto improvement. However, none of these papers investigates tax incentives for the purchase of life annuities, but each restricts attention to a situation, when a social security system is totally eliminated or introduced. We, instead, determine separately the effect of each reform measures on the equilibrium price. This allows us to discuss, which of them should be given priority for reasons of efficiency of the annuity market and which of them is more effective in increasing private old-age provision.

In order to analyse the question, we consider N types of individuals, who differ in their life expectancy. Moreover, we allow for the possibility of heterogeneous income, which is assumed to have a non-negative impact on the survival probability.[4] Further, we assume price competition among the annuity companies, which implies that only a pooling equilibrium, where all individuals pay the same price per unit of annuity payoff, is possible.[5] We find that in this framework a cut in the social security benefits reduces the adverse-selection problem in the private annuity market, while an increase in the social security contributions exacerbates the problem of adverse selection. Both results hold unambiguously and highlight that for reasons of efficiency in the private annuity a cut in the social security benefits should be the preferred reform instrument to assure future financing of the social security system. On the other hand, we find that the effect of a tax incentive for life annuities on adverse selection is ambiguous and

[3] Another simulation study about the privatisation of the social security system is that of Kotlikoff, Smetters and Walliser (1998). The authors compare two methods, mandatory participation in the new privatised system versus allowing the individuals to choose between entering the new privatised system or remaining in social security. Using a large-scale rational-expectations OLG simulation model, they find that both methods lead to long-run gains for all individuals. However, in the short run, the latter method may, despite adverse selection, produce more favourable macroeconomic and distributional outcomes than the former method.

[4] This approach is similar to that chosen by Walliser (1998, 2000). Abel (1986) and Eckstein et al. (1985) considered a model with two types of individuals with identical income and/or wealth.

[5] Price competition is usually adopted for the analysis of annuity markets; see Pauly (1974), Abel (1986), Brugiavini (1993), Walliser (2000), Brunner and Pech (2000, 2002).

depends on the degree of risk aversion of the individuals. The problem of adverse selection is reduced, if the coefficient of relative risk aversion does not exceed a critical value, which is shown that to be higher in case that the tax incentives are financed by a reduction in public goods compared to the case where they are financed by an increase in the income tax. Numerical calculations give some evidence that in the first case adverse selection is reduced for reasonable degrees of risk aversion, while in the second case the possibility that adverse selection is exacerbated cannot be excluded. Obviously this result reduces sharply the appeal of tax incentives as an instrument to stimulate private old-age provision.

This paper is as organized as follows. In Section 2 the basic model is developed and the effects of the three instruments of the social security reform on annuity demand and consumption behaviour are discussed. In Section 3 it is analysed, how adverse selection, i.e. the difference between the equilibrium price and the actuarially fair price, is affected by these reform instruments. Section 4 summarises and concludes the paper.

2. Annuity demand and consumption behaviour

2.1. The basic model

Consider an economy with M individuals who live for a maximum of two periods t = 0,1. In the working period 0, an individual i earns a fixed labour income w_i, which is taxed at a proportional rate τ_w. The tax revenue used to finance government spending for public goods. At the end of the working period 0 the individual retires. Survival to the retirement period 1 is uncertain and occurs with probability π_i, $0 < \pi_i < 1$. Provision for future consumption is guaranteed by a social security system, organized according to the pay-as-you-go method. The individual pays a proportional social security tax rate τ_S on income and receives a benefit $S_i(w_i)$, which depends on income and can thus be regarded to be calculated according to a defined benefit formula.[6]

Preferences of an individual i for lifetime consumption of private goods c_{ti} and public goods g_t are represented by expected utility. That is

$$U_i = u(c_{0i}) + \frac{\pi_i}{1+\alpha} u(c_{1i}) + E\left[v(g_0, g_1)\right], \qquad (1)$$

where α denotes the pure rate of time preference. u is the per-period utility function depending on private consumption, with $u' > 0$, $u'' < 0$ and $\lim_{c \to 0} u'(c) = \infty$. The specification in (1) means that the individual discounts old-age consumption c_{1i} for two reasons, risk aversion and time

[6] Note that this specification means that social security contributions are not deductible from income tax and benefits are tax exempt.

preference. v is the utility function derived from government spending of public goods, which enters (1) in an additively separable fashion. This assumption implies that the choice of private consumption is independent of public spending, which allows a partial-equilibrium analysis. Further note that the individual has no bequest motive.

To smooth consumption over the uncertain lifetime appropriately, the individual can make private old-age provision in addition to the social security system. She can purchase an amount A_i of annuity payouts in the retirement period 1 (conditional on the individual's survival), which the annuity companies supply at a price Q per unit of the payout. Due to the lack of a bequest motive, she will decide for life annuities against holding wealth in the form of bonds, since the former can offer a higher rate of return than the latter (see Yaari, 1965). Individuals may receive a tax incentive for the purchase of life annuities. In this case, the price Q paid to the annuity companies differs from the consumer price $R \equiv Q(1-b)$, with b as the subsidy rate. Note that in this section we take the producer price Q as constant. The budget constraint in each period t = 0,1 reads

$$c_{0i} = w_i(1-\tau_w-\tau_S)-RA_i, \qquad (2)$$

$$c_{1i} = A_i + S_i. \qquad (3)$$

In addition, we assume $A_i \geq 0$. By this, we rule out the possibility that the individual can sell annuities or raise a loan in the working period, whose redemption is guaranteed through a life insurance.[7] The individual decides on her consumption plan over the uncertain lifetime by maximizing (1) subject to (2) and (3). Substituting (2) and (3) into (1) and differentiating with respect to A_i, we obtain the Kuhn-Tucker conditions of this maximization problem,

$$A_i > 0 \quad \text{and} \quad -Ru'(w_i(1-\tau_w-\tau_S)-RA_i)+\frac{\pi_i}{1+\alpha}u'(A_i+S_i)=0, \text{ or} \qquad (4a)$$

$$A_i = 0 \quad \text{and} \quad -Ru'(w_i(1-\tau_w-\tau_S)-RA_i)+\frac{\pi_i}{1+\alpha}u'(A_i+S_i)\leq 0, \qquad (4b)$$

which determine annuity demand $A_i(R, \pi_i, \alpha, \tau_w, \tau_S, w_i, S_i(w_i))$ for an individual i. The interior solution (4a) will hold, as long as the social security benefits S_i are sufficiently small. In case that an individual i is over-annuitized due to high social security benefits, annuity demand is equal to zero.

[7] See Yaari (1965). The same assumption is made by Friedman and Warshawsky (1988, 1990) and Walliser (1998, 2000). Abel (1986), on the other hand, makes sure that all individuals have a positive annuity demand by restricting the range of survival probabilities in the population and the level of the social security benefits.

2.2 The effects of a social security reform on consumption behaviour

In this section we analyse the effects of a social security reform on annuity demand and on consumption behaviour for a given producer price Q. We consider the following reform measures: a cut of the social security benefits, an increase of the contribution rates and tax incentives for the purchase of private life annuities. While the first two measures are implemented to assure future financing of the public pension system[8], the intention of the latter measure is to encourage individuals to compensate the cut of the social security benefits by an increase of private old-age provision. First we show that both, a cut in the social security benefits and an increase in the contribution rate reduce consumption in old-age. Then we investigate whether tax incentives for the purchase of life annuities indeed counteract this effect, where we distinguish between two alternatives concerning the way they are financed: The government can either reduce the expenditures for public goods or can increase the income tax to keep the budget balanced. It turns out that this distinction in the method of financing is crucial for the results.

Lemma 1: A cut in social security benefits increases annuity demand of an individual i, while an increase in the contribution rate reduces annuity demand, i.e. $dA_i/dS_i < 0$ and $dA_i/d\tau_S < 0$.[9] A cut in social security benefits and an increase in the contribution rate reduce consumption in both periods t = 0,1, i.e. $dc_{ti}/dS_i > 0$ and $dc_{ti}/d\tau_S < 0$ for t = 0,1.

Proof: $dA_i/dS_i < 0$ and $dA_i/d\tau_S < 0$ follow directly from implicit differentiation of (4a). Use (2) and (3), together with the formulas for dA_i/dS_i and $dA_i/d\tau_S$, to show that $dc_{ti}/dS_i > 0$ and $dc_{ti}/d\tau_S < 0$, t = 0,1. [10] Q.E.D.

The results of Lemma 1 are illustrated in figures 1 – 2. The consumption possibility curve in (c_{0i},c_{1i})-space is derived by eliminating A_i in the budget constraints (2) and (3) of both periods t = 0,1, which yields

$$c_{1i} = S_i + \frac{w_i(1-\tau_w-\tau_S)}{R} - \frac{c_{0i}}{R} \qquad \text{for } c_{0i} \leq w_i(1-\tau_w-\tau_S).$$

This relation describes the feasible consumption levels for an individual i in both periods under the given social security system (S_i,τ_S) for any annuity level $A_i \geq 0$. If an individual demands no annuities, she has a consumption level of $w_i(1-\tau_w-\tau_S)$ in the working period 0 and a consumption level of S_i in the retirement period 1. Any unit of her net income invested into life

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[8] Since this assumption suffices for our analysis, we do not model explicitly the budget constraint for the social security system.

[9] For shortness, we write A_i instead of $A_i(R, \pi_i, \alpha, \tau_w, \tau_S, w_i, S_i(w_i))$ from now on.

[10] Detailed proofs of this Lemma and also of the Propositions 1 and 2 are available on request.

annuities guarantees her an annuity payoff (and thus additional consumption) of A_i = 1/R in the retirement period. An individual i chooses annuity demand A_i and thus the consumption levels in both periods by maximizing (1) subject to the consumption possibility set. The optimal consumption vector is indicated by C'.

