Financial Contagion and Network Analysis

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Abstract
Network models of interbank exposures allow the mapping of the complex web of financial linkages among many institutions and address issues of system stability and contagion risk. Although existing models cover a fair amount of ground in explaining how network structure can lead to default cascades and in quantifying the likelihood and the impact of default cascades through balance-sheet mechanics, the literature has shortcomings in explaining how shocks are potentially amplified through the network of exposures. These amplification mechanisms seem to be very important in financial crises. This review discusses the main conceptual ideas behind network models of contagion, the major findings of this literature, as well as some limitations of existing models.
1. INTRODUCTION

Will the failure of a financial institution trigger the failure of others and perhaps even escalate into a widespread default cascade? Is the complicated network of mutual debt exposures between financial institutions and between the financial and the real sector of the economy a fragile web of promises that can easily collapse in a domino effect? Or does this network have structural features that would allow us to conclude that it is fairly robust and resilient? These are key questions that need to be answered both in crisis management as well as in policy discussions aimed at safeguarding the stability of the financial system.

Concepts and tools from network analysis have advanced recently the analysis of financial exposure webs and its implications for financial fragility and stability. Although there is not yet a unified framework that is able to fully embed the literature on financial networks into the wider field of economic theory, there seem to be some recurring economic intuitions that are mentioned in many of the papers written on this topic: A highly interconnected network of financial exposures might facilitate the sharing of aggregate risks and make the financial system more resilient to shocks. A shock to a highly interconnected system might be more easily dissipated because it can be absorbed by more parties. Thus, the share of the shock borne by each individual agent or institution is small enough to prevent, for instance, fire sales and the resulting distress in market and funding liquidity (Brunnermeier & Pedersen 2009, Geanakoplos 2009). Of course, this risk-sharing intuition depends on what the networks look like precisely and which agents are connected by financial exposures. The actual network of exposures may be conducive to the sharing of aggregate risk, or it may provide an obstacle to risk-sharing. How exposure networks actually look and how they function are key questions in that regard.

Another frequently recurring intuition is that highly interconnected systems might be more prone to the contagion of shocks. This is particularly relevant when individual agents have incomplete information about the precise pattern of exposures. Thus, a highly interconnected exposure network with incomplete information about exposures might be particularly prone to runs and thus be more fragile than a network not so tightly interconnected. Shin (2010) discusses another economic mechanism by which a more tightly connected system of exposures may be more prone to fragility: He shows how longer intermediation chains make financial institutions more dependent on capital markets and thus amplify the systemic feedback loops between leverage, asset prices, and risk-taking that seem to underlie so many recent and past financial crises. From this perspective, financial networks seem to be connected to multiplier effects that make initially small exogenous shocks bigger through an endogenous mechanism that is a function of the network structure.

The growing literature on financial networks and contagion contributes to the analysis of these economic intuitions and to the deeper understanding of the exact role of the exposure networks in the origin and spread of financial crisis. Although many papers in the literature study only partial aspects of these intuitions, they have to be understood against the backdrop of these underlying economic ideas.

Network concepts seem particularly relevant when questions of contagion are the focus. Contagion of distress through direct or indirect interbank exposures is of course only one, albeit a potentially important, mechanism by which distress at one institution can spread in the financial system. The precise mechanism that spreads distress can in general take many different forms ranging from bank runs to asset pricing (see Upper 2011). Obviously, not all of them need network concepts as a tool for analysis. For a deeper understanding of circumstances and conditions under which default cascades can occur, however, we need a precise map of how the network of financial promises is structured. In this review, I survey the recent literature on this form of interbank
contagion and discuss some of the conceptual ideas behind network models of default cascades, their major findings as well as their limitations.

When we talk about network models we need to be more precise: A network is an abstract concept that can be applied to many different contexts, which can be described by a set of nodes and a set of links between them. Newman (2012) gives an impressive overview on the diversity of applications that may be covered. In the context of finance, the most important distinction is whether we study the network of financial exposures (a network of stocks) or the network of payments (a network of flows). Both perspectives are dealt with in the literature but for the analysis of contagion and default cascades, studies of exposure networks have been more important. In this review, I focus on the literature on exposure networks and refer to other papers (see for instance Jaramillo et al. 2012 or Kyriakopoulos et al. 2009) for literature on payment flow networks.

I start by surveying the literature on the empirical description of exposure networks of interbank liabilities in Section 2. This research established some facts that are both relevant in terms of contagion risk analysis as well as for network modeling. Section 3 discusses the small theoretical literature in economics that made attempts to bridge traditional economic models of equilibrium coordination problems and the network literature. The literature on financial contagion and networks that was perhaps most prolific is the literature on simulation models of contagion discussed in Section 4. I give an overview of the concepts and findings of this literature. Section 5 reports on papers that looked deeper into the structural features of the dependence of cascading behavior and network topology. Section 6 puts the network literature into the broader context of the economic discussion about systemic risk. Sections 7 and 8 go one step further and discuss the use of network models of contagion for economic policy as well as the difficulties to connect the network literature with the rest of the recent economic literature on the financial crisis. Section 9 concludes.

2. FINANCIAL NETWORKS: WHAT DO THEY LOOK LIKE?

The structure of interbank exposure networks has been studied in various papers, starting with Boss et al. (2004). In an interbank exposure network, the nodes are banks. If banks have a debt exposure to another bank, there is a link between them. If information on the size of the exposure is included, these links can also be weighted by the value of the liabilities. Only few papers study the richer networks of debt and equity exposures between banks (see, e.g., Boss et al. 2006 and Elsinger 2009).

The specific interest in exposures of debt or debt-like claims comes from the interest in the analysis of contagion risk, when at least one bank becomes unable to service its debts. In this case, the claims to these debts as well as the control rights of the assets that are financed by them have to be redefined. The complexity of this task is legally regulated by bankruptcy procedures and can cause many disruptions with spillovers to the real economy. Liquidation of assets as a consequence of disruptions in debt service may have price impacts and constitute a source of systemic risk. There are thus good economic reasons to focus on networks of debt exposures.