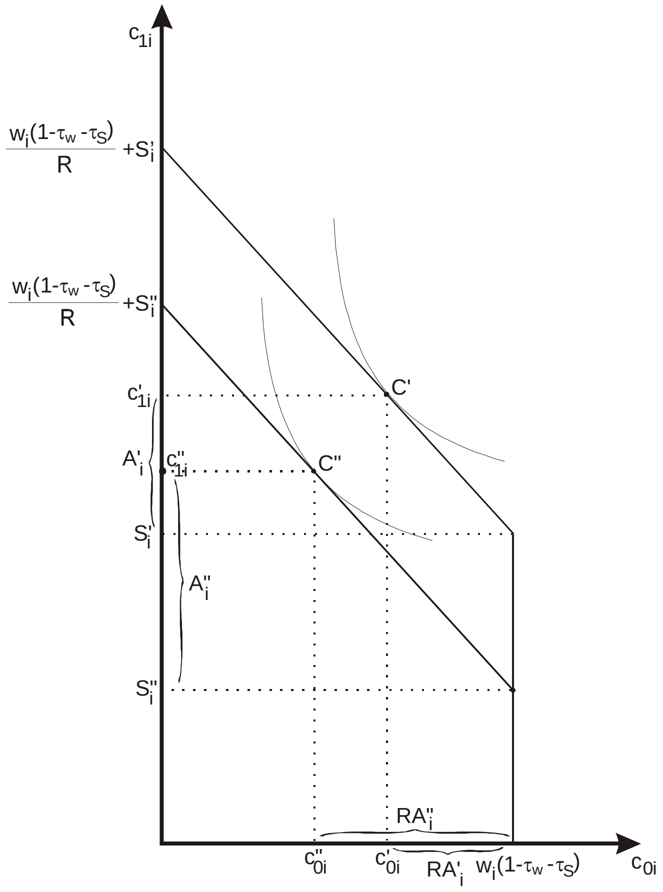

Figure 1: The effect of a cut in social security benefits

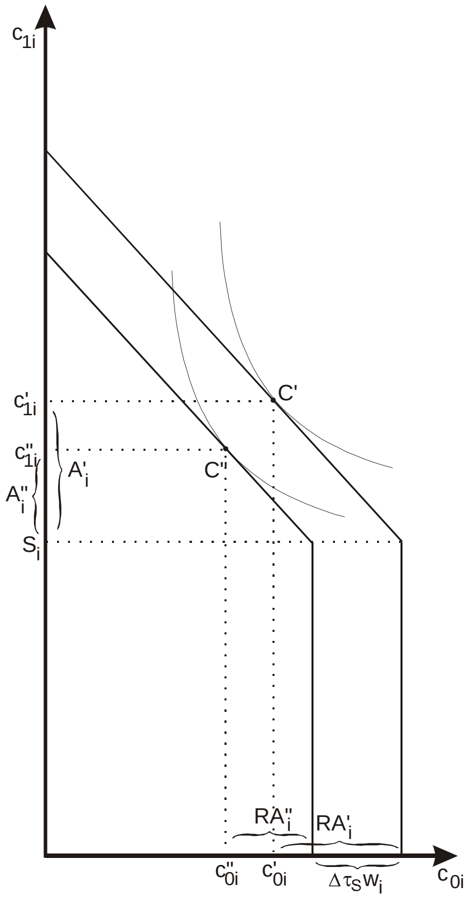

Figure 2: The effect of an increase in the social security tax

A cut in the social security benefits S_i for any given labour income w_i, illustrated in figure 1, shifts the consumption possibility curve downwards (by the amount ΔS_i) and induces an individual i to consume less in both periods (see point C''). For the working period 0, this effect follows immediately from a higher annuity demand. Since the reduction of the social security benefits is larger than the increase in annuities, the overall effect on consumption in the retirement period is negative too. Thus consumption in both periods are normal goods, which is a well-known consequence of additively separable utility functions. For the same reason an increase in the social security tax also reduces consumption in both periods (see point C'' in figure 2). In this case, the consumption possibility curve shifts to the left (by the amount $\Delta\tau_S w_i$). An individual i chooses a lower level of annuities $A_i^{''}$ and, thus, a lower consumption level in the retirement period. She also consumes less in the working period, since the decrease in net income $\Delta\tau_S w_i$ is larger than the reduction in annuity expenditures $R\Delta A_i$.

Finally consider a corner solution, where originally the optimal consumption levels of the individual are $w_i(1-\tau_w-\tau_S)$ in the working period and S_i in the retirement period. Such a situation occurs, if the public pension system offers the individual more than enough for old-age consumption (relatively to consumption in the working period). Obviously, a cut in the social security benefits reduces this over-consumption in old-age and induces the individual to buy annuities, if the cut is sufficiently large. On the other hand, an increase in the contribution rate will raise this relative over-consumption in old-age, thus the best the individual can do is to continue to demand no annuities.

Next we investigate whether tax incentives for the purchase of life annuities indeed counteract the negative effects on old-age consumption of the other two reform instruments. We introduce the public budget constraint in a rudimentary way that will suffice for the upcoming analysis. In each period t, revenues from income tax must balance the government spending for public goods and for the subsidies for life annuities.[11] We denote average labour income by $\bar{w}$ and average annuity demand by $\bar{A}$. Then the public budget constraint can be written as

$$\tau_w \bar{w} - bQ\bar{A} - \frac{g_0}{M} = 0 \,. \qquad (5)$$

First we consider the case that the financing is provided by a cut of the expenditures for public goods. As this reduction in government spending does not influence the private consumption decision of the individuals, we regard this method of public financing as a benchmark scenario, which is then compared to the case, where the tax incentives are financed by an increase in the income tax, which in turn reduces disposable lifetime income and consequently private consumption.

Proposition 1: Assume that the tax incentives for the purchase of private life annuities are financed by a reduction of government spending for public goods. Then a tax incentive for annuities increases annuity demand and thus consumption in the retirement period 1, i.e. $dA_i/db > 0$, $dc_{1i}/db > 0$. The effect of the tax incentive on consumption in the working period 0 is ambiguous and can be characterized as follows: $\frac{dc_{0i}}{db} \gtreqless 0$, if $u'(c_{0i}) \lesseqgtr -\frac{\pi_i A_i}{(1+\alpha)R} u''(c_{1i})$.

Proof: Use $R \equiv (1-b)Q$ and proceed as in the proof of Lemma 1. Q.E.D.

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[11] This means that each generation pays for the subsidies that it receives and for the public goods provided in its working period. However, the public goods which it consumes in its retirement period are paid by the next generation.

The introduction of a subsidy for private life annuities reduces the consumer price R' = Q to R'' = (1 – b)Q. Thus the consumption possibility curve rotates upwards (see figure 3) and individual i moves to the new optimum, point C''. She chooses a higher level of annuities $A_i^{''}$ and thus a higher consumption level $c_{1i}^{''}$ in the retirement period 1. She may also consume more in the working period 0. Such a situation is drawn in figure 3. Annuity expenditures decrease, i.e. $QA_i^{'} > (1-b)QA_i^{''}$, and the positive income effect of the price decrease on c_{0i} outweighs the negative substitution effect.[12]

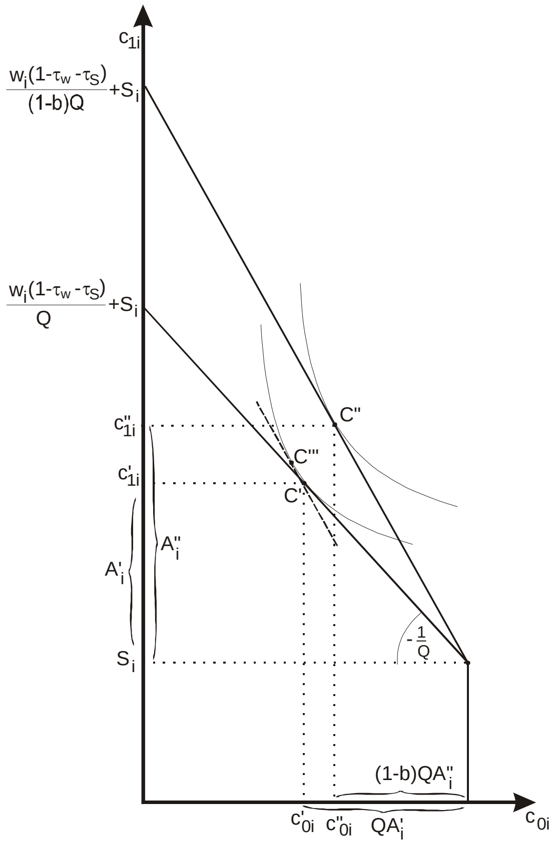

Figure 3: The effect of a tax incentive for life annuities

Consequently, financing tax incentives for private life annuities by a cut in government spending on public goods stimulates annuity demand and indeed counteracts the negative effects of the other two reform measures on old-age consumption. This effect is in accordance with the intention of the policymakers to avoid a gap in old-age provision due to a change in the parameters of the public pension system.

To show how these results depend on the assumption that this reduction of the supply of public goods has no effect on annuity demand we consider another method of financing, namely the increase in the proportional income tax τ_w which, obviously, has the same negative effect on

[12] Note that the opportunity costs of consumption in the working period have increased.

annuity demand as the proportional social security tax τ_S, discussed above. In this case, the positive effect on annuity demand and old-age consumption is smaller and can even turn negative for some individuals, as will be shown in the following Proposition 2. There we confine attention to the introduction of a subsidy, i.e. to a small increase in the subsidy rate b, starting from an initial situation in which b = 0. This increase in b has to be financed by an increase in τ_w, such that the public budget constraint (5) remains fulfilled. Implicit differentiation of (5) yields

$$\left.\frac{d\tau_w}{db}\right|_{b=0} = \frac{Q\bar{A}}{\bar{w}}. \qquad (6)$$

(6) demonstrates that given a small increase Δb of the subsidy rate, the tax rate τ_w has to be increased by $\Delta\tau_w = \Delta bQ\bar{A}/\bar{w}$ to keep the public budget balanced. Thus, each individual i pays an additional income tax in the amount of $\Delta bQ\bar{A}w_i/\bar{w}$. On the other hand, she saves ΔbQA_i in annuity expenditures. As one expects, it is the ratio of $\bar{A}w_i/\bar{w}$ to A_i, which is decisive for the effects of introducing a subsidy for life annuities.[13]

Proposition 2: Assume that a tax incentive for the purchase of private life annuities is introduced and financed by an increase in the income tax rate τ_w, such that the public budget constraint (5) remains fulfilled. Then the effects on annuity demand, consumption in both periods and on indirect utility, are ambiguous and can be characterized as follows:

$$\left.\frac{dA_i}{db}\right|_{b=0} \gtreqless 0 \text{ and } \left.\frac{dc_{1i}}{db}\right|_{b=0} \gtreqless 0, \quad \text{if } u'(c_{0i}) \gtreqless (A_i - \frac{w_i}{\bar{w}}\bar{A})Ru''(c_{0i}),$$

$$\left.\frac{dc_{0i}}{db}\right|_{b=0} \gtreqless 0, \quad \text{if } u'(c_{0i}) \lesseqgtr -(A_i - \frac{w_i}{\bar{w}}\bar{A})\frac{\pi_i}{(1+\alpha)R}u''(c_{1i}).$$

$$\left.\frac{dU_i}{db}\right|_{b=0} \gtreqless 0, \quad \text{if } A_i - \frac{w_i}{\bar{w}}\bar{A} \gtreqless 0$$

Proof: Use (6) and proceed as in the proof of Proposition 1. To determine $dU_i/db|_{b=0}$, use of (1) – (3) and apply the Envelope Theorem. Q.E.D.

To explain the effects of introducing a tax incentive for life annuities, we assume for the moment that labour income is identical for all individuals, i.e. $w_i = \bar{w}$. Then for each individual, the

[13] Note that the RHS of (6) can be interpreted as describing the necessary increase of τ_w under the assumption that the public budget constraint (5) remains fulfilled for a fixed average annuity demand $\bar{A}$, ignoring the second-round effects $bQ\partial\bar{A}/\partial b$ on τ_w for an initial situation b > 0. By this, the results of Proposition 2 can also be regarded as a characterization of first-round effects of an increase of b, where the adjustments of aggregate annuity demand are neglected.

additional tax payment adds up to $\Delta bQ\bar{A}$ (see above), which is traded off against the received subsidy ΔbQA_i. First, let annuity demand of an individual i be equal to the average annuity demand, i.e. $A_i = \bar{A}$. Then she is as well-off as before the introduction of the subsidy; the amount the individual pays in form of a higher income tax corresponds exactly to her maximum willingness-to-pay for the introduction of the tax subsidies. Since only the substitution effect remains, the individual chooses a higher level of annuities and of old-age consumption, and a lower level of consumption in the working period. Such a situation is drawn in Figure 3, where the consumption possibility curve rotates around the indifference curve through the original consumption bundle C'. In this way, the relative price for old-age consumption as well as disposable income decrease such that the individual can afford a consumption bundle C''' that is just indifferent to her original bundle C'.