From the papers on financial networks of interbank debt, we have gained some understanding of the structural features of typical exposure networks in some countries around the globe and also internationally. Typically, these networks show a pronounced core-periphery structure with few banking hubs with several connections and many other banks with only a few connections.

Before we describe the findings in the literature in more detail, I point out why these empirical illustrations of the interbank exposure network are important. First, the structural features may contain some interesting information about the form of systemic failure, which such a network is
prone to. Second, the empirical picture also gives insights into particular structural features that have to be included into network models.

An early study of the network topology of a national interbank exposure network is Boss et al. (2004). The authors study approximately ten interbank exposure matrices reconstructed from quarterly data sources of the Austrian Central Bank (OeNB) from 2000 to 2003. The exposure network maps interbank debts between 900 Austrian banks, the OeNB, and an aggregate node representing all exposures outside Austria. The Austrian banking system has a two-tiered structure with core or head institutions of various banking groups, placed mainly in the capital Vienna, and a periphery of local banks belonging to these groups, placed mainly in the countryside and the Austrian federal states. Figure 1 shows the Austrian network as of 2002.

Another important feature of this network is that there are only a few institutions with many exposures and many institutions with only a few. Such a property can be expressed in technical terms by the distribution of so-called node degrees, the number of links in each node. This picture can be refined by specifying additionally a direction for each link and distinguishing whether the link leads out of a node or points into a node; this is referred to as out-degree or in-degree, respectively. That there are only few institutions with many exposures and many institutions with

Figure 1
The Austrian interbank network. Clusters are grouped and colored. The core of the sectors’ head institutions are connected to each other and connected to their peripheral affiliated institutions. This figure is reproduced from Boss et al. (2004).
only a few is reflected in in- and out-degree distributions that are skewed to the left and heavy tailed. The out- and in-degree distributions both show power laws with an exponent between 2 and 3. This is a feature of many other networks across different contexts (see Newman 2012). The other structural feature Boss et al. (2004) describe is that there is only a small degree of clustering in the interbank network. This means that the probability that two banks that have a link to any given bank are also themselves connected is small. This reflects the tiering structure of the interbank network. The average path length of links between banks is small. In the Austrian exposure network, it takes on average two links to connect any two banks in the network.

Cont, Moussa & Santos (2013) study the Brazilian interbank exposure network with a dataset of considerably finer resolution than the data used in Boss et al. (2004). The network is analyzed for the period from January 2007 until November 2008 and leads to similar structural findings as those in the previous study. Cont, Moussa & Santos (2013) show, using formal statistical tests, that although the precise pattern of exposures does of course change over time, the degree distribution of the network stays stationary. It also has power law features that are very similar to the Austrian network. They show also that the distribution of exposures is fat tailed. They find that institutions with larger (smaller) exposures are also more (less) connected. In terms of clustering, they find that financial institutions with a small degree (few links) have counterparties that are very strongly connected to each other, whereas financial institutions with a large degree (many links) have counterparties with sparsely connected neighbors. Core-periphery structures and power laws in the degree distributions are found also in other studies; for Germany, see Craig & von Peter (2010), the United Kingdom, Langfield, Liu & Ota (2012), and in an international context, von Peter (2007).

Jaramillo et al. (2012) study in detail a comprehensive dataset of interbank exposures of the Mexican banking system. Although in their paper they are mainly interested in the pros and cons of network centrality measures as a way to empirically determine the systemic importance of an institution, they establish an important empirical fact: The authors provide evidence that one popular network formation model, the preferential attachment model (Barabási & Albert 1999), does not apply to the formation of interbank exposure networks.

Overall, the descriptive evidence on interbank exposure networks is that they have a core-periphery structure with heavy-tailed degree and exposure-size distributions. Networks with such a structure are quite resilient to random disruptions but highly vulnerable to disruptions in hubs. Finally, in terms of modeling exposure networks they seem neither very well captured by random graph models nor by network structures arising from preferential attachment dynamics.

3. THEORETICAL STUDIES

During the past decade, several papers have tried to deal theoretically with the financial stability issues in the context of exposure networks. The main motivation behind this research is that for issues of financial stability, in particular problems of insolvency and illiquidity contagion, it is not sufficient to study institutions in isolation. From the viewpoint of this literature, it is also not sufficient to study financial institutions simultaneously as is done in general equilibrium or in game theory. This literature takes the perspective that the precise topology of exposure linkages, the particular way the banking system is wired at a particular moment in time, might play an important role for the risk of the system as a whole.

As a general approach, these papers are all in the spirit of the literature on games on networks (see Jackson & Zenou 2013), where the network topology is taken as a constraint on behavior, interaction opportunities, and equilibrium. The focus is on the consequences of different topologies of the exposure network for the impact of a particular institution’s failure.
3.1. Insolvency Contagion

An early important contribution to this strand of literature was Allen & Gale (2000). The authors study an example in which banks in four different regions face negatively correlated liquidity shocks (Figure 2). In such a situation, it can be optimal for banks to hold part of their deposit base in other regions to provide liquidity in times of elevated liquidity demand in their own region. In the case of a banking crisis in one region, the claims of other banks on this region lose value and can cause a banking crisis in the adjacent regions. The crisis can gain momentum and spread in a contagion effect to all regions. The main theoretical result is that in their example, contagion can spread more easily if the network topology is such that the exposure network has the form of a circular exposure of one region against its neighboring region. Allen & Gale (2000) call this topology an incomplete network. If the regions are fully connected (Allen & Gale call this a complete network), the network is more resilient in absorbing losses.

Although cast in a highly stylized example, this paper has been very influential because it combined a classical model by Diamond & Dybvig (1983) of bank runs as coordination failures in depositor liquidity demand with issues of exposure network topology and insolvency contagion. The results turned out difficult to generalize, and it remains unclear which results depended on the specific setup and the symmetry of the example. For instance, the claim that a complete network is more resilient to insolvency contagion could not be reproduced in simulation studies as reported in Boss et al. (2004).