An individual with above-average annuity demand pays a smaller amount in form of additional income tax than she would be willing to pay in order to receive the subsidy for tax incentives. Thus, she is better off and consumes more in the retirement period. The effect on consumption in the working period is ambiguous and may be positive as well. On the other hand, in case that annuity demand of an individual is below average, the additional payment of income tax exceeds her maximum willingness-to-pay for receiving the subsidy for life annuities.[14] Thus, this individual is worse off. Moreover, she consumes less in the working period, and the effect on annuity demand is ambiguous. It may be optimal for her to reduce also annuity demand and thus old-age consumption.

This result means that for some individuals tax incentives for life annuities may have an opposite effect than intended, if they are financed by an increase in the income tax. In particular, this may be the case, if the income effect of the subsidy is smaller than the offsetting income effect due to an increase in the income tax. Besides the effects on consumption behaviour, financing tax incentives for life annuities by an increase in the income tax has a redistributive impact. Those individuals who pay a higher income tax than they acquire subsidies are made worse-off. In case of identical income these are those with below-average annuity demand. The intuition is obvious: They have to finance not only the subsidies they receive themselves, but also part of the subsidies for those individuals with above-average annuity demand. The latter in turn are better off. However note that with uniform pricing of annuities the individuals with above-average annuity demand are those, who on average live longer (see next section). Consequently, the introduction of tax incentives financed by the income tax would redistribute from individuals with a low life expectancy to individuals with high life expectancy. Note however that similar applies in case that the tax incentives are financed by

[14] In this case, the "new" consumption possibility curve with slope -1/(1-b)Q would lie below the indifference curve through the original optimum C'.

a cut in the supply in public goods: Individuals with high annuity demand benefit more, since they receive more subsidies than those with a lower annuity demand, while the utility loss due to the lower supply of public goods is the same for all individuals.

Altogether, when looking at the combined effect of a social security reform, which introduces tax incentives for private life annuities in addition to a cut in the social security benefits (or/and an increase in the contribution rates), we conclude that it is crucial which assumptions concerning the financing of the tax incentives are considered: Using a partial equilibrium approach, where a cut in the government spending of the public goods has no influence on the private old-age provision, it turn outs that tax incentives stimulate annuity demand and indeed counteract the negative effects of the other two reform measures on old-age consumption. However, assuming an increase in the income tax, which has a negative effect on annuity demand, the positive effect on annuity demand and old-age consumption is lower and can even turn negative for some individuals.

3. Adverse Selection in the private annuity market

3.1 Equilibrium

In the previous section we focused on the demand side and analysed the consumption behaviour of an individual for any given annuity price Q. Now we introduce the supply side to study the equilibrium outcomes, i.e. how the price of annuities adjusts in order to make demand and supply decisions compatible, when there is asymmetric information between the economic agents. For this analysis, we make the following assumptions: The population with a total number of M individuals consists of N groups, $N \leq M$. Each group i = 1,2, …, N, is characterized by a different survival probability π_i and has a share γ_i in total population, with $0 < \gamma_i < 1$ and $\sum_{i=1}^{N} \gamma_i = 1$. The groups are ordered according to their survival probabilities: $0 < \pi_1 < \pi_2 < \ldots < \pi_N < 1$. Besides, we allow for heterogeneous income, where the assumption is made that $w_1 \leq w_2 \leq \ldots \leq w_N$. From this it follows that survival probability and income are not negatively correlated, which is in accordance with empirical evidence.[15] Note that each type i is characterized by the pair (π_i, w_i). The survival probabilities π_i and the group shares γ_i are public information, known by the annuity companies. But it is the private information for each individual to know her type, i.e. her probability of survival. As a consequence, there is an adverse-selection problem in the annuity market: Because of asymmetric information the first-best

[15] It is well known from many empirical studies that the survival probability is correlated positively with income besides other indicators of socioeconomic status, as wealth and education; see e.g. Attanasio and Hoynes (2000), Feinstein (1993), Hurd and McGarry (1995), Lillard and Panis (1998), Lillard and Waite (1995), Menchik (1993) among others.

outcome, in which each type i can buy annuities at her individually fair price according to her survival probability, i.e. $Q_i = \pi_i$, cannot be realized.[16]

Moreover, we assume that insurance companies cannot monitor whether consumers hold annuities from other insurance companies. It follows that there is price competition among the annuity companies.[17] In equilibrium only one selling price Q can exist, which is offered to all individuals. Such a situation is called a pooling equilibrium. Since the annuity companies behave perfectly competitive, the expected profits of a pooling contract with price Q must be equal to zero. It is obvious that the equilibrium price must lie between π_1, the individually fair price for type 1 with the lowest life expectancy, and π_N, the fair price for type N with the highest life expectancy. For any price lower than π_1, annuity companies would suffer a loss and for any price higher than π_N, an annuity company could slightly reduce price and profitably attract all types. We write $A_i(R(Q,b))$ as annuity demand which depends on the consumer price R, which in turn is determined by the producer price Q and the subsidy rate b. P(Q) denotes the expected profits, which we obtain by subtracting total expected annuity payoffs from total revenues[18], i.e.

$$P(Q) \equiv Q\sum_{i=1}^{N}\gamma_i A_i\big(R(Q,b)\big) - \sum_{i=1}^{N}\pi_i\gamma_i A_i\big(R(Q,b)\big). \qquad (7)$$

The equilibrium price $\tilde{Q}$ is implicitly defined by the zero-profit condition $P(\tilde{Q}) = 0$. Let ε_i be the demand share of group i in aggregate annuity demand, defined by $\varepsilon_i \equiv \gamma_i A_i \big/ \sum_{j=1}^{N}\gamma_j A_j$. Then the zero-profit condition can be written as

$$\tilde{Q} - \sum_{i=1}^{N}\pi_i\varepsilon_i\left(R(\tilde{Q},b)\right) = 0\,, \qquad (8)$$

where we assume that $\varepsilon_i(R(\tilde{Q},b)) \geq 0$ for all i = 1,…,N, and $\varepsilon_j(R(\tilde{Q},b)) > 0$ for at least two $j \in \{1,...,N\}$. This assumption ensures that there is an adverse selection problem in the annuity market, because at least two types indeed buy annuities.

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[16] Note that it is usually assumed that income is observable and verifiable. In this case the insurance companies could deduct from the income level to the survival probability. This would put them in the position to differentiate prices on the basis of income. However, this is not common practice in real world. Particularly, as far as we know, in no country price differentiation on the basis of income is utilized by the insurance companies. The following reasons to explain this behaviour come into considerations: First, it might be the case that income is not verifiable and only imperfectly observable, as e.g. in Germany and Austria due to protection of data privacy. Second, insurance companies might worry about that such a practice could not withstand the legal challenge and/or could be sanctioned by costumers, because it offends the social norms and fails the demand for justice. There is some evidence that the latter reason explains why we do not observe price discrimination on racial lines in the U.S, although empirical studies show a lower life expectancy of ethnical minorities like African Americans.

[17] Price competition appears to be a more plausible assumption than price and quantity competition, which requires that individuals can buy only one insurance contract, but generates the possibility of a separating equilibrium (see Rothschild and Stiglitz, 1976; Wilson, 1977).

[18] For simplicity it is assumed that the interest rate is zero, which has no influence on the qualitative results.

Note that the equilibrium price is only unique, if (7), a continuous function of Q, is strictly increasing, i.e. $P'(Q) > 0$. In general however, multiple equilibria are possible; their occurrence depends on the specifics of the per-period utility function u and, accordingly, on the behaviour of the annuity demand functions. Since $\tilde{Q}$ must be a weighted average of all survival probabilities, i.e. $\pi_1 < \tilde{Q} < \pi_N$, we have $P(\pi_1) < 0 < P(\pi_N)$. It follows that there is at least one root of (8) for which $P'(\tilde{Q}) \geq 0$, while those roots of the zero-profit condition (8), for which $P'(Q) < 0$, cannot constitute an equilibrium by the following reasoning: If such a price prevailed, an annuity company could offer a slightly lower price and profitably attract all annuity purchases.[19] Henceforth, we assume that $P'(\tilde{Q}) > 0$.[20]

In a first step, we show that the equilibrium price $\tilde{Q}$ is higher than the actuarially fair price, which corresponds to the average survival probabilities $\sum_{i=1}^{N} \gamma_i \pi_i$ of the individuals. This is due to the adverse-selection effect: The fact that individuals have more information about their survival probability than annuity companies induces higher annuity demand of those individuals with long life expectancy. As a consequence of this over-representation of annuities bought by high-risk individuals, insurance companies offer a price which is higher than the fair price in order to avoid losses.

Lemma 2: Consider a price $\tilde{Q}$ which, together with $A_i \geq 0$ for i = 1,…,N and $A_j > 0$ for at least two $j \in \{1,...,N\}$, fulfils the zero- profit condition (8). The equilibrium price $\tilde{Q}$ is higher than the actuarially fair price, characterized by $\bar{Q} \equiv \sum_{i=1}^{N} \gamma_i \pi_i$, if $A_j \geq A_i$ for all j > i and $A_j > A_i$ for some j > i.

Proof: We determine the difference $\tilde{Q} - \bar{Q}$. By use of (8) and $\bar{Q} \equiv \sum_{i=1}^{N} \gamma_i \pi_i$, we obtain

$$\tilde{Q} - \bar{Q} = \sum_{i=1}^{N} \pi_i (\varepsilon_i - \gamma_i) . \tag{9}$$

Inserting $\varepsilon_i \equiv \gamma_i A_i \Big/ \sum_{j=1}^{N} \gamma_j A_j$, one gets from (9)

$$\tilde{Q} - \bar{Q} = \frac{1}{\bar{A}} \sum_{i=1}^{N} \pi_i \gamma_i \left(A_i - \sum_{j=1}^{N} \gamma_j A_j \right), \tag{10}$$

where $\bar{A} \equiv \sum_{j=1}^{N} \gamma_j A_j$. Because $\sum_{i=1}^{N} \gamma_i = 1$, (10) can be written as

$$\tilde{Q} - \bar{Q} = \frac{1}{\bar{A}} \sum_{i=1}^{N} \sum_{j=1}^{N} \pi_i \gamma_i \gamma_j \left(A_i - A_j \right), \tag{11}$$

[19] See also Abel (1986), Walliser (1998).

[20] It can be easily shown that for the CRRA per-period utility function (17), profits P(Q) are strictly concave for a coefficient of relative risk aversion $\rho = 1$ (logarithmic utility). In this case there is a unique equilibrium price $\tilde{Q}$ between π_1 and π_N and $P'(\tilde{Q}) > 0$.

which can be rearranged to

$$\tilde{Q} - \bar{Q} = \frac{1}{\bar{A}} \sum_{j=1}^{N} \sum_{i=1}^{j} (\pi_j - \pi_i)\gamma_i\gamma_j(A_j - A_i)\,. \tag{12}$$

Since for any two types j > i, $\pi_j > \pi_i$, the RHS of (12) is positive, if $A_j \geq A_i$ for all j > i and $A_j > A_i$ for some j > i. Q.E.D.

Lemma 2 has demonstrated that the equilibrium price $\tilde{Q}$ is above the actuarially fair price $\bar{Q}$, if the following condition holds: Some high-risk types demand more annuities than any type with a lower risk and none of the former demands less annuities than the latter.[21] In a next step, we give attention to this condition. Under the assumption that types differ only in their survival probability, but have identical income, the result that individuals with a higher life expectancy demand more annuities, holds unambiguously. This has been shown and employed in various contributions about adverse selection; see e.g. Abel (1986), Eckstein et. al (1985), Rothschild and Stiglitz (1976). In a framework where individuals may differ also in their income, Walliser (1998) obtained the same (unambiguous) result, given the assumption of fixed social security benefits, which means that they do not depend on income. However, this issue becomes more complex, if the more plausible case that social security benefits vary with income is taken into account. This issue is investigated in the next two Lemmas.

Lemma 3: Assume that $A_i, A_j > 0$, $i, j \in \{1,...,N\}$. For any consumer price R, an individual of type i chooses a lower annuity demand than any individual of type j > i, where $\pi_j > \pi_i$ and $w_j \geq w_i$, if

$$\frac{\pi_i}{1+\alpha} u''(c_{1i}) \frac{\partial S_i(w_i)}{\partial w_i} \geq R(1-\tau_w - \tau_S)u''(c_{0i})\,. \tag{13}$$

Proof: First, we show that $dA_i/d\pi_i > 0$. The effect of a marginal change in the survival probability π_i, is determined by implicit differentiation of the first-order condition for annuity demand (4a) with respect to π_i as

$$\frac{dA_i}{d\pi_i} = -\frac{\partial^2 U_i / \partial A_i \partial \pi_i}{\partial^2 U_i / \partial A_i^2}\,. \tag{14}$$

[21] Note that $\tilde{Q} - \bar{Q} > 0$ holds even in the case that only one type buys annuities at the equilibrium price $\tilde{Q}$ (see (12)). Obviously this must be type N, who is then charged a price $\tilde{Q} = \pi_N$ (see (8)). This price corresponds to the individually fair price of group N. However, due to adverse selection the individually fair prices of any other type i ≠ N will not be offered, since then type N would buy the annuity at the lower price π_i, i ≠ N, and the annuity companies would suffer a loss.

Since the denominator of the RHS of (14) is negative due to the second-order condition of the maximization problem, $dA_i/d\pi_i$ has the same sign as the numerator of the RHS of (14). The latter reads

$$\frac{\partial^2 U_i}{\partial A_i \partial \pi_i} = \frac{1}{1+\alpha} u'(c_{1i}), \tag{15}$$

which is positive.

Next, we determine dA_i/dw_i by implicit differentiation of (4a). Since the denominator is the same as in (14) and the numerator is

$$\frac{\partial^2 U_i}{\partial A_i \partial w_i} = -R(1-\tau_w-\tau_S)u''(c_{0i}) + \frac{\pi_i}{1+\alpha} u''(c_{1i}) \frac{\partial S_i(w_i)}{\partial w_i}, \tag{16}$$

dA_i/dw_i is nonnegative, if (13) is fulfilled.

Consider annuity demand $A_i(w_i,\pi_i) > 0$ and $A_j(w_j,\pi_j) > 0$ for $i < j$, where $\pi_i < \pi_j$ and $w_i \leq w_j$, given that (13) is fulfilled. In this case, $dA_i/d\pi_i > 0$ and $dA_i/dw_i \geq 0$, and it follows that $A_i(w_i, \pi_i) < A_j(w_j, \pi_j)$. Q.E.D.

Note from the first part of the proof (see (15)) that, as already mentioned, a higher survival probability induces higher annuity demand, given that different risk-types have identical income. Moreover, in case of heterogeneous income, it is condition (13), which implies $dA_i/dw_i \geq 0$ and, thus, guarantees that annuity demand is higher for higher types $j > i$ (remember that w_i and π_i are taken as non-negatively correlated). One observes that the condition (13) is certainly fulfilled, if $\partial S_i(w_i)/\partial w_i \leq 0$, i.e. social security benefits do not increase with income, because in this case the LHS of (13) is non-negative, while the RHS is negative. To explain this result, consider the case when social security benefits do not depend on income, i.e. $\partial S_i/\partial w_i = 0$: For fixed S_i, higher income induces higher annuity demand, since part of the additional income, received in the working period, is shifted to the retirement period.[22] Further, recall from Lemma 1 that a cut in social security benefits increases annuity demand. Thus, $\partial A_i/\partial w_i$ is also positive, if $\partial S_i/\partial w_i < 0$.

Finally, note that $\partial S_i(w_i)/\partial w_i \leq 0$ is a sufficient, but not a necessary condition. (13) is also fulfilled, if $\partial S_i/\partial w_i$ is positive and not too large. In the next Lemma we show for a specific class

[22] Note that $\partial S_i/\partial w_i = 0$ correspond to the assumption used by Walliser (1998), mentioned above. Regulations, which realise $\partial S_i/\partial w_i = 0$ below and above a certain threshold of income, are indeed in force: On the one hand there are flat-rate pensions, which guarantee a minimum retirement income, on the other hand often an assessment ceiling for calculating benefits and contributions exists.

of per-period utility functions that annuity demand increases with higher types j > i, when social security benefits increase proportionally (or less) with income. For this, we consider the utility function with a constant Arrow-Pratt coefficient of relative risk aversion ρ,

$$u(c_{ti}) = \frac{c_{ti}^{1-\rho} - 1}{1-\rho} \tag{17}$$

with $\rho = -c_{ti} u''(c_{ti}) / u'(c_{ti})$. Since individuals are assumed to be risk avers, the CRRA-utility function (CRRA abbreviates Constant Relative Risk Aversion) exhibits $\rho > 0$. In case of an interior solution $A_i > 0$ we can derive from the condition (4a) together with (17) an explicit formula for annuity demand (for the case that the rate of time preference $\alpha = 0$)[23]

$$A_i = \frac{\pi_i^{1/\rho}(1-\tau_w-\tau_S)w_i - R^{1/\rho}s_i w_i}{R^{1/\rho} + \pi_i^{1/\rho}R}, \tag{18}$$

where $s_i(w_i)$ is defined as $s_i(w_i) \equiv S_i(w_i)/w_i$, that is the ratio of the benefits to labour income. If s_i is equal for all types i, then benefits S_i rise proportionally with income. Thus we call s_i the average benefit rate in the following.

Lemma 4: Let $A_i, A_j > 0$ be the interior solution (18) given CRRA utility, $i, j \in \{1,...,N\}$. For any consumer price R, an individual of type i has a lower annuity demand than any individual of type j > i, as long as social security benefits are not rising more than proportionally with income, i.e. $A_i < A_j$ for any two types i < j, if $s_i \geq s_j$.

Proof: Substituting (18) into the difference $A_j - A_i$ gives

$$A_j - A_i = \psi\left[\left(R^{1/\rho} + \pi_i^{1/\rho}R\right)w_j\Phi_j - \left(R^{1/\rho} + \pi_j^{1/\rho}R\right)w_i\Phi_i\right], \tag{19}$$

where $\psi \equiv \left[\left(R^{1/\rho} + \pi_i^{1/\rho}R\right)\left(R^{1/\rho} + \pi_j^{1/\rho}R\right)\right]^{-1}$, $\Phi_i \equiv \pi_i^{1/\rho}(1-\tau_w-\tau_S) - R^{1/\rho}s_i$ and analogous for Φ_j, with $\psi > 0$ and Φ_i, $\Phi_j > 0$ for A_i, $A_j > 0$. Rearranging (19), we obtain

$$A_j - A_i = \psi w_i\left[R^{1/\rho}\left(\frac{w_j}{w_i}\Phi_j - \Phi_i\right) + R\left(\frac{w_j}{w_i}\pi_i^{1/\rho}\Phi_j - \pi_j^{1/\rho}\Phi_i\right)\right], \tag{20}$$

We know that $\pi_j > \pi_i$. Thus $\Phi_j > \Phi_i$, if $s_i \geq s_j$. From this, together with $\frac{w_j}{w_i} \geq 1$, it follows that the first term of the RHS of (20), $R^{1/\rho}\left(\frac{w_j}{w_i}\Phi_j - \Phi_i\right)$, is positive, if $s_i \geq s_j$. The same holds for the

[23] Note that α has the same effect as π_i. So, for simplicity, we assume a time preference rate α of zero.

second term of the RHS of (20), $R\left(\frac{w_j}{w_i}\pi_i^{1/\rho}\Phi_j - \pi_j^{1/\rho}\Phi_i\right)$, because $\pi_i^{1/\rho}\Phi_j > \pi_j^{1/\rho}\Phi_i$, i.e. $\left(\pi_i^{1/\rho}\pi_j^{1/\rho}(1-\tau_w-\tau_S)-R^{1/\rho}\pi_i^{1/\rho}s_j\right) > \left(\pi_i^{1/\rho}\pi_j^{1/\rho}(1-\tau_w-\tau_S)-R^{1/\rho}\pi_j^{1/\rho}s_i\right)$, if $s_i \geq s_j$. Q.E.D.

Altogether, we can conclude that Lemmas 3 and 4 provide a strong indication for the existence of an adverse-selection problem in the annuity market, also in the case of heterogeneous income. In general, annuity demand is higher for types j > i, when social security benefits do not increase too much with income. In particular this holds for the CRRA-utility function, when benefits increase proportionally (or less) with income. Probably every existing social security system has this property.

As a consequence of the over-representation of annuities bought by high-risk types, $\tilde{Q}$ is above the actuarially fair price $\overline{Q}$, which corresponds to the average survival probabilities of the individuals, as shown in Lemma 2. Further, it follows from this Lemma that the difference $\tilde{Q} - \overline{Q}$ increases, when the difference in annuity demand of different risk-types increases. From Lemma 3 and 4 it is obvious that this difference is greater, the higher the (positive) correlation between survival probabilities and income and the less social security benefits increase relative to income. These two factors aggravate the problem of adverse selection.

3.2 The effect of the social security reform on adverse selection

In this section we turn our attention to the question of how the problem of adverse selection is affected by the three policy measures of the social security reform, introduced in section 2.2. This is an important issue, since adverse selection is regarded as a major reason for the fact that there is so little trade of life annuities. If a reform instrument exacerbates adverse selection, then even less annuities will be traded. As already mentioned in the introduction, Abel (1986) and Walliser (1998, 2000) have shown that under price competition the problem of adverse selection in the private annuity market is more severe in an economy with a social security system than without. Their findings, however, do not give advice concerning the problem in the centre of the current debates, how to reform social security to maintain its future solvency and simultaneously to ensure adequate old-age provision for the individuals. The aim of this section is to contribute to this debate by answering the following two questions, concerning the effects on adverse selection in the annuity market: If a reform is necessary to assure financing of social security, should it be a cut in the benefits or an increase in the contributions to social security? If a tax incentive for the purchase of life annuities is introduced to counteract the negative effects of the other two reform measures on old-age consumption, can it indeed serve its purpose?

Remember from the previous section that it is the over-representation of annuities bought by high-risk individuals, which is responsible for the fact that the equilibrium price $\tilde{Q}$ is inefficiently high. Thus, for the effect of a change of any exogenous parameter X on $\tilde{Q}$, it is crucial to which extent the different risk-types adjust their annuity demand. If the demand share of high-risk types increases (decreases), $\tilde{Q}$ increases (decreases). This is shown in the following Lemma 5, which then allows us to conclude in Proposition 3 and 4 whether the adverse-selection problem is alleviated or aggravated by each of the three reform instruments.