3.2. Liquidity Gridlock

Another influential paper from around the same time is Freixas, Parigi & Rochet (2000). These authors also made the point that exposure networks can matter for the equilibrium outcomes in a financial system of banks providing liquidity. As in Allen & Gale (2000), there is a system of banks each located in different regions and modeled individually in the Diamond & Dybvig (1983) framework. There is depositor mobility between locations, and depositors can only invest their funds in a bank at their home location. At the system level, banks can improve on the liquidity provision they would choose in isolation by granting each other credit lines so that consumers who travel and have a demand for liquidity at another location can get a claim written on their home bank and credited at the bank of their travel destination.

![Figure 2](https://example.com/figure2.png)

The two network topologies discussed in the example of Allen & Gale (2000). Nodes are banks, and links are exposures. The arrows symbolize the direction of exposure. The network on the left is called incomplete; the network on the right is called complete.
A coordination problem leads to multiple equilibria: one in which an efficient liquidity provision on the system level emerges as a self-enforcing arrangement with intraregional credit lines and one where bank runs occur. The network begins to play a crucial role when one bank is insolvent and efficiency would require a closure. This might be impossible without triggering a systemic crisis, and there is under certain conditions an incentive to absorb the losses in the system. Thus, the exposure network leads to a decline in market discipline. Unlike in Allen & Gale (2000), the network topology itself is fixed, and Freixas, Parigi & Rochet (2000) do not compare different exposure network topologies.

Although both papers built a bridge between game theoretic models of liquidity provision and exposure networks, a more general analysis turned out difficult. Although attempts were made to generalize results (see Lagunoff & Schreft 2001, Dasgupta 2004, Leitner 2005) or to pursue this research agenda more systematically, the theoretical literature on networks and financial contagion remains sparse. Recently, there is renewed interest in theoretical network modeling of financial systems. Zawadowski (2013), for instance, analyzes how exposure networks in OTC markets induce banks to risk-shifting behavior that results in the spread of idiosyncratic bank failures into a systemic crisis.

4. SIMULATION STUDIES

For researchers who want to study domino effects empirically, there is one fundamental difficulty: lack of observations. In a severe banking crisis, the authorities usually do not stand passively at the sidelines and watch the unraveling of the financial system. Usually, there are various forms of rescue operations, and there are not enough observations where the spillover of difficulties at one institution to the system as a whole can actually be observed. Thus many researchers interested in empirical approaches to contagion analysis turned to simulation studies. They are a potentially interesting way to get an estimate and quantification of the risks involved in a default cascade. This field was perhaps also the field in which the research agenda on networks and contagion was most prolific in the past.

Although the pioneering paper by Furfine (2003) was on counterparty default and contagion in payment systems, the interest of the literature turned quickly to banking systems, starting with Upper & Worms (2004); Wells (2004); and Elsinger, Lehar & Summer (2006a,b). Meanwhile, there are many simulation studies of insolvency contagion or domino effects for many datasets around the world. These papers have recently been surveyed excellently by Upper (2011). I refer the readers who are interested in a fairly complete list of recent contributions to this survey. Here, I give a selective overview of the main ideas and findings of this literature.¹

4.1. Simulation Models

Simulation studies start from mapping the exposure network of a given banking system. The financial network in these studies consists of \( n \) banks, which have for each institution \( i \) non-interbank-related assets \( a_{i}^{NIB} \) and interbank assets \( a_{i}^{IB} \) on their balance sheet. On the liabilities side, there are interbank liabilities \( d_{i}^{IB} \), as well as liabilities to creditors outside the network \( d_{i}^{NIB} \) and of course as a residual equity \( e_{i} \). The value of the noninterbank assets \( a_{i}^{NIB} \) can be interpreted as an exogenous random variable. The values of all the other parts of the balance sheet are determined endogenously within the network conditional on a particular draw of \( a^{NIB} = (a_{1}^{NIB}, \ldots, a_{n}^{NIB}) \).

¹In this section, I draw extensively on Elsinger, Lehar & Summer (2013).
Not all the liabilities will be of the same seniority, and some of the interbank liabilities may be more senior than others. Network models are able to take this into account correctly, as is shown in Elsinger (2009). To keep the description of the model as simple as possible, assume in this context that there is only one seniority class.

The structure of the interbank liabilities is represented by an \( n \times n \) matrix \( L \), where \( l_{ij} \) represents the nominal obligation of bank \( i \) to bank \( j \). These liabilities are nonnegative, and the diagonal elements of \( L \) are zero as banks are not allowed to hold liabilities against themselves. Evidently,

\[
\sum_{j=1}^{n} l_{ij} = d_i^{IB} \quad \text{and} \quad \sum_{i=1}^{n} l_{ij} = a_j^{IB},
\]

where \( d_i^{IB} \) and \( a_j^{IB} \) denote the nominal values of the interbank claims and liabilities in contrast to the endogenously determined market values \( d_i^{IB} \) and \( a_j^{IB} \).

A bank is defined to be in default whenever exogenous income plus the amounts received from other nodes are insufficient to cover the bank’s nominal liabilities.\(^2\) It is often assumed that bank defaults do not change the prices outside of the network; i.e., \( a_{NIB} \) is independent of defaults and exogenous. Some recent papers, in particular Alessandri et al. (2009), following Cifuentes, Ferrucci & Shin (2005), relax this assumption.

In the case of default, three criteria have to be respected:

1. Limited liability: Requires that the total payments made by a node must never exceed the sum of exogenous income and payments received from other nodes.
2. Priority of debt claims: Requires that stockholders receive nothing unless the bank is able to pay off all of its outstanding debt completely.
3. Proportionality: Requires that in the case of default, all claimant nodes are paid off in proportion to the size of their claims on bank assets.