Lemma 5: Consider a price $\tilde{Q}$ which, together with $A_i \geq 0$ for all i = 1,…,N and $A_i, A_j > 0$ for some $i, j \in \{1,...,N\}$, fulfils the zero-condition profit condition (8). The effect of a marginal change in any exogenous parameter X on the equilibrium price $\tilde{Q}$ depends on the percentage change of annuity demand of any type i, compared to that of any other type j, in the following way:

$$d\tilde{Q}/dX \lesseqgtr 0, \text{ if } \frac{\partial A_i/\partial X}{A_i} \gtreqless \frac{\partial A_j/\partial X}{A_j} \quad \text{for all types } i < j \text{ with } A_i, A_j > 0 .$$

Proof: The effect of a marginal change of exogenous parameter X on the equilibrium price $\tilde{Q}$ is obtained by implicit differentiation of the zero-profit condition (8), where annuity demand and, consequently, the demand share ε_i of group i in aggregate annuity demand depends on $\tilde{Q}$ and on the exogenous variable X, that is

$$\frac{d\tilde{Q}}{dX} = -\frac{\partial P/\partial X}{\partial P/\partial Q} . \tag{21}$$

Since the denominator of the RHS of (21) (see the considerations following (7)) is positive, the sign of $\partial\tilde{Q}/\partial X$ is determined by $-\partial P/\partial X$. Differentiating the profit function $P = \sum_{i=1}^{N}\gamma_i A_i\left(\tilde{Q} - \sum_{j=1}^{N}\pi_j\varepsilon_j\right)$ with respect to X yields

$$\frac{\partial P}{\partial X} = \sum_{i=1}^{N}\gamma_i \frac{\partial A_i}{\partial X}\left(\tilde{Q} - \sum_{j=1}^{N}\pi_j\varepsilon_j\right) - \sum_{i=1}^{N}\gamma_i A_i \sum_{j=1}^{N}\pi_j \frac{\partial \varepsilon_j}{\partial X} , \tag{22}$$

where the first term $\tilde{Q} - \sum_{i=1}^{N}\pi_i\varepsilon_i$ of the RHS of (22) is equal to zero, due to zero-profit condition (8). Thus $\partial P/\partial X$ simplifies to

$$\frac{\partial P}{\partial X} = -\bar{A}\sum_{i=1}^{N}\pi_i \frac{\partial \varepsilon_i}{\partial X} , \tag{23}$$

with $\bar{A} \equiv \sum_{j=1}^{N}\gamma_j A_j$. Using the definition of $\varepsilon_i \equiv \gamma_i A_i \Big/ \sum_{j=1}^{N}\gamma_j A_j$, $\frac{\partial \varepsilon_i}{\partial X}$ may be rewritten as

$$\frac{\partial \varepsilon_i}{\partial X} = \frac{\gamma_i}{\bar{A}^2}\left(\frac{\partial A_i}{\partial X}\sum_{j=1}^{N}\gamma_j A_j - A_i \sum_{j=1}^{N}\gamma_j \frac{\partial A_j}{\partial X}\right), \tag{24}$$

which in turn can be transformed to

$$\frac{\partial \varepsilon_i}{\partial X} = \frac{\gamma_i}{\bar{A}^2}\sum_{j=1}^{N}\gamma_j\left(\frac{\partial A_i}{\partial X}A_j - \frac{\partial A_j}{\partial X}A_i\right) \tag{25}$$

and further, by use of the definitions of ε_i and $\bar{A}$, to

$$\frac{\partial \varepsilon_i}{\partial X} = \varepsilon_i\sum_{j=1}^{N}\varepsilon_j\left(\frac{\partial A_i/\partial X}{A_i} - \frac{\partial A_j/\partial X}{A_j}\right). \tag{26}$$

Substituting (26) into (23) yields

$$\frac{\partial P}{\partial X} = -\bar{A}\sum_{i=1}^{N}\sum_{j=1}^{N}\pi_i\varepsilon_i\varepsilon_j\left(\frac{\partial A_i/\partial X}{A_i} - \frac{\partial A_j/\partial X}{A_j}\right). \tag{27}$$

Rearranging the RHS of (27) gives

$$\frac{\partial P}{\partial X} = -\bar{A}\sum_{j=1}^{N}\sum_{i=1}^{j}(\pi_j - \pi_i)\varepsilon_i\varepsilon_j\left(\frac{\partial A_j/\partial X}{A_j} - \frac{\partial A_i/\partial X}{A_i}\right). \tag{28}$$

For any two types j > i, we have $\pi_j > \pi_i$ and $\varepsilon_i,\varepsilon_j \geq 0$. From this together with (28) it follows that the sign of $\partial P/\partial X$ is determined by $(\partial A_j/\partial X)/A_j - (\partial A_i/\partial X)/A_i$ of all those pairs of types i, j with positive demand shares $\varepsilon_i,\varepsilon_j > 0$. Consequently, if $\frac{\partial A_i/\partial X}{A_i} \gtreqless \frac{\partial A_j/\partial X}{A_j}$ for any two types i < j with $A_i,A_j > 0$, then $\partial P/\partial X \gtreqless 0$ and, due to (21), $d\tilde{Q}/dX \lesseqgtr 0$. Q.E.D.

Thus, adverse selection remains unchanged in case that the relative change of annuity demand is equal for all types i, j. On the other hand, adverse selection is alleviated, i.e. $\tilde{Q}$ decreases, if as a reaction to a marginal change in any exogenous variable, the percentage increase (decrease) of annuity demand of a low-risk type i is higher (lower) than the percentage increase (decrease) of annuity demand of a high-risk type j. In this case, the demand share of the low-risk types increases. For unchanged $\tilde{Q}$, this shift in the composition of aggregate annuity demand to the "profitable" types would lead to an increase in profits. In order to restore the zero profits, the equilibrium price $\tilde{Q}$ must fall. By the same argument, it can be explained that $\tilde{Q}$ rises, if the percentage increase (decrease) of annuity demand of a low-risk type is lower (higher) than the percentage increase (decrease) of annuity demand of a high-risk type.

With this result, we are ready to determine the effect of the social security reform on adverse selection by comparing the percentage change of annuity demand of any two types i and j due to a marginal change in each of the reform instruments. In order to obtain clear-cut results, we consider the per-period CRRA-utility function (17) introduced in section 3.1, which is characterized by a constant coefficient of relative risk aversion ρ. First, we consider a cut in the average benefit-rate $s_i(w_i) \equiv S_i(w_i)/w_i$ and an increase in the social security tax τ_S. Then, we turn to tax incentives for the purchase of life annuities, assuming the same two alternatives of financing as in section 2.2.

Proposition 3: Let $A_i, A_j > 0$ be the interior solution (18) given CRRA utility, $i, j \in \{1,...,N\}$ and assume that social security benefits do not increase more than proportionally with income. The percentage change of annuity demand of any type i compared to any other type j > i is characterized as follows:

$$\text{For any } \rho > 0: \quad \frac{\partial A_i/\partial s_i}{A_i} < \frac{\partial A_j/\partial s_j}{A_j}, \quad \frac{\partial A_i/\partial \tau_S}{A_i} < \frac{\partial A_j/\partial \tau_S}{A_j}.$$

As a consequence, a cut in the average benefit rate $s_i \equiv S_i/w_i$ alleviates adverse selection in the private annuity market, while an increase in the social security tax τ_S exacerbates adverse selection.

Proof: See the Appendix.

Proposition 3 shows that an increase in the social security tax τ_S raises the demand share of the high-risk individuals, since the percentage decrease of annuity demand is higher for the low-risk types than for the high-risk types. This, together with Lemma 5, implies that the equilibrium price rises. On the contrary, a cut in the average benefit rate s_i reduces the over-representation of annuities bought by high-risk individuals and thus adverse selection. As a consequence, the equilibrium price decreases.[24]

Concerning old-age consumption, we can conclude to the overall effects: We know from Lemma 1 that both reform instruments reduce old-age consumption for any given price Q. However, due to adverse selection in the annuity market, there is a second effect: An increase in the social security tax raises the equilibrium price, which in turn leads to an even stronger decline in old-age consumption (since $\partial c_{i1}/\partial Q < 0$). On the other hand, a cut in the average benefit rate reduces the distortion of equilibrium price. By this, the overall decline in old-age consumption will occur to a lower degree compared to the case of a fixed price (without adverse selection).

[24] However, in case that not the benefit-income-ratio s_i, but the social security benefits S_i are considered as the relevant measure, the effects on adverse selection are ambiguous, which is shown in the Appendix. Unambiguous effects are only obtained in case that individuals have identical income (see also Abel (1986)) or in case that social security benefits are introduced (see also Walliser (2000)).

Finally we investigate how tax incentives for the purchase of life annuities affect adverse selection. This is an important issue not only for reasons of efficiency, but also for the success of stimulating private old-age provision. In case that tax incentives for the purchase of life annuities reduce adverse selection, the consumer price R = Q(1 – b) decreases for two reasons: There is the direct effect of the increase in the subsidy rate Δb and the indirect effect of the decrease in the equilibrium price $\Delta\tilde{Q}$. However, if adverse selection is aggravated, the equilibrium price $\tilde{Q}$ increases, which then counteracts the positive influence of the tax incentive on the consumer price R. Obviously this reduces sharply the attractiveness of tax incentives as an instrument to stimulate private old-age provision.

As in section 2.2., we distinguish between two ways to keep the public budget balanced: The tax incentive can be financed (i) by a reduction in the government expenditures for public goods or (ii) by an increase in the income tax. Note that for the latter case (ii) we restrict to show the effects of an increase in the subsidy rate b starting from an initial situation in which b = 0, while for the former case (i) the effects are shown for any initial $b \geq 0$.

Proposition 4: Let $A_i, A_j > 0$ be the interior solution (18) given CRRA utility, $i, j \in \{1,...,N\}$, and assume that social security benefits do not increase more than proportionally with income. Let the subsidy rate b for life annuities be financed

(i) by a reduction in public goods or (ii) by an increase in the income tax

such that the public budget constraint (5) remains fulfilled. The percentage change of annuity demand of any type i, compared to any other type j > i, depends on the constant coefficient of relative risk aversion ρ as follows:

In case (i): There exists $\rho^* > 1$ such that for any ρ, $\rho^* > \rho > 0$: $\frac{\partial A_i/\partial b}{A_i} > \frac{\partial A_j/\partial b}{A_j}$.

In case (ii): There exists $\rho^{**} < \rho^*$ such that for any ρ, $\rho^{**} > \rho > 0$: $\left.\frac{\partial A_i/\partial b}{A_i}\right|_{b=0} > \left.\frac{\partial A_j/\partial b}{A_j}\right|_{b=0}$.

As a consequence, a tax incentive b for private life annuities alleviates adverse selection in the annuity market, if the constant coefficient of relative risk aversion ρ is smaller than a critical value, which is higher in case (i) than in case (ii). Otherwise, the effect is indeterminate.

Proof: See the Appendix.