To operationalize proportionality, let \( \bar{d}_i \) be the total nominal obligations of node \( i \), i.e.,

\[
\bar{d}_i = d_i^{IB} + d_N^{NIB} = \sum_{j=1}^{n} l_{ij} + d_N^{NIB},
\]

and define the proportionality matrix \( \Pi \) by

\[
\Pi_{ij} = \begin{cases} \frac{l_{ij}}{\bar{d}_i} & \text{if } \bar{d}_i > 0 \\ 0 & \text{otherwise} \end{cases}
\]

A network model in the form presented above is used in Elsinger, Lehar & Summer (2006a,b). All the papers discussed in Upper (2011) use variants of such a framework. Although there are quite a few inessential differences in the details, two differences are essential: They come from the way insolvency events are modeled and the way the consequences of insolvency events are treated within the network model.

As to the modeling of insolvency events, most simulation studies assume an idiosyncratic failure of a single institution. In contrast, the studies by Elsinger, Lehar & Summer (2006a,b); Cont, Moussa & Santos (2013); and Alessandri et al. (2009) assume that the value of \( a_i^{NIB} \) is an

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\(^2\)A bank is in default if the value of the liabilities exceeds the value of the assets. Using a violation of capital requirements as a default threshold does not change the main results.
the actual payments made do not cover the liabilities $F_i$ defined as

Alternatively, a clearing vector $\mathbf{d}$ can be characterized as a fixed point of the map $\Phi^1(\cdot; \mathbf{d}, \mathbf{a}_{\text{NIB}}, \Theta): [0, \overline{d}] \rightarrow [0, \overline{d}]$ defined by

$$\Phi^1(\mathbf{d}; \mathbf{d}, \mathbf{a}_{\text{NIB}}) = \left\{ \mathbf{a}_{\text{NIB}} + \mathbf{d} \right\} \wedge \overline{d}. \tag{4}$$

$\Phi^1$ returns the minimum of the maximum possible payment and the promised payment $\overline{d}$. Hence, any supersolution $\mathbf{d} \geq \Phi^1(\mathbf{d})$ is compatible with absolute priority but not necessarily with limited liability.

Eisenberg & Noe (2001) prove that a clearing vector exists for each realization of $\mathbf{a}_{\text{NIB}}$. Under mild regularity conditions on the network structure, the clearing vector is unique.

...
Clearing vectors can be calculated in different ways using relatively simple and fast algorithms. Elsinger, Lehar & Summer (2013) describe some of the technical details. That the clearing algorithms reveal a sequence of defaults makes this approach attractive for contagion analysis. The default sequence cannot be interpreted as showing an order in a dynamic default process. The sequence, however, allows for the distinction of whether a default occurred directly as a consequence of shocks to the given exposures or indirectly through contagion in the interbank network.

This allows one to conduct contagion risk analysis based on idiosyncratic failure scenarios. It can be furthermore combined with a richer analysis of loss scenarios based on Monte Carlo simulation of shocks to the asset values $a^{NIB}$. The simulation of loss scenarios can be based on bank exposure data, if appropriate data are available. Especially countries with central loan registers, such as, for example, Germany, Italy, Spain, and Austria, allow for a fairly detailed mapping of the domestic exposure network. International exposures can usually be included only in the form of an aggregate node that takes into account the aggregate nondomestic exposures.

The papers that simulate aggregate shocks usually first simulate a macroeconomic scenario that affects all banks. Common variation in credit risk, which is often the most significant risk for banks, can be captured by scenarios of industry probabilities of default (PD). Gauthier, Lehar & Souissi (2012) use draws from a distribution of industry PDs under a macro-stress scenario specified by the Bank of Canada and the International Monetary Fund (IMF) during one of its financial stability assessments. Elsinger, Lehar & Summer (2006a) assume that average default rates are gamma distributed with parameters estimated from historical data. Similarly, other risk factors such as foreign exchange rates and interest rates can be simulated conditional on the macroeconomic scenario.

Individual losses for banks can then be derived conditional on the macro scenario. For credit risk, one draw of the macro scenario specifies the average PD per industry. Each of the bank’s loans can be seen then as a Bernoulli-distributed random variable that either gets repaid in full or defaults, in which case only a fraction of the loan can be recovered. Total loan losses for each bank are then just the sum of outcomes of the Bernoulli variables and can be obtained via Monte Carlo simulation. For computational convenience, Elsinger, Lehar & Summer (2006a) and Gauthier, Lehar & Souissi (2012) use a simplified CreditRisk+ model to derive each bank’s distribution of loan losses conditional on the macro scenario and then take draws from this distribution to create loss scenarios.

If loan registers are not available as a data source, there is still the possibility to model loss scenarios for banks using stock market data. This approach has the advantage that it can incorporate information beyond that contained in central bank reports by including the market’s belief on the state of the banks. Stock prices reflect a bank’s exposure to all risks including those not captured in central banks’ reports. Market data is also available at a higher frequency, allowing almost instantaneous assessments of financial stability. The problem with market data is that it not only includes the market’s view on bank losses but also the market’s view on regulatory action. Some banks may be perceived as less risky because they are too big to fail and therefore would be bailed out by the government in the case of distress. Anticipated government intervention might thus contaminate exactly those observations that are most valuable for measuring systemic risk, the ones in the left tail of the distribution.

To generate bank loss scenarios most authors follow Merton (1973), who assumes that the market value of the assets follows a geometric Brownian motion and interprets equity as a call option on assets with a strike price equal to liabilities. Although there is no obvious choice for the maturity of debt, it can be interpreted as the time until the next audit of the bank, when the regulator can observe the asset value and close the bank if it is undercapitalized.
The maximum likelihood estimator developed by Duan (1994, 2000) allows one to estimate the market values of banks’ assets, their volatilities, the drift parameters, and the correlation matrix from stock price data. Lehar (2005) uses this information to simulate scenarios for future asset values. To capture banks’ systematic risk, it is important to include the correlation structure in bank asset returns. Elsinger, Lehar & Summer (2006b) combine these simulated scenarios with a network model to estimate systemic risk for the British banking system.