Proposition 4 shows that in both cases (i) and (ii) the effects of a tax incentive for private life annuities on adverse selection depend on the degree of risk aversion. For sufficiently low values of ρ adverse selection is reduced. Otherwise the effect is indeterminate and can turn negative for high values of ρ. In the Appendix 5.2 we provide numerical computations illustrating Proposition 4 for two risk-types, when in both cases (i) and (ii) a tax incentive for life annuities are introduced. We distinguish between two scenarios: Scenario 1 (see Table 1) considers a more generous social security system (with higher values of τ_S, s_1, s_2) than Scenario 2 (see Table 2). Besides we allow for a variation of the values of π_1, γ_1, s_1 and w_1 (see "sub-scenarios" a – e) and of the coefficient of the relative risk aversion ρ. These calculations should exemplify what degrees of risk aversion are required for a mitigation, aggravation, resp., of adverse selection, i.e. for $(\partial A_1/\partial b)/A_1 - (\partial A_2/\partial b)/A_2$ to be positive, negative, resp. (see the shaded columns).[25]

First consider case (i) where the tax incentive is financed by a reduction of public goods. Proposition 4 shows that if ρ is smaller than a critical value ρ^*, which is greater than one, the percentage increase of annuity demand is higher for low-risk types than for high-risk types (remember that $\partial A_i/\partial b > 0$, see Proposition 1). Thus, the over-representation of annuities bought by the high-risk individuals decreases, which implies, together with Lemma 5, that the equilibrium price $\tilde{Q}$ decreases. The numerical computations in Appendix 5.2 show that given the generous social security system in Scenario 1, the critical value ρ^* is above 16 for the parameter constellations of 1a – 1d), only for the parameter constellation of 1e (identical income) ρ^* takes a value of 9.85. Given the moderate social security system in Scenario 2, the critical value ρ^* is lower, but still far above one. It varies from 13.3 (for the parameter constellation of 2b) to 3.4 (for the parameter constellation of 2e).

Second, consider case (ii) where the tax incentive is financed by an increase in the income tax. Proposition 4 shows that adverse selection is alleviated for any $\rho < \rho^{**}$, where the critical value ρ^{**} is smaller than the critical value ρ^* in case (i). This result is reproduced in the numerical computations of Appendix 5.2. Further, recall from Proposition 2 that the introduction of a tax incentive may have a negative effect on annuity demand for individuals with below-average annuity-demand. From Lemma 3 and 4 it follows that these individuals are the low-risk types. For high-risk individuals (those with above-average annuity demand) the effect is positive. Obviously, if such a situation occurs, then the demand share of the high-risk types increases, thus the problem of adverse selection is aggravated and the equilibrium price $\tilde{Q}$ increases. Such a situation is reproduced in Appendix 5.2 for the parameter constellations of 1a and 2a, if ρ equals 16. Moreover, adverse selection is also aggravated, if annuity demand of both risk-types increases by the introduction of a tax incentive, yet that of the low-risk individuals by a lower percentage than that of the high-risk individuals. Given the moderate social security system (see Table 2) such a situation occurs for any $\rho \geq 3$ and adverse selection is alleviated

[25] Note that given two risk-groups only, adverse selection rises for any $\rho > \rho^*$. This follows from the fact that in this case the critical value ρ^* coincides with $\hat{\rho}_i^j$, defined in Appendix 5.1.

only for $\rho \leq 2$. Given the more generous system (see Table 1) adverse selection is alleviated for lower values of risk aversion $\rho \leq 4$ (for parameter constellations of 1a – 1d), $\rho \leq 3$ (for parameter constellations of 1e).

Altogether, we can conclude that the degree of risk aversion is crucial for the effects on adverse selection. Numerical calculations give some evidence that, in case (i) where the reduction of the supply of public goods has no impact on annuity demand, adverse selection is reduced for reasonable degrees of risk aversion, at least for a wide range of parameter constellations. However in case (ii) where the increase of the income tax has a negative effect on annuity demand, one should not be confident and eliminate the possibility that the introduction of a tax incentive exacerbates the problem of adverse selection. Numerical computations, although exemplarily, show that this can be the case for reasonably high coefficients of risk aversion ($\rho \geq 3$).

4. Conclusion

Governments of many developed nations are now looking to reform their social security systems to respond to the anticipated development in demography. They strive for an increase in self-provision for retirement in order to compensate the reduction of the legal responsibility to provide financial support for the retired. Tax incentives for the purchase of private life annuities enjoy great popularity as one obvious policy option to achieve this purpose. The main justification for having them implemented is based on paternalism: Tax incentives should keep myopic individuals from making to little old-age provision and thus from ending up in poverty when retired. This is regarded as an imminent danger by governments that want to shift responsibility for old-age provision away from the public sector towards the individuals.[26] However, how effective are tax incentives in stimulating the purchase of life annuities? This paper highlights this question regarding causes which might inhibit the desired effect. Besides, it has focused on a cut of the social security benefits and on increase of the social security tax, two potential reform measures to maintain the long-run solvency of the social security system.

In a partial-equilibrium framework with a constant producer price, where the impact of the budgetary costs of state subsidies on private consumption decision is neglected, the

[26] Some empirical studies tried to answer the difficult question whether individuals actually do make too little private old-age provision. In a survey, Disney (2000) concludes that there is some evidence from the UK and the USA. Especially for low-earners, saving rates are below those required for appropriate smoothing of lifetime consumption according to the life cycle hypothesis. Recent studies for Great Britain confirm these findings (Disney et al., 2001a; Disney et al., 2001b). On the other hand, there is contradictory evidence for Germany: Schnabel (2000) finds that even in the periods of retirement there is positive saving, although lower than in the working periods. This indicates that at present elderly people have (more than) enough retirement income at their disposal.

introduction of tax incentives for life annuities leads to stimulation of annuity demand. This result was shown in a two-period model based on individual utility maximisation subject to uncertainty about life expectancy, where a cut of the government spending for public goods to finance the tax incentives does not influence private old-age provision. However, tax incentives for the purchase of life annuities are less effective in increasing self-provision, when partial-equilibrium analysis is dropped. In particular, we incorporated an increase in the proportional income tax to keep the public budget in balance. In this case, the positive effect on annuity demand is smaller and it may even be optimal for some individuals to reduce their annuity demand. This counterintuitive result may occur, if the negative income effect due to an increase in the income tax is higher than the positive income effect of the subsidy for life annuities. Hence, the individuals most likely concerned are those with low life expectancy, who purchase only a small amount of annuities, but pay a high income tax. Obviously these are also the ones, who are made worse-off by the introduction of tax incentives.

These considerations bring us to the question about the distributional effects of the benefits and costs associated with tax incentives. Because individuals with a high life expectancy demand more annuities and thus acquire a higher total of subsidies than individuals with a low life expectancy, there is an unequal distribution of the benefits in favour of individuals who on average live longer. But to what extent are the individuals affected negatively in their well-being by the budgetary costs? First, suppose that the budgetary costs arise in the form of a cut in the supply of public goods. Under the assumption of identical preferences for public goods (as in our model), it is straightforward to conclude that individuals with a higher life expectancy benefit at the expense of those with lower life expectancy. However, when allowing individuals to differ in their preferences for public goods (with no systematic correlation to life expectancy), no conclusive statement can be given. Second, suppose that tax incentives are financed by an increase in the income tax and that the individuals earn identical income. Then all individuals pay the same amount of additional income tax and tax incentives for life annuities induce again redistribution from individuals with high life expectancy to individuals with low life expectancy. This redistributive effect is alleviated, if the individuals with low survival probability are assumed to be those who earn a lower income.[27] In this context, it is worth to recall that uniform pricing of annuities already implies redistribution towards the high-risk types compared to the first-best outcome where each risk-type receives a price according to his survival probability. In view of this fact the question arises whether it is desirable to intensify this redistribution.

However, all previous considerations are based on the assumption of a constant producer price of annuities. When focusing on the equilibrium outcomes in the private annuity market, one can identify a second cause which may hamper the effectiveness of tax incentives for life annuities.

[27] As already cited in footnote 15, there is empirical evidence for this positive correlation.

It is the problem of adverse selection, which leads to inefficiently high equilibrium prices charged by the annuity companies. If one of the reform instruments aggravates the problem of adverse selection, the equilibrium price rises that in turn diminishes the stimulation of annuity demand.

We introduced asymmetric information in the annuity market, with heterogeneous individuals, who differ in their life expectancy and may differ in labour income, where the assumption was made that survival probability and income are not negatively correlated. We found unambiguous effects of the following two reform instruments on adverse selection. A cut in the average benefit rate alleviate the adverse selection problem in the private annuity market, while an increase in the contribution rate aggravates it. These results suggests that for reasons of efficiency of the private annuity market, governments should stick to a cut in the social security benefits instead of an increase in the social security tax, when a reform is required to solve the financial difficulties of the social security system. Till now, such considerations have hardly entered into the political debate. However, they should be of special interest in view of the recent trend at the political level to stimulate the purchase of private life annuities by state subsidies, since only a cut in the social security benefits has the virtue to reduce the equilibrium price.

On the other hand, we found that the effect of tax incentives for private life annuities on adverse selection is ambiguous. Adverse selection is shown to be reduced, if the coefficient of relative risk aversion does not exceed a critical value. Otherwise the effect is indeterminate and can turn negative for higher degrees of risk aversion. We found that this critical value is greater than one, when the partial-equilibrium approach is chosen, where the cut in the supply of public goods to finance the tax incentives has no impact on annuity demand. In case that they are financed by an increase in the income tax, this critical value of risk aversion turns out to be lower. Numerical computations were made to exemplify what degrees of risk aversion are required for a mitigation, aggravation, resp., of adverse selection. They provide some indication that in the first case adverse selection is reduced for a wide range of reasonable degrees of risk aversion, while in the second case one cannot rule out the possibility that the introduction of tax incentives exacerbates the problem of adverse selection.

Altogether we can conclude that for the effectiveness of tax incentives it is crucial whether or not the influence of the budgetary costs of tax incentives on private old-age provision is taken into account: The results suggest that only if the influence is assumed to be negligible, tax incentives are an effective instrument to increase self-provision for retirement. However, if the budgetary costs are taken as a relevant influencing variable for private old-age provision, then we may observe little additional annuity demand: Given a constant producer price the positive effect on annuity demand is smaller than in the former case and it may even turn negative for individuals with low life expectancy. Moreover, the producer price may rise due to an aggravation of adverse selection in the private annuity market which would dampen further the stimulation of annuity demand.

5. Appendix

5.1. Proofs

Let $A_i, A_j > 0$ be the interior solution (18) given CRRA utility, $i, j \in \{1,...,N\}$, and assume that $s_i \geq s_j$ for any type i < j.