A practical difficulty is usually the availability of data on bilateral interbank exposures contained in the entries of the L matrix. Although quite a few countries maintain central loan registers, other countries do not have such data sources. Some simulation studies have thus resorted to methods attempting to estimate the bilateral exposures from total exposure data plus some known constraints. One such constraint is for instance that no bank has exposures to itself. These estimation techniques are also known as maximum entropy estimation. They usually tend to distribute the aggregate exposures as evenly as possible across institutions in the network given the constraints imposed by the exact matrix entries known. A detailed description of how such estimation techniques work technically can be found in Elsinger, Lehar & Summer (2013). The systematic errors brought into contagion simulation studies by estimating rather than observing the L matrix were investigated by Mistrulli (2007), Degryse & Nguyen (2007), and Lelyveld & Liedorp (2006). The general consensus seems to be that estimation of the L matrix typically leads to underestimation of contagion.

4.2. Empirical Findings

The empirical assessment of simulation studies of contagion finds, in general, the following:

1. Contagion of insolvency due to interbank exposures is rare.
2. It is very hard to create quantitatively realistic scenarios that will lead to a significant amount of contagion.

The first finding has been observed in numerous simulation studies on interbank markets around the world (see Upper 2011). The second finding is reported mainly in Elsinger, Lehar & Summer (2006a,b). This is because these papers build simulations that do not rely on idiosyncratic hypothetical failure scenarios for individual institutions but work with a simulated profit and loss distribution for the entire banking portfolio.


The studies working with correlated exposures and shocks to the asset values \( a_{NIB} \), such as Elsinger, Lehar & Summer (2006a,b); Cont, Moussa & Santos (2013); and Alessandri et al. (2009), also find that contagion is likely to be rare. A contagion model based on the ideas in Elsinger, Lehar & Summer (2006a) has been implemented at the OeNB (Boss et al. 2006) and is used by financial stability analysts. Data are updated every quarter, and regular simulations are run. That the majority of defaults in the simulation come from direct exposure to risk factors and that domino effects of contagion are likely to be rare turned out to be a robust feature of the simulations over time. This fact remained robust even when data from the financial crisis entered the simulation.

Although most of these studies conclude that contagion is likely to be rare, they do not reach a clear-cut result about the potential impact of contagion. Although some studies found that the
worst case domino effects could destroy between 15% and 20% of the banking system, others found only minor impacts. An exception is a study based on Swedish data (Frisell et al. 2007) using aggregate shocks to bank portfolios along the lines of Elsinger, Lehar & Summer (2006b). They find a high probability of domino effects, occurring in approximately one-half of the cases in which one of the top four Swedish banks fail. In summary, these studies find that domino effects are unlikely, but when they occur they may affect a substantial part of the banking system.

The second finding, that contagion is likely to occur only in doomsday scenarios, seems to be mainly reported in Elsinger, Lehar & Summer (2006a). There, introducing the assumption that in a default assets are partially destroyed due to bankruptcy costs helps to determine under which circumstances contagion becomes widespread. It is found that there is little contagion for low bankruptcy costs, but as total assets are destroyed up to an amount of 30% and more, the number of contagious defaults increases sharply.

This loss rate on the value of total assets beyond which domino effects start playing a significant role could be taken as a benchmark. If we look at the loss distributions resulting from simulations of losses due to market and credit risk reported in Elsinger, Lehar & Summer (2006a), we see that in 2006 even the extreme quantiles of the loss distribution for market risk would destroy only \( \sim 1.6\% \) of total assets in the entire system in the case of market risk and \( \sim 1\% \) in the case of credit risk. Even if default costs were added and the losses from shocks were much worse, we were not near the threshold of 30%, above which domino effect contagion becomes a significant issue.

### 4.3. Lessons from Simulation Studies

All the papers discussed here were written before the financial crisis, and none of the models predicted the crisis with its actual scope and depth. So, do the empirical findings reported here suggest that simulation studies of contagion were just another contribution to professional pre-crisis complacency with respect to the actual risks borne by the financial system?

Let me give another, perhaps more positive, interpretation of these findings. The empirical findings help to refocus the research agenda on systemic risk. This is because they settle the issue of how important domino effects of insolvency working through the balance-sheet mechanics of the banking system really are. The answer is that they are just not very important.

Before the financial crisis, many institutions were preoccupied with stories of domino effects and systemic risk. But, what we learn from the research is that there is no way that the losses from the US subprime crises that were predicted by the IMF in April 2008 at approximately $945 billion (IMF 2008)—although huge as an absolute number—would be able to bring down the world financial system. The dynamics of a financial crisis come from powerful amplification mechanisms that have to do with the interaction of behavior and the pricing of risk (see Shin 2010). The crisis showed and reminded us quite clearly that the core mechanisms at work in the buildup and the unfolding of a financial crisis lie in the interaction between leverage, asset prices, and portfolio decisions. Models of domino effects arising from the balance-sheet mechanics of a complicated web of interbank debt can in principle be extended to take these dynamics into account. This research remains, however, yet to be done.

The qualitative aspects of the amplification mechanisms were analyzed and described in quite a few books and papers that have been published since the financial crisis. These include, among others, Shin (2010), Geanakoplos (2009), Kiyotaki & Moore (2008), Brunnermeier & Pedersen (2009), Brunnermeier (2009), Holmstöm & Tirole (2011), and Hellwig (2008). How the insights of this literature can be applied in the quantitative analysis of systemic risk is still an open and largely unresolved issue.
4.4. Next Steps Forward

However, the literature on the quantitative analysis of financial amplification mechanisms is moving very quickly, and some recent contributions in both theory and empirical work show the way to progress.

Cont & Wagalath (2013) propose a way to quantify the influence of fire sales on both prices and the risk factor distribution. Starting from assumed deleveraging schedules for banks, and assuming that in the course of deleveraging assets are sold proportionally, they show that realized correlations between returns of assets increase further in bad scenarios, due to deleveraging. Such an approach could be the basis of stress test procedures taking into account endogeneity of risk and feedback effects of market participants’ reaction to adverse scenarios. They apply this approach to the analysis of fire sales and the quantification of their impact. Cont & Wagalath (2012) apply these techniques to devise a test for the presence of fire sales and the estimate of their magnitude and impact. Both papers build on the theory of stochastic processes and abstract away from network modeling. They do, however, provide guidance on how quantitative models for fire sale feedback could perhaps be combined with an analysis of an exposure network in future research.