Proof of Proposition 3: We show that, for any $\rho > 0$, $(\partial A_j/\partial s_j)/A_j - (\partial A_i/\partial s_i)/A_i > 0$ and $(\partial A_j/\partial \tau_S)/A_j - (\partial A_i/\partial \tau_S)/A_i > 0$. Differentiation of (18) with respect to s_i, τ_S, resp., gives

$$\frac{\partial A_i}{\partial s_i} = -\frac{R^{1/\rho} w_i}{R^{1/\rho} + \pi_i^{1/\rho} R}, \tag{29}$$

$$\frac{\partial A_i}{\partial \tau_S} = -\frac{\pi_i^{1/\rho} w_i}{R^{1/\rho} + \pi_i^{1/\rho} R}. \tag{30}$$

By use of (18), together with (29), (30), resp., one obtains

$$\frac{\partial A_j/\partial s_j}{A_j} - \frac{\partial A_i/\partial s_i}{A_i} = \theta R^{1/\rho}\left((\pi_j^{1/\rho}(1-\tau_w-\tau_S) - R^{1/\rho}s_j) - (\pi_i^{1/\rho}(1-\tau_w-\tau_S) - R^{1/\rho}s_i)\right), \tag{31}$$

$$\frac{\partial A_i/\partial \tau_S}{A_i} - \frac{\partial A_j/\partial \tau_S}{A_j} = \theta\left(\pi_i^{1/\rho}(\pi_j^{1/\rho}(1-\tau_w-\tau_S) - R^{1/\rho}s_j) - \pi_j^{1/\rho}(\pi_i^{1/\rho}(1-\tau_w-\tau_S) - R^{1/\rho}s_i)\right). \tag{32}$$

where $\theta \equiv \left[\left(\pi_i^{1/\rho}(1-\tau_w-\tau_S) - R^{1/\rho}s_i\right)\left(\pi_j^{1/\rho}(1-\tau_w-\tau_S) - R^{1/\rho}s_j\right)\right]^{-1}$ with $\theta > 0$ for A_i, $A_j > 0$. Since $\pi_j > \pi_i$, $s_i \geq s_j$ and $\theta > 0$, (31) and (32) are positive. This, together with Lemma 5, implies that $d\tilde{Q}/ds_i > 0$, $d\tilde{Q}/d\tau_S > 0$. Q.E.D.

Proof of the remark given in footnote 24: We calculate the effects of the social security benefits S_i on adverse selection in analogous steps as above: By use of the definition of $s_i \equiv S_i/w_i$ and the first derivative of (18) with respect to S_i, which is

$$\frac{\partial A_i}{\partial S_i} = -\frac{R^{1/\rho}}{R^{1/\rho} + \pi_i^{1/\rho} R}, \tag{33}$$

we get

$$\frac{\partial A_j/\partial S_j}{A_j} - \frac{\partial A_i/\partial S_i}{A_i} = \frac{\theta R^{1/\rho}}{w_i w_j}\Big((\pi_j^{1/\rho}(1-\tau_w-\tau_S)w_j - R^{1/\rho}s_j w_j) - (\pi_i^{1/\rho}(1-\tau_w-\tau_S)w_i - R^{1/\rho}s_i w_i)\Big) \quad (34)$$

with $s_i w_i \equiv S_i$, $s_j w_j \equiv S_j$. We know that $\pi_j > \pi_i$, $w_j \geq w_i$, $s_j \leq s_i$ and $\theta > 0$. From this it follows that the LHS of (34) is ambiguous, if $s_j w_j > s_i w_i$. If $s_j w_j \leq s_i w_i$, $(\partial A_j/\partial S_j)/A_j - (\partial A_i/\partial S_i)/A_i$ is positive. Obviously, this holds in case of identical income, i.e. $w_i = w_j$, or starting from a situation, in which $s_i = s_j = 0$. Q.E.D.

Proof of Proposition 4

Case (i): Assume that the tax incentives for life annuities are financed by a reduction of government spending for public goods. We show that there exists $\rho^* > 1$, such that the difference $(\partial A_i/\partial b)/A_i - (\partial A_j/\partial b)/A_j$ is positive for any ρ, $\rho^* > \rho > 0$.

By use of $R \equiv (1 - b)Q$, the first derivate of annuity demand (18) with respect to b is equal to

$$\frac{\partial A_i}{\partial b} = \frac{Q}{\rho}\left(\frac{R^{-1+1/\rho}s_i w_i}{R^{1/\rho} + \pi_i^{1/\rho}R} + (R^{-1+1/\rho} + \rho\pi_i^{1/\rho})\frac{\pi_i^{1/\rho}(1-\tau_w-\tau_S)w_i - R^{1/\rho}s_i w_i}{(R^{1/\rho} + \pi_i^{1/\rho}R)^2}\right). \quad (35)$$

By use of (35) and (18) one obtains

$$\frac{\partial A_i/\partial b}{A_i} = \frac{Q}{\rho}\left(\frac{R^{-1+1/\rho}s_i}{\pi_i^{1/\rho}(1-\tau_w-\tau_S) - R^{1/\rho}s_i} + \frac{R^{-1+1/\rho} + \rho\pi_i^{1/\rho}}{R^{1/\rho} + \pi_i^{1/\rho}R}\right) \quad (36)$$

and further, by use of (36) and some simple transformations,

$$\begin{aligned} &\frac{\partial A_i/\partial b}{A_i} - \frac{\partial A_j/\partial b}{A_j} = \\ &\quad \frac{Q}{\rho}\Big(\theta R^{-1+1/\rho}\Big(s_i(\pi_j^{1/\rho}(1-\tau_w-\tau_S) - R^{1/\rho}s_j) - s_j(\pi_i^{1/\rho}(1-\tau_w-\tau_S) - R^{1/\rho}s_i)\Big) + \\ &\quad + \psi\Big(\Big(R^{-1+1/\rho} + \rho\pi_i^{1/\rho}\Big)\Big(R^{1/\rho} + \pi_j^{1/\rho}R\Big) - \Big(R^{-1+1/\rho} + \rho\pi_j^{1/\rho}\Big)\Big(R^{1/\rho} + \pi_i^{1/\rho}R\Big)\Big)\Big) \end{aligned} \quad (37)$$

where $\psi \equiv \left[\left(R^{1/\rho} + \pi_i^{1/\rho}R\right)\left(R^{1/\rho} + \pi_j^{1/\rho}R\right)\right]^{-1}$, $\psi > 0$. Further computation of (37) yields

$$\frac{\partial A_i/\partial b}{A_i} - \frac{\partial A_j/\partial b}{A_j} = \frac{QR^{-1+1/\rho}}{\rho}\Big(\theta(s_i\pi_j^{1/\rho} - s_j\pi_i^{1/\rho})(1-\tau_w-\tau_S) + \psi R(1-\rho)(\pi_j^{1/\rho} - \pi_i^{1/\rho})\Big). \quad (38)$$

Note first that (38) is positive for $\rho \leq 1$, since $\pi_j > \pi_i$ and $s_i \geq s_j$. But (38) is also positive for any $\rho < \hat{\rho}_i^j$, where $\hat{\rho}_i^j$ represents the smallest root when (38) is set equal to zero. Thus for each pair of risk-types i, j a critical value $\hat{\rho}_i^j > 1$ exists, which depends on π_i, π_j, s_i, s_j, τ_w, τ_S, and R.

Finally, recall from Lemma 4 that annuity demand is increasing in the risk-type. Let $\bar{k}$ be the index of the first type with strictly positive annuity demand and define ρ^* as the smallest value of all $\hat{\rho}_i^j$, $i, j = \bar{k}, \ldots N$, $i < j$, i.e. $\rho^* \equiv \min\left\{\hat{\rho}_{\bar{k}}^{\bar{k}+1}, \hat{\rho}_{\bar{k}}^{\bar{k}+2}, \ldots, \hat{\rho}_i^j, \ldots, \hat{\rho}_{N-1}^N\right\}$, $\rho^* > 1$. It follows that (38) is positive for any two types i, j for any $\rho < \rho^*$. This, together with Lemma 5, implies that $d\tilde{Q}/db < 0$.

Case (ii): Assume that a tax incentive for life annuities is introduced and financed by an increase in the income tax rate τ_w, such that the public budget constraint (5) remains fulfilled. We show that there exists $\rho^{**} < \rho^*$, such that the difference $(\partial A_i/\partial b)/A_i\big|_{b=0} - (\partial A_j/\partial b)/A_j\big|_{b=0}$ is positive for any ρ, $\rho^{**} > \rho > 0$. We proceed as above: Differentiating (18) with respect to b yields

$$\frac{\partial A_i}{\partial b} = \frac{Qw_i}{\rho(R^{1/\rho} + \pi_i^{1/\rho} R)}\left((R^{-1+1/\rho} s_i - \frac{\rho\pi^{1/\rho}}{Q}\frac{\partial \tau_w}{\partial b}) + (R^{-1+1/\rho} + \rho\pi_i^{1/\rho})\frac{\pi_i^{1/\rho}(1-\tau_w-\tau_S) - R^{1/\rho} s_i}{R^{1/\rho} + \pi_i^{1/\rho} R}\right). \quad (39)$$

By use of (6), (18) and (39) one obtains

$$\left.\frac{\partial A_i/\partial b}{A_i}\right|_{b=0} = \frac{Q}{\rho}\left(\frac{R^{-1+1/\rho} s_i - \rho\pi_i^{1/\rho}\bar{A}/\bar{w}}{\pi_i^{1/\rho}(1-\tau_w-\tau_S) - R^{1/\rho} s_i} + \frac{R^{-1+1/\rho} + \rho\pi_i^{1/\rho}}{R^{1/\rho} + \pi_i^{1/\rho} R}\right). \quad (40)$$

Calculating the difference $(\partial A_i/\partial b)/A_i\big|_{b=0} - (\partial A_j/\partial b)/A_j\big|_{b=0}$ analogously as above yields

$$\left.\frac{\partial A_i/\partial b}{A_i}\right|_{b=0} - \left.\frac{\partial A_j/\partial b}{A_j}\right|_{b=0} = \frac{QR^{-1+1/\rho}}{\rho}\Big(\theta(s_i\pi_j^{1/\rho} - s_j\pi_i^{1/\rho})(1-\tau_w-\tau_S-\rho R\frac{\bar{A}}{\bar{w}}) + \\ + \psi R(1-\rho)(\pi_j^{1/\rho} - \pi_i^{1/\rho})\Big) \quad (41)$$

First note, that the sign of (41) is ambiguous, since $1-\tau_w-\tau_S > 0$ and $R\bar{A}/\bar{w} > 0$ and for any $j > i$, $\pi_j > \pi_i$, $s_i \geq s_j$. In analogous manner as above, we define $\breve{\rho}_i^j$ as the smallest root when (41) is set equal to zero, where $\breve{\rho}_i^j$ is smaller than $\hat{\rho}_i^j$, because the RHS of (41) is smaller than the RHS of (38). Thus for each pair of risk-types i, j, (41) is positive for any $\rho < \breve{\rho}_i^j$, $\breve{\rho}_i^j < \hat{\rho}_i^j$. Finally, we define ρ^{**} as the smallest value of all $\breve{\rho}_i^j$, $i, j = \bar{k}, \ldots N$, $i < j$, i.e. $\rho^{**} \equiv \min\left\{\breve{\rho}_{\bar{k}}^{\bar{k}+1}, \breve{\rho}_{\bar{k}}^{\bar{k}+2}, \ldots, \breve{\rho}_i^j, \ldots, \breve{\rho}_{N-1}^N\right\}$, where $\bar{k}$ indicates the index of the first type with strictly positive annuity demand. It follows that (41) is positive for any two types i, j for any $\rho < \rho^{**}$, where $\rho^{**} < \rho^*$, because $\breve{\rho}_i^j < \hat{\rho}_i^j$. It follows from Lemma 5 that $d\tilde{Q}/db\big|_{b=0} < 0$ for any $\rho < \rho^{**}$.