The network literature also led to interesting new empirical research that tries to provide evidence for the importance of certain channels of contagion assumed and studied in both the theoretical and the simulation literature. Duarte & Kolasinski (2012), using credit default swap spread data on broker-dealers, devise an econometric test to decide whether direct interconnections, i.e., the network topology, or endogenous feedback and liquidity spirals explain contagion of distress between them. They find that direct interconnection effects account for \(\sim 86\%\) of the total contagion effect.

Two recent contributions by Cohen-Cole, Kirilenko & Patacchini (2013) and Cohen-Cole, Patacchini & Zenou (2012) directly tackle the challenge of combining the analysis of financial networks with the analysis of economic behavior. In Cohen-Cole, Kirilenko & Patacchini (2013), the authors draw on the literature on social interactions and apply it to the analysis of financial trading networks. They suggest a definition of systemic risk in terms of how the network topology will amplify impacts of individual behavior through strategic interaction on the network. They apply this approach to empirically study the network of realized trades on the Chicago Mercantile Exchange.

In Cohen-Cole, Patacchini & Zenou (2012), oligopolistic competition in an interbank market is studied by modeling strategic interaction on a network directly. Using again results from the literature on social interactions, the authors are able to analyze how uncertainty, risk, or behavior is propagated through the network, even without any default occurring during the interaction. In this model, there is a direct link between network topology, behavioral incentives, financial market prices, and volatility. In this framework, the authors are able to study theoretically and empirically the aggregate liquidity cost of a reduction in interbank lending by an individual financial institution. They find multipliers that grow as large as 2.5 times the initial shocks.

Although there is still no unifying framework that links models of financial networks with behavioral models that lead both to empirical frameworks as well as to quantitative simulation tools, these papers clearly show what the next steps forward in this literature might look like.

5. NETWORK TOPOLOGY AND FINANCIAL FRAGILITY

The theoretical literature as well as the simulation studies have raised the question of how contagion of insolvency, the cascading behavior of default, is related to the underlying network
topology. Although Nier, Yorulmazer & Alenthorn (2007) tried to look at this question from a simulation perspective, more recently Amini, Cont & Minca (2010) have made substantial analytical progress. These authors apply powerful tools from modern graph theory and probability. This paper introduced new concepts to the literature on financial networks and contagion and established a connection to the literature in economics and sociology that was interested in cascades on networks in a generic context, such as Morris (2000), Kleinberg (2007), Jackson & Yariv (2012), and Watts (2002). Applying to exposure networks a more general understanding of cascading behavior and network topology allows for the identification of nodes that pose a high cascading risk on the system. What are the exact criteria that give an institution systemic importance in this sense?

Amini, Cont & Minca (2010) study a network of interlinked balance sheets in which losses flow through the asset side of the balance sheet. Using the notation used in Section 4.1, there are \( n \) institutions with assets separated between noninterbank and interbank assets \( a_{i}^{\text{NIB}} \) and \( a_{i}^{\text{IB}} \) and interbank and noninterbank liabilities \( d_{i}^{\text{IB}} \) and \( d_{i}^{\text{NIB}} \). The residual equity \( e_{i} \) or the net worth of a bank is defined by a capital ratio \( \gamma_{i} \). In this way, the financial network can be described by banks, an exposure matrix, and a set of capital ratios for the institutions. For a given recovery ratio, a default cascade can be defined as the sequence of institutions that default following the initial insolvency of at least one institution.

The network is then modeled as a random graph (see Newman 2005) with statistical properties that correspond to the properties of real-world exposure networks as described in Section 2. With this construction, asymptotic results for critical quantities, such as the fraction of nodes defaulting as the network gets large as well as the resilience of the network to small shocks, can be characterized.

Let \( \mu(i, j) \) be the fraction of banks that have \( i \) obligors and \( j \) obligees within the network, and let \( p(i, j, 1) \) be the probability that a bank with \( i \) obligors and \( j \) obligees is vulnerable, i.e., dragged into default by the default of a single counterparty. Amini, Cont & Minca (2010) define network resilience by

\[
1 - \sum_{i,j} \frac{ij}{\lambda} \mu(i,j)p(i,j,1),
\]

where \( \lambda = \sum_{i,j} \mu(i,j) \). If network resilience is larger than zero and the fraction of initial defaults is sufficiently small, then the probability is high that only a small fraction of the banks will default. On the contrary, if network resilience is smaller than zero, then there exists with high probability a subset of banks such that the default of any bank in this subset triggers the default of all other banks in the subset. This is a resilience concept that allows one to quantify the contribution of each node in a large network to systemic risk in terms of its connectivity and its local characteristics. In particular, it allows the computation of a minimal capital ratio above which a network becomes resilient.

This generalizes previous results from Gai & Kapadia (2010). They assume that the interbank claims are evenly distributed across the counterparties. Amini, Cont & Minca’s (2010) paper allows one to relax this assumption and to refine these results. In line with Gai & Kapadia (2010), Amini, Cont & Minca (2010) find that (asymptotically) the probability of contagion is not monotone in the average degree. Gai & Kapadia (2010) found that for capital buffers exceeding 4%, the probability that at least 5% of the banks default due to contagion is highest for average degrees in the range of 3 to 6. For high average degrees, contagion becomes a very rare phenomenon but if it happens, all banks default.
6. NETWORK ANALYSIS AND SYSTEMIC RISK

After the discussion of the literature on financial networks and contagion, is there an emerging story on how the network literature discussed in this review relates to what is commonly termed systemic risk? This is hard to answer because the concept of systemic risk is an elusive term that is itself not precisely defined. I nevertheless try to connect more broadly the results of the network literature to the systemic risk discussion.