Q.E.D.

5.2. The effects of the introduction of a tax incentive on adverse selection: Numerical illustration of Proposition 4 for two risk-types i = 1,2

Table 1: ***Scenario 1:*** "Generous" social security system

$b = 0$, $\tau_w = 0.3$, $\tau_S = 0.2$, $w_2 = 150$, $s_2 = 0.24$ $\pi_2 = 0.8$, $\gamma_2 = 1 - \gamma_1$

		$\rho = 2$	$\rho = 3$	$\rho = 4$	$\rho = 5$	$\rho = 8$	$\rho = 16$
1a) $w_1 = 100$, $s_1 = 0.28$, $\pi_1 = 0.2$, $\gamma_1 = 0.2$							
$\tilde{Q}$		0.8	0.784	0.768	0.760	0.747	0.736
A_1		0	2.48	4.98	6.50	8.82	10.78
A_2		21.67	22.09	22.39	22.51	22.61	22.64
$\partial A_1/\partial b$	Case (i)	--	7.61	7.09	6.76	6.24	5.80
	Case (ii)	--	3.30	2.30	1.68	0.74	*-0.05*
$\partial A_2/\partial b$	Case (i)	25.65	20.57	18.00	16.39	13.90	11.73
	Case (ii)	17.39	11.98	9.15	7.41	4.76	4.50
$\partial A_1/\partial b/A_1 - \partial A_2/\partial b/A_2$	Case (i)	--	2.14	0.62	0.31	0.09	0.02
	Case (ii)	--	0.79	0.05	**-0.07**	**-0.12**	**-0.12**
1b) $w_1 = 100$, $s_1 = 0.28$, $\boldsymbol{\pi_1} = \mathbf{0.4}$, $\gamma_1 = 0.2$							
$\tilde{Q}$		0.789	0.769	0.764	0.762	0.757	0.754
A_1		5.03	7.54	8.80	9.58	10.69	11.63
A_2		22.37	22.48	22.48	22.47	22.42	22.36
$\partial A_1/\partial b$	Case (i)	12.40	10.20	9.04	8.33	7.24	6.32
	Case (ii)	7.57	4.88	3.49	2.64	1.34	0.26
$\partial A_2/\partial b$	Case (i)	26.18	20.80	18.05	16.38	13.84	11.71
	Case (ii)	17.24	11.65	8.82	7.11	4.54	2.38
$\partial A_1/\partial b/A_1 - \partial A_2/\partial b/A_2$	Case (i)	1.29	0.43	0.22	0.14	0.06	0.02
	Case (ii)	0.73	0.13	0.004	**-0.04**	**-0.08**	**-0.08**
1c) $w_1 = 100$, $s_1 = 0.28$, $\pi_1 = 0.2$, $\boldsymbol{\gamma_1} = \mathbf{0.4}$							
$\tilde{Q}$		0.8	0.755	0.720	0.701	0.675	0.655
A_1		0	2.77	5.45	7.06	9.46	11.46
A_2		21.67	22.86	23.58	23.84	24.03	24.01
$\partial A_1/\partial b$	Case (i)	--	7.81	7.36	7.03	6.44	5.87
	Case (ii)	--	4.07	3.05	2.38	1.30	0.34
$\partial A_2/\partial b$	Case (i)	25.65	21.03	18.59	16.94	14.25	11.83
	Case (ii)	18.98	13.59	10.58	8.67	5.68	3.09
$\partial A_1/\partial b/A_1 - \partial A_2/\partial b/A_2$	Case (i)	--	1.90	0.56	0.29	0.09	0.02
	Case (ii)	--	0.88	0.11	**-0.03**	**-0.10**	**-0.10**
1d) $w_1 = 100$, $\mathbf{s_1} = \mathbf{0.26}$, $\pi_1 = 0.2$, $\gamma_1 = 0.2$							
$\tilde{Q}$		0.8	0.782	0.766	0.756	0.743	0.732
A_1		0	2.50	5.00	6.53	8.86	10.81
A_2		21.67	20.45	20.75	20.88	20.98	20.98
$\partial A_1/\partial b$	Case (i)	--	7.62	7.10	6.77	6.25	5.80
	Case (ii)	--	3.63	2.65	2.05	1.13	0.35
$\partial A_2/\partial b$	Case (i)	25.65	20.09	17.48	15.83	13.27	11.05
	Case (ii)	17.39	12.13	9.24	7.47	4.76	2.45
$\partial A_1/\partial b/A_1 - \partial A_2/\partial b/A_2$	Case (i)	--	2.07	0.58	0.28	0.07	0.01
	Case (ii)	--	0.86	0.08	**-0.04**	**-0.10**	**-0.08**
1e) $\mathbf{w_1} = \mathbf{150}$, $\mathbf{s_1} = \mathbf{0.24}$, $\pi_1 = 0.2$, $\gamma_1 = 0.2$							
$\tilde{Q}$		0.792	0.750	0.732	0.721	0.705	0.693
A_1		1.21	8.27	11.92	14.14	17.51	20.35
A_2		21.93	22.99	23.27	23.37	23.41	23.35
$\partial A_1/\partial b$	Case (i)	13.65	12.64	11.96	11.50	10.76	10.10
	Case (ii)	8.59	6.12	4.69	3.79	2.40	1.21
$\partial A_2/\partial b$	Case (i)	25.85	21.10	18.44	16.75	14.10	11.79
	Case (ii)	17.97	12.41	9.45	7.62	4.82	2.43
$\partial A_1/\partial b/A_1 - \partial A_2/\partial b/A_2$	Case (i)	10.10	0.61	0.21	0.10	0.01	**-0.01**
	Case (ii)	6.27	0.20	**-0.01**	**-0.06**	**-0.07**	**-0.05**

Table 2: ***Scenario 2:*** "Moderate" social security system

$b = 0$, $\tau_w = 0.3$, $\tau_S = 0.1$, $w_2 = 150$, $s_2 = 0.14$ $\pi_2 = 0.8$, $\gamma_2 = 1 - \gamma_1$

		$\rho = 2$	$\rho = 3$	$\rho = 4$	$\rho = 6$	$\rho = 7$	$\rho = 16$
2a) $w_1 = 100$, $s_1 = 0.18$, $\pi_1 = 0.2$, $\gamma_1 = 0.2$							
$\tilde{Q}$		0.768	0.752	0.744	0.737	0.734	0.727
A_1		9.07	13.87	16.40	19.01	19.77	22.36
A_2		39.74	40.09	40.18	40.22	40.21	40.17
$\partial A_1/\partial b$	Case (i)	12.28	11.68	11.32	10.95	10.84	10.48
	Case (ii)	5.52	3.57	2.49	1.29	1.07	*-0.01*
$\partial A_2/\partial b$	Case (i)	34.49	28.93	26.02	23.03	22.16	19.17
	Case (ii)	18.67	12.71	9.66	6.56	5.66	2.61
$\partial A_1/\partial b/A_1 - \partial A_2/\partial b/A_2$	Case (i)	0.49	0.12	0.04	0.003	**-0.003**	**-0.008**
	Case (ii)	0.14	**-0.06**	**-0.09**	**-0.09**	**-0.09**	**-0.07**
2b) $w_1 = 100$, $s_1 = 0.18$, $\boldsymbol{\pi_1 = 0.4}$, $\gamma_1 = 0.2$							
$\tilde{Q}$		0.763	0.758	0.755	0.752	0.751	0.749
A_1		16.39	18.91	20.19	21.47	21.83	23.07
A_2		39.96	39.88	39.81	39.73	39.70	39.59
$\partial A_1/\partial b$	Case (i)	16.91	14.81	13.72	12.59	12.26	11.15
	Case (ii)	7.95	5.13	3.67	2.19	1.76	0.30
$\partial A_2/\partial b$	Case (i)	34.64	28.82	25.88	22.91	22.05	19.16
	Case (ii)	18.07	12.17	9.20	6.21	5.36	2.47
$\partial A_1/\partial b/A_1 - \partial A_2/\partial b/A_2$	Case (i)	0.16	0.06	0.03	0.01	0.006	**-0.001**
	Case (ii)	0.03	**-0.04**	**-0.05**	**-0.05**	**-0.05**	**-0.05**
2c) $w_1 = 100$, $s_1 = 0.18$, $\pi_1 = 0.2$, $\boldsymbol{\gamma_1 = 0.4}$							
$\tilde{Q}$		0.719	0.687	0.671	0.656	0.652	0.638
A_1		9.90	14.96	17.60	20.30	21.08	23.73
A_2		42.06	42.79	42.93	42.92	42.88	42.68
$\partial A_1/\partial b$	Case (i)	12.84	12.23	11.78	11.25	11.08	10.47
	Case (ii)	6.66	4.61	3.42	2.13	1.75	0.42
$\partial A_2/\partial b$	Case (i)	36.07	30.29	29.09	23.69	22.69	19.18
	Case (ii)	21.54	14.79	11.49	7.90	6.85	3.27
$\partial A_1/\partial b/A_1 - \partial A_2/\partial b/A_2$	Case (i)	0.44	0.11	0.04	0.002	**-0.003**	**-0.008**
	Case (ii)	0.16	**-0.04**	-0.07	**-0.08**	**-0.08**	**-0.06**
2d) $w_1 = 100$, $\boldsymbol{s_1 = 0.16}$, $\pi_1 = 0.2$, $\gamma_1 = 0.2$							
$\tilde{Q}$		0.763	0.748	0.740	0.733	0.731	0.723
A_1		10.59	15.29	17.77	20.33	21.07	23.61
A_2		39.96	40.25	40.32	40.33	40.32	40.46
$\partial A_1/\partial b$	Case (i)	12.53	12.01	11.69	11.35	11.25	10.94
	Case (ii)	5.70	3.82	2.79	1.72	1.41	0.38
$\partial A_2/\partial b$	Case (i)	34.64	29.01	26.08	23.06	22.19	19.18
	Case (ii)	18.62	12.64	9.58	6.46	5.56	2.51
$\partial A_1/\partial b/A_1 - \partial A_2/\partial b/A_2$	Case (i)	0.32	0.06	0.01	**-0.01**	**-0.02**	**-0.01**
	Case (ii)	0.07	**-0.06**	**-0.08**	**-0.08**	**-0.07**	**-0.05**
2e) $\boldsymbol{w_1 = 150}$, $\boldsymbol{s_1 = 0.14}$, $\pi_1 = 0.2$, $\gamma_1 = 0.2$							
$\tilde{Q}$		0.739	0.719	0.710	0.700	0.697	0.687
A_1		18.66	25.68	29.38	33.19	34.30	38.08
A_2		41.07	41.39	41.44	41.40	41.38	41.24
$\partial A_1/\partial b$	Case (i)	19.50	18.79	18.32	17.78	17.62	17.07
	Case (ii)	9.34	6.57	5.02	3.38	2.90	1.27
$\partial A_2/\partial b$	Case (i)	35.40	29.59	26.52	23.33	22.40	19.19
	Case (ii)	19.50	13.26	10.03	6.75	5.80	2.55
$\partial A_1/\partial b/A_1 - \partial A_2/\partial b/A_2$	Case (i)	0.18	0.02	**-0.02**	**-0.03**	**-0.03**	**-0.02**
	Case (ii)	0.03	**-0.06**	**-0.07**	**-0.06**	**-0.06**	**-0.03**

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