One aspect of systemic risk to which the network literature has contributed strongly is the intuition that the risk exposure of a banking system cannot be meaningfully analyzed at the level of single institutions in isolation. The way the institutions are financially connected is crucial for understanding the risk exposure of the system as a whole. In the system as a whole, shocks to some of its parts can develop into a wider breakdown of banks and financial institutions through cascades of insolvencies and domino effects (see Gai, Haldane & Kapadia 2011). Although this effect is important, the network literature on domino effects, in particular the various simulation studies, have shown that the domino view of systemic risk is perhaps too mechanical. Without a mechanism for how initial shocks gain momentum while spreading in the system, mechanical domino effects cannot explain the widespread damage experienced in historical financial crises.

The term systemic risk is also used to describe the economy-wide effects of a financial crisis that eventually hit the real economy. This has been a classical theme of macroeconomics since the Great Depression. The debate in this literature revolves around the question of whether recessions can be caused by financial instability, rather than financial instability being the result of a recession. The knock-on effects of financial instability on the real sector are the issues in this debate. The network literature has not yet contributed significantly to this side of the systemic risk story. Most of the papers seem to take the perspective that the analysis of financial contagion, insolvency cascades, risk spillovers, and volatility amplification in the financial system is interesting because it is known from the macroeconomic literature that the real economy impact of financial instability will be huge and costly. The precise interaction between financial exposure networks and the real economy is, however, not modeled in any detail.

One of the most powerful stories of financial instability and the interaction between finance and the real economy comes from papers that model systemic risk through the interaction of leverage, asset prices, and behavior. For instance, papers such as Brunnermeier & Pedersen (2009), Shin (2010), or Geanakoplos (2009) suggest the following narrative of a typical buildup and unraveling of financial distress: A crisis usually begins in good times. People become more optimistic and become convinced that fundamental structural changes in the economy will allow them to take on greater financial risk. This is usually the beginning of a leverage cycle, in which borrowers lower their lending standards and collateral requirements, allowing the most upbeat investors to conduct leveraged asset purchases and feeding an asset price boom. If risks are measured from historical data, perceived risk decreases, allowing for yet more leverage in the system. At the peak of a leverage cycle, minor decreases in asset values are enough to drive the most leveraged investors into default. The assets serving as collateral go to other investors, who value them less, reinforcing the decline in asset prices and potentially driving more investors into default. At this stage, borrowers step up their lending standards and liquidity evaporates, forcing fire sales of leveraged institutions and individuals and feeding the negative spiral even further.

Interestingly, this story can be consistently told without referring to any network concepts at all within a more or less standard general equilibrium framework. Intuitively, network concepts should play an important role in describing the microstructure of exposures between agents in the financial and in the real sector. The combination of the network literature with the economic analysis of boom and bust cycles and its impact on the real economy has not yet been made.
7. DOES NETWORK ANALYSIS CONNECT TO ECONOMIC POLICY?

Using network concepts to analyze financial networks and systemic risk is a fairly recent practice. Although the idea seems very compelling that financial stability should be closely related to the deeper understanding of this network’s structure, the use of network analysis in the design of financial stability-oriented policies has been extremely limited (see, e.g., Chan-Lau, Espinosa & Sole 2009, as well as IMF 2009, chapter 2, for examples that provide policy-oriented network concepts). Several reasons might account for this fact. First, the concept of a network is an entirely abstract concept. Networks can be found all over the economy and their interpretation can differ depending on context. The literature has so far concentrated mainly on networks of debt exposures. Although there are good reasons to take this perspective, it is not clear whether this is key to understanding financial fragility. Second, networks can be studied as a given constraint on individual and collective behavior or they can be thought of as the aggregate result of individual decisions (see for instance König, Tessone & Zenou 2012). Both perspectives are important for policy questions. The first perspective, which could be labeled the ex post perspective, is important for issues such as the identification of critical institutions or substructures in the network that are not immediately obvious. The latter perspective is relevant for questions of regulation, such as whether there are reasons to believe that individual decision-making by financial institutions leads to a collectively fragile structure. In this section, I discuss briefly two fields in which the analysis of exposure networks could make an important contribution to economic policy.

7.1. Mapping the Financial Network to Decrease Uncertainty

Although most of the models reviewed here were in place before the financial crisis, they did not predict the imminent breakdown of the world financial system, nor did they play a major role in shaping policy decisions during the financial crisis. Although the first problem is a deeper problem of models used across economics and finance not specific to network models, in the latter case it is obvious, with the benefit of hindsight, how network models could have played a very useful role in crisis management had they been in place.

The failure of Lehman Brothers in September 2008 was a key event in the financial crisis. Although no major institution failed due to its direct exposure to Lehmann, the uncertainty about these exposures and the indirect exposures of other institutions led to a credit freeze in the world financial system. If a more precise map of the exposure network were available at that time, this uncertainty and the gridlock it induced might have been substantially reduced. The traditional technique for collecting financial statistics is to collect a large amount of information about individual institutions within national jurisdictions, but the network map cannot be constructed easily from these data. Creating international standards and cooperation to allow for such exposure maps would be of considerable benefit potentially in resolving gridlocks due to uncertainty, such as in the fall of 2008.

7.2. Identifying Critical Institutions

The work on network topology and default cascades could be helpful in identifying critical institutions in a banking system. The work of Amini, Cont & Minca (2010), for instance, provides a measure of resilience that allows for a given network to predict the spread of distress. How this can be used as a supervisory tool, for instance, is developed in Amini, Cont & Minca (2012). Billo et al. (2012) show—using a combination of network concepts and econometric techniques—that the banking sector plays a larger role in transmitting shocks than other financial institutions such as hedge funds, broker-dealers, or insurance companies.
Demange (2011) uses the framework of Eisenberg & Noe (2001) to construct a measure of systemic threat imposed by a single institution in the system. The so-called threat index measures the decrease in payment within the banking system following a reduction in net worth at one institution. The threat index is shown to differ substantially from the default level of an institution in the system. This has important implications for creating liquidity support policies. For instance, it generally might not be optimal to target the weakest institutions with such a policy but rather the institutions with the highest threat index. Thus, an optimal liquidity injection policy that targets the weakest institutions would in general not be optimal.

The concept of a threat index is related to the Bonacich-Katz index of network centrality (Bonacich 1987, Katz 1953) and to a class of network models in which individual actions have complementary effects on other players (see Ballester, Calvó-Armengol & Zenou 2006). These connections have yet to be systematically investigated, but they are highly relevant to designing optimal liquidity as well as recapitalization policies.

8. DOES NETWORK THEORY CONNECT TO STANDARD MODELS OF MARKETS?

In the literature on financial stability and financial crisis, the idea that there are externalities by which one institution in the system by its actions influences other institutions is key. The exact mechanism by which these externalities occur is, however, less clear.

The network literature emphasizes the problem of cascading defaults. Although there has been progress in understanding the structure and the risk of default cascades more deeply, many simulation studies have concluded that the risk from domino effects has to be considered small, as I have discussed at length in Section 4.1.

The economic literature on the recent financial crisis interestingly discusses very powerful mechanisms by which the portfolio decisions of market participants and the pricing of risks interact to create strong boom and bust cycles in asset prices even without any network structure (Shin 2010) and even with risk-free debt (Geanakoplos 2009). These mechanisms are not only of theoretical interest. Adrian & Shin (2010) provide empirical evidence that bears on the pricing of risks on the balance sheets of intermediaries. Interestingly, the essential economic reading list on the financial crisis compiled by Gorton & Metrick (2012) does not contain one single network paper.

A key insight from the analysis of the recent crisis as discussed, for instance, in Hellwig (2008), Brunnermeier & Pedersen (2009), and Shin (2010) comes from the interaction of leverage of households and financial institutions and capital markets. It has so far turned out difficult to take appropriately into account the link between exposure networks and capital markets. Although some network papers such as in Alessandri et al. (2009) made attempts to include fairly mechanical fire sale mechanisms, a systematic investigation of how networks interact with our more traditional models of markets is still only beginning (see, for instance, Thurner, Farmer & Geanakoplos 2012). It is, however, at this link where behavior and amplification mechanisms kick in. A mechanic domino effect analysis as applied in most simulation studies can show how an exogenous shock will be distributed in the system. But in a pure domino story, there is no mechanism that allows contagion to gain momentum as it spreads in the system. For such amplifiers to kick in, there has to be some form of interaction between prices and behavior.

While in the broader literature on networks (Jackson 2008) models of behavior are combined with models of networks, it is not obvious how to combine network models with our traditional models of markets. In the standard model of markets, we have individual behavior and constraints. Prices are parameters that adjust such that the decisions of individuals become aligned with
aggregate resource constraints in equilibrium. Market exchange in this model is anonymous, and bilateral exposures and their topology play no role at all. If one thinks of the standard general equilibrium model as a network model at all, it should be considered a model with a central node through which all trade takes place once equilibrium prices have been found through some unspecified process of price adjustment.

If models of markets and market prices are combined with models of exposure networks, there is also a fundamental question of how precisely these perspectives should be combined. Should an exposure network be thought of as a physical constraint, where the topology of the network restricts behavior, opportunities, and equilibrium, or is the structure of the exposure network something that has to be explained from more fundamental principles? No matter which of these perspectives is regarded as more important, combining networks and markets not only raises many difficult conceptual issues, but also takes a big step away from a purely mechanical contagion analysis based on pure balance-sheet mechanics without taking into account behavior and pricing. In my view, the weak connection between models of financial networks and models of market exchange and pricing is the reason why network models so far have played only a minor role in the economic analysis of the recent financial crisis.

9. CONCLUSION

Models of interbank exposure networks provide interesting and useful tools to map exposure data in a way that allows for the detection of the system’s vulnerable areas as well as the assessment of the likelihood of potential default cascades. However, most of the models developed in the literature so far are confined to a pure analysis of domino effects through the mechanics of balance-sheet interlinkages and abstract away from behavior and asset pricing effects. Most simulation studies find that this mechanical approach seems to lead to the conclusion that contagion risk is small. What is perhaps misleading in this assessment is the neglect of some mechanisms that seem to be very powerful during financial crises and which go beyond the pure mechanics of balance-sheet linkages: amplification of losses by behavior and asset pricing. The papers surveyed in this literature seem to cover a fair amount of ground in analyzing and explaining when a default cascade is likely to spread. The amplification mechanisms at work during a financial crisis, however, are likely to be missed by this approach. Once a shock has hit the system, the balance-sheet-mechanics approach can explain only how the given losses are distributed among the agents but not how the losses accumulate and get magnified through the process of cascading.

Although some of the shortcomings of the literature on financial networks and contagion have become very clear with the benefit of hindsight, the literature has also opened some potentially very fruitful research areas and has also contributed some important insights to our knowledge of financial crises. Perhaps one of the most important contributions of the literature so far is tools to map supervisory and regulatory data in a more systemic way, revealing how institutions depend on each other through exposures in a system. This is a big step forward from single-institution-based reporting of financial data to a more systemic or macroprudential perspective. Another important contribution of the network papers, in particular of the simulation studies, to the literature was that they established the importance of amplifier mechanisms in a financial crisis. This helps to refocus the research agenda on systemic risk. Before the network literature was available, the academic discussion gave perhaps too much weight to domino effect stories of default cascades and too little weight to the amplifiers. The network literature has shown the need to shift this weight.

Finally, network models provide concepts and perspectives that are very compelling descriptions of the structure of real-world financial systems. If they can be combined usefully with more
traditional economic concepts of markets, individual behavior, and pricing, they can potentially bridge the gap between qualitative conceptual ideas in the economic literature about how crisis amplification mechanisms work with actual quantitative models of financial fragility.

**DISCLOSURE STATEMENT**

